A Handbook of
Reinforced Concrete Structures

Part - II

- Theory analysis
- Basic Concepts
- Short Techniques
- Problem Solutions
- Question Bank Solution

Concept
Publication

Md. Jaber Hossain
BSc.(Civil),KUET

A Handbook of
Reinforced Concrete Structures
(Part-II)

Theory Analysis
Basic Concepts
Problem Solutions
Question Solution (2013-2019)

1st Edition: August 2021

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Publication

A Handbook of
Reinforced Concrete Structures
(Part-II)

[Especially for the students of Civil Engineering, BECM and URP department]

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Most Deeply Dedicated to
My Beloved Parents

[The sacrifices and contributions of every parents
for their child are immense and undeniable]

"O Allah! Bestow on my parents The Mercy even as they cherished me in childhood"
[The Holy Quran; 17:24]

Preface

First of all, the book is written for those students who love to study neatly. In fact, in the beginning there was no intention to write a book like this approach. The author studied on Reinforced Concrete Structures during third year of undergraduate in civil engineering. However, there were many books on this subject but there were no well-arranged books which was easily fathomed. So the author personally combines different textbooks as well as notes the theories and problem solutions under the chapters according to the class lecture. That is why the book is written during fourth year of undergraduate of the author to save the valuable notes from being lost inexpertly as well as for the students who are thirst for knowledge. The book is arranged stepwise with chapter-based theory analysis, basic concepts, problem solving techniques and examination useful questions. It is better to say that the problems on the both design and analysis of concrete structures are solved in the both practicing method of WSD and USD. The author has tried to make it students friendly by providing neat sketches and illustrations with practical problems, wherever necessary. Hopefully the book can be useful to those-

- who wants to pass in the examination in a short time.
- who wants to get a letter mark easily.
- for all civil engineering students.
- for all civil engineering job seeker.

The author is very optimist that this book will be received with open arms by the students and educators. As well as this book will motivate the learners to be more devoted to the subject.

In this work as a human, it is possible that errors may occur. Any comments and suggestions about the book are always welcome. There will be a request to the users, if there is any mistakes and omissions, it will be thankfully acknowledged. The author will be glad to consider them for inclusion in the next edition of the book. Insha-Allah.

Just write to the author via mail.

Khulna, Bangladesh
13 August, 2020

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Acknowledgement

At the beginning, I would like to thank and my immense gratefulness to The Almighty ALLAH for his kindness and grace to make me capable to compose the handbook. The handbook is the compilation and composed of valuable class lectures of a semester long delivered by the honorable teachers Prof. Dr. Abu Zakir Morshed and Prof. Dr. Muhammad Harunur Rashid of KUET, engineering university. As well as combining the different textbooks and semester hand notes, the book has been transformed into a complete easy and smooth handbook.

Finally I would like to say that the gratitude of my parents behind my coming so far is undeniable. As well as I sincerely thank my friends of the civil engineering department whose help me much in every steps of my study journey.

Especially I am so grateful to my close younger brother SM Tanvir Ahmed for his outstanding cover design. With that I also would like to give endless thanks to my elder brother Jamir Uddin Mamun as well as my close friends Rakib Hasan, Ahnaf Hasan and Ashif Adnan Khan Aoyon for giving me advice and encourage to complete the work.

In addition, the author is very grateful to the writers of all the textbooks cited in reference section for the acquiring basic knowledge, these helped the author to make an easy handbook.

Khulna, Bangladesh
13 August, 2020

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Chapter - 1
Design of Columns

1.1 Theory Analysis

Q-1. What are the types of columns? Briefly describe them.

Ans: Types of columns: Columns may be classified based on the following different categories:

1. Based on shape of the cross-section, columns may be classified as follows:
(i) Square column, (ii) Rectangular column, (iii) Triangular column, (iv) Circular column.

2. Based on column ties, columns may be classified as follows:
(i) Tied column: Tied columns containing steel ties to confine the main longitudinal bars in the columns.
(ii) Spiral column: Spiral columns containing spiral (spring-type reinforcement) to hold the main longitudinal bars and to help increase the column ductility before failure.

3. Based on slenderness ratio, columns may be classified as follows:
(i) Short column: A column is considered to be short if the ratio of effective length to its least lateral dimension is less than or equal to 12.
(ii) Long column: A column is considered to be long if the ratio of effective length of column to its least lateral dimension is greater than 12.

4. Based on loading, columns may be classified as follows:
(i) Axially loaded column: It is the column, where loads are assumed acting at the center of the column section.
(ii) Eccentrically loaded column: It is the column, where loads are acting at a distance e from the center of the column section.
(iii) Biaxially loaded column: It is the column, where the load is applied at any point on the column section, causing moments about both the x and y-axes simultaneously.

Q-2. What are the functions of tie bars or spirals in a reinforced concrete column?

Ans: Functions of tie bars or spirals in a reinforced concrete column are described below:-
(i) To hold the longitudinal bars in position while the concrete is being placed.
(ii) To prevent the slender longitudinal bars from buckling outward.
(iii) To take the horizontal shear during wind/seismic forces.

Q-3. What are the differences between short and long column?

Ans: The differences between short and long column are described below:

Short column	Long column
(i) A column is considered to be short if the ratio of effective length to its least lateral dimension is less than or equal to 12.	(i) A column is considered to be short if the ratio of effective length to its least lateral dimension is less than or equal to 12.
(ii) The ratio of effective length of a short column to its least radius of gyration is less than or equal to 40.	(ii) The ratio of effective length of a long column to its least radius of gyration is greater than 40.
(iii) Buckling tendency is very low.	(iii) Long and slender columns buckle easily.
(iv) The load carrying capacity is high as compared to long column of the same cross sectional area.	(iv) The load carrying capacity is less as compared to short column of the same cross sectional area.
(v) The failure of the short column is by crushing.	(v) The column generally fails in buckling.

Q-4. Why the strength reduction factor for a column is more conservative than those for a beam?

Ans: The strength reduction factor for a column is much lower than a beam. The causes are described below:

(i) The concrete in columns is subjected to more segregation than in the case of beams.

(ii) Columns are cast vertically in long, narrow forms in where beams are cast in shallow, horizontal forms and so casting uncertainty is more in columns.

(iii) The failure of a column in a structure is more critical than that of a floor beam.

(iv) The failure strength of a column is closely related to the concrete's ultimate strength, a value that is quite variable.

So the strength reduction factor for a column ($\varphi = 0.65$) is more conservative than those for a beam ($\varphi = 0.90$).

Q-5. Why spiral column can support more loads than tied column?

Ans: Concrete columns reinforced with spiral reinforcement can withstand more loads than tied column. This phenomenon happens because when load eccentricities are small, spirally reinforced columns shows greater toughness, greater ductility than the columns with ties.

Q-6. What is an interaction diagram? Describe interaction diagram for an eccentrically loaded column. [*CE-2017, 2015*]

Ans: ***Interaction diagram:*** Interaction diagram in a column is a graph which shows a plot for the axial load, P_n that a column could carry versus its moment capacity, M_n.

The interaction diagram for an eccentrically loaded column is described below:

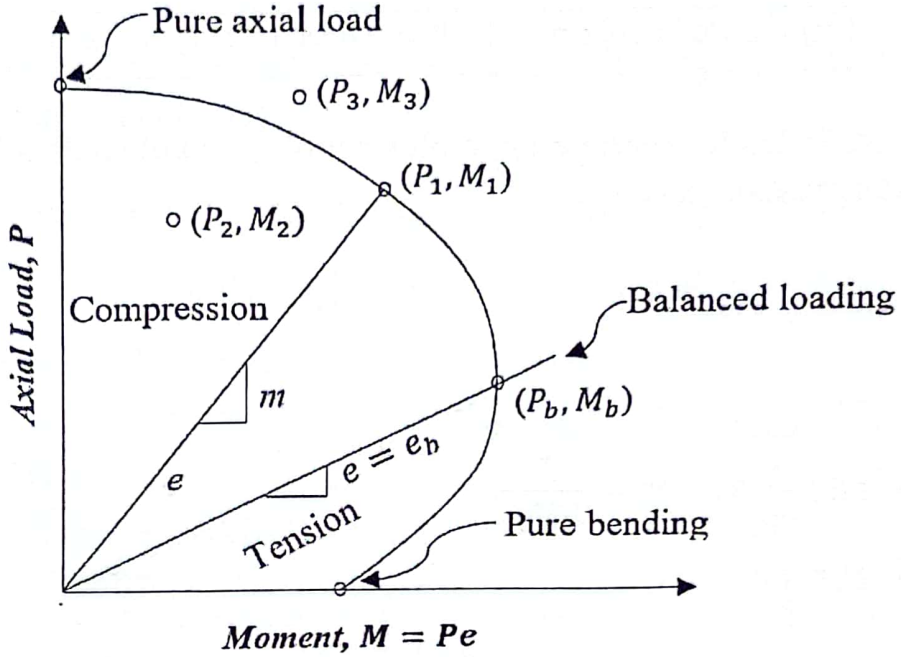


Figure 1.1: Interaction diagram for a column

(i) The X-axis represents the bending moment and Y-axis represents the axial load.

(ii) The ordinate on Y-axis means the eccentricity is equal to zero and also represents the truly concentric column.

(iii) The ordinate on X-axis means the eccentricity is infinity and also represents the pure bending column.

(iv) The ordinate (P_1, M_1) is acceptable combination of load & moment and optimized design. The ordinate (P_2, M_2) is acceptable but overdesign. The ordinate (P_3, M_3) is unacceptable and under design. And (P_b, M_b) is the balanced loading.

(v) The slope of the radial line from origin is m and the reciprocal of the slope is eccentricity e.

(vi) From balanced eccentricity (e_b), towards clockwise represents tension zone and towards counter-clockwise represents compression zone.

1.2. Basic Concepts

1. For axially loaded (concentric) column:

Axial force is given by-

i) For spiral column	$P = A_g(0.25f_c' + f_s\rho_g)$
ii) For tied column	$P = 0.85A_g(0.25f_c' + f_s\rho_g)$

2. For eccentrically loaded (compression plus bending or uniaxial) column (*WSD*):

(a) Concrete compression governs:

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1$$

Here,

$$f_a = \frac{P}{A_g}$$

$$F_a = 0.34f_c'(1 + m\rho_g) \quad and \quad m = \frac{f_y}{0.85f_c'}$$

$$f_b = \frac{M}{S_{ut}} \quad and \quad M = Pe$$

$$F_b = 0.45f_c'$$

Rectangular, $S_{ut} = \frac{1}{c}\left[\frac{bt^3}{12} + (2n-1)A_s\left(\frac{t}{2} - d'\right)^2\right]$

Circular, $S_{ut} = \frac{1}{c}\left[\frac{\pi d^4}{64} + (2n-1)\frac{A_s}{8}(g_t)^2\right]$

(b) Steel tension governs:

$$\frac{P}{P_b} = \frac{M - M_o}{M_b - M_o}$$

Here,

For symmetrically tied columns,

$M_o = 0.4A_{st}f_y(d - d')$

$and \quad e_b = (0.67\rho_g m + 0.17)d$

For unsymmetrically tied columns,

$M_o = 0.4A_{st}f_y jd$

$$and \quad e_b = \frac{P'm(d - d') + 0.1d}{(P' - P)m + 0.6}$$

For spiral columns,
$M_o = 0.12 A_s f_y D_s$
and $e_b = 0.43 \rho_g m D_s + 0.14t$

Here, A_s= Total steel; A_{st}= Steel on tension side; A_g= Gross concrete area

3. Small eccentrically (near axially) loaded column – Strength Design Method (*USD*): [For both tied & spiral column]

Nominal axial load, $P_n = 0.85 f_c'(A_g - A_{st}) + f_y A_{st}$

$$\therefore P_n = A_g[0.85 f_c'(1 - \rho_g) + f_y \rho_g]$$

Now, Ultimate axial load, $P_u = \alpha \varphi P_n$

Factor \ Column		*Spiral*	*Tied*
Inherent (accidental) eccentricity	α	0.85	0.80
Strength reduction factor	φ	0.75	0.65

4. Design of Tie:
Spacing of ties is the least of –
i) 16 times diameter of longitudinal bar = $16 d_{bL}$
ii) 48 times diameter of tie bar = $48 d_{bT}$
iii) Least column dimension

Specification of ties:

Longitudinal bar size	*Tie bar size*
Up to #10	#3
Above #10	#4
	Max. #5

5. Design of Spiral:
Core diameter, $D_c = D - 2 * cc$

Area of core, $A_c = \frac{\pi {D_c}^2}{4}$

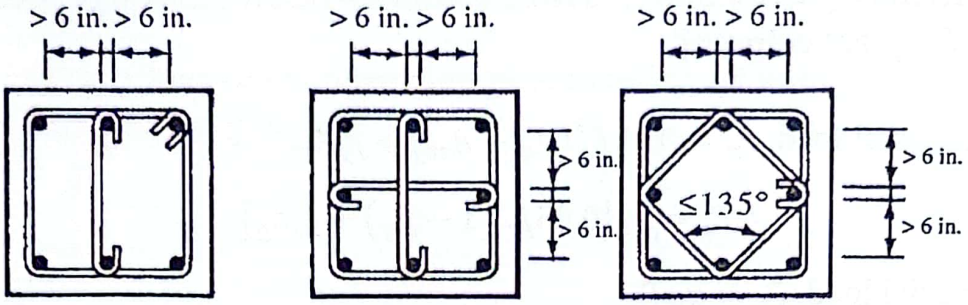
Minimum spiral ratio, $\rho_{s(min)} = 0.45 \left(\frac{A_g}{A_c} - 1\right) \frac{f_c'}{f_y}$

Spiral ratio, $\rho_s = \frac{4a_s}{gD_c}$

Pitch of spiral, $g = \frac{1}{6}D_c$; $but\ 1'' \leq g \leq 6''$

6. Extra ties requirement:

When clear space between longitudinal bars exceeding 6 in. then extra ties are required.

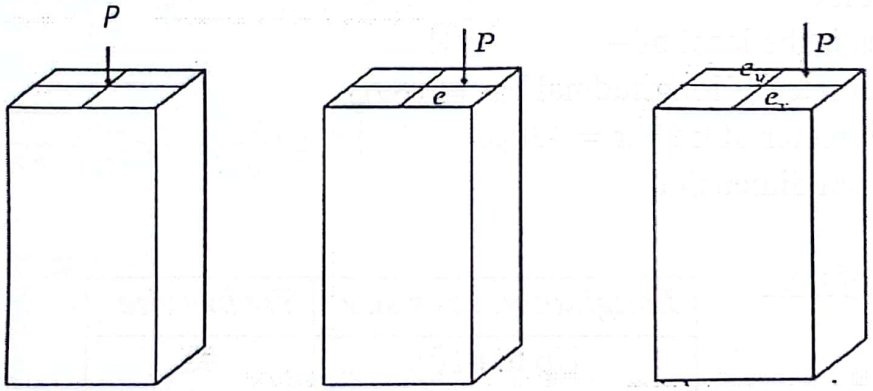


7. Classification of columns based on type of loading and bending:

I. Column with axial loading: Loading is applied concentrically and no bending.

II. Column with uniaxial eccentric loading: Loading is applied eccentrically (e) and causes uniaxial bending. In Uniaxial Bending, column will tends to bend in minor axis.

III. Column with biaxial eccentric loading: Loading is applied eccentrically (e_x & e_y) and causes biaxial bending. In Biaxial Bending, column will tends to bend in both major and minor axis.



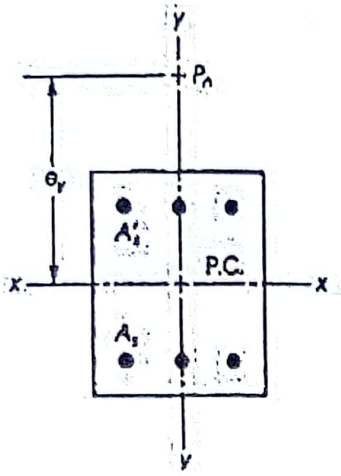
I. Axial compression

II. Axial compression with **uniaxial bending**

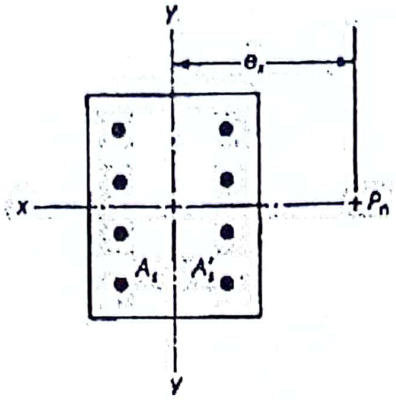
III. Axial compression with **biaxial bending**

8. Biaxial bending column:

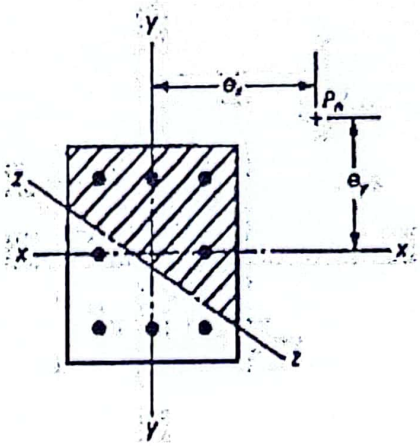
8.1. Column bending and eccentricity:



(a) Uniaxial bending with load P_n along the y-axis with eccentricity e_y.



(b) Uniaxial bending with load P_n along the x-axis with eccentricity e_x.



(c) Biaxial bending.

8.2. Simple approximate methods to analyze biaxial bending:

(i) Load Contour Method:

$$\left(\frac{M_{nx}}{M_{nx_o}}\right)^{\alpha_1} + \left(\frac{M_{ny}}{M_{ny_o}}\right)^{\alpha_2}$$

Here, $M_{n_x} = P_n e_y$ ----- (c)

$M_{nx_o} = M_{n_x}$ when $M_{n_y} = 0$ ----- (b)

$M_{n_y} = P_n e_x$ ----- (c)

$M_{ny_o} = M_{n_y}$ when $M_{n_x} = 0$ ----- (a)

$\alpha_1 = \alpha_1 = \alpha =$ (1.15~1.55) values near the lower end range are more conservative.

So normally $\alpha = 1.15$ is considered.

(ii) Reciprocal Load Method:

$$\frac{1}{P_n} = \frac{1}{P_{nx_o}} + \frac{1}{P_{ny_o}} - \frac{1}{P_o}$$

Here, P_n = Normal load in biaxial bending

P_{nx_o} = Normal load when only e_y is present

P_{ny_o} = Normal load when only e_x is present

P_o = Normal load in concentrically column.

1.3 Solution Techniques

1.3.1 Axial Compression (Concentric)

1. Reinforcement ratio
Assume, $\rho_g = 2\% = 0.02$ (1%~8%)

2. Column dimension
For spiral column, $P = A_g(0.25f_c' + f_s\rho_g)$
or,
For tied column, $P = 0.85A_g(0.25f_c' + f_s\rho_g)$

For circular column, $A_g = \frac{\pi D^2}{4}$; then D is calculated
or,
For rectangular column, $A_g = b^2$ or $A_g = b * h$; then b or h is calculated

3. Steel calculation
Load contribution of concrete, $P_c = 0.25A_gf_c'$
Load to carried by steel, $P_s = P - P_c$
$\Rightarrow f_sA_g\rho_g = P - P_c$
$\Rightarrow f_sA_g * \frac{A_s}{A_g} = P - P_c$
$\therefore A_s = \frac{P - P_c}{f_s}$

4. Checking Reinforcemenr ratio, $\rho_g = \frac{A_s}{A_g}$ [Between 0.01 & 0.08; ok]

5. Spiral design
Core diameter, $D_c = D - 2 * cc$
Area of core, $A_c = \frac{\pi {D_c}^2}{4}$
Minimum spiral ratio, $\rho_{s(min)} = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_y}$
Pitch of spiral, $g = \frac{1}{6}D_c$ $[1'' \le g \le 6'']$ (ok)
6. Reinforcement detailing

1.3.2 Uniaxia

I. Method: *W*

1. Reinforcem
Assume, ρ_g =
2. Column dim
For spiral colu
or,
For tied colun

For circular c
or,
For rectangul
3. Steel calcul
$\rho_g = \frac{A_s}{A_g}$ $\therefore$ A

4. Check for u
For uniaxial e
$\therefore \frac{f_a}{F_a} + \frac{f_b}{F_b} =$

Now, F_a = 0
$M = P * e$
$F_b = 0.45f_c'$

i) For rectan
or,
ii) For circul

Trial No.	(i

Provide nec

5. Design of
a) Design o
Spacing of t
i) 16 times

1.3.2 Uniaxial Bending (Compression plus Bending)

I. Method: *WSD*

1. Reinforcement ratio

Assume, $\rho_g = 2\% = 0.02$ (1%~8%)

2. Column dimension

For spiral column, $P = A_g(0.25f_c' + f_s\rho_g)$

or,

For tied column, $P = 0.85A_g(0.25f_c' + f_s\rho_g)$

For circular column, $A_g = \frac{\pi D^2}{4}$; then D is calculated

or,

For rectangular column, $A_g = b^2$ or $A_g = b * h$; then b or h is calculated

3. Steel calculation (A_s)

$$\rho_g = \frac{A_s}{A_g} \therefore A_s = \rho_g A_g$$

4. Check for uniaxial column

For uniaxial eccentric loading, $\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1$

$$\therefore \frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{P}{A_g * F_a} + \frac{M}{S_{ut} * F_b}$$

Now, $F_a = 0.34f_c'(1 + m\rho_g)$; Here, $m = \frac{f_y}{0.85f_c'}$

$M = P * e$

$F_b = 0.45f_c'$

i) For rectangular column, $S_{ut} = \frac{1}{c}\left[\frac{bt^3}{12} + (2n-1)A_s\left(\frac{t}{2} - d'\right)^2\right]$

or,

ii) For circular column, $S_{ut} = \frac{1}{c}\left[\frac{\pi D^4}{64} + (2n-1)\frac{A_s}{8}(g_t)^2\right]$

Trial No.	D (in.)	A_g (in^2)	Steel No.	A_s (in^2)	$\rho_g = \frac{A_s}{A_g}$	S_{ut} (in^3)	$\frac{f_a}{F_a} + \frac{f_b}{F_b}$	Remarks

Provide necessary reinforcement as longitudinal bars.

5. Design of tie or spiral

a) Design of tie

Spacing of tie is the least of –

i) 16 times diameter of longitudinal bar $= 16d_{bL}$

ii) 48 times diameter of tie bar (bar no.) $= 16d_{bT}$
iii) Least column dimension $= b$

Check for extra tie:

$\therefore$ Clear distance between longitudinal bars, S

$$= \frac{b - 2 * cc - 2 * d_{bT} - \text{Bar no. of each side} * d_{bL}}{\text{Bar no. of each side} - 1}$$

$\therefore 1.5d_b < S < 6''$ (ok)

So, no extra tie is required.
or,
b) Design of spiral
$\therefore$ Clear distance between longitudinal bars

$$= \frac{\pi D_c - A_s}{Bar\ No.} > 1.5d_b$$

Core diameter, $D_c = D - 2 * cc$

Area of core, $A_c = \frac{\pi {D_c}^2}{4}$

Minimum spiral ratio, $\rho_{s(min)} = 0.45 \left(\frac{A_g}{A_c} - 1\right) \frac{f_c'}{f_y}$

Spiral ratio, $\rho_s = \frac{4a_s}{gD_c} > \rho_{s(min)}$ (ok)

Pitch of spiral, $g = \frac{1}{6} D_c$; $but\ 1'' \leq g \leq 6''$

6. Reinforcement detailing

II. Method: *Stren*

1. Load calculation,

2. Reinforcement r
Assume, $\rho_g = 3\%$

3. Column dimensi
Ultimate axial load

For circular colum

or,
For rectangular co

4. Steel calculation
Load contribution
Load to carried by
$\Rightarrow \alpha\varphi A_g f_y \rho_g = P_u$
$\Rightarrow \alpha\varphi A_g f_y A_g * \frac{A_s}{A_g}$
$\therefore A_s = \frac{P - P_c}{\alpha\varphi f_y}$

5. Checking Reinf

6. Design of tie
Spacing of tie is t
i) 16 times diame
ii) 48 times diam
iii) Least column

Check for extra t

$\therefore$ Clear distance

$= \frac{b - 2 * cc - 2}{B}$

$\therefore 1.5d_b < S <$
So, no extra tie

7. Reinforceme

II. Method: *Strength Design Method or USD*

1. Load calculation, $P_u = 1.2\,DL + 1.6\,LL$

2. Reinforcement ratio
Assume, $\rho_g = 3\% = 0.03$ (1%~8%)

3. Column dimension
Ultimate axial load, $P_u = \alpha\varphi A_g\left[0.85f_c'(1-\rho_g) + f_y\rho_g\right]$
For circular column, $A_g = \frac{\pi D^2}{4}$; then D is calculated
or,
For rectangular column, $A_g = b^2$ or $A_g = b * h$; then b or h is calculated
4. Steel calculation
Load contribution of concrete, $P_{uc} = \alpha\varphi A_g\left[0.85f_c'(1-\rho_g)\right]$
Load to carried by steel, $P_{us} = P_u - P_{uc}$
$\Rightarrow \alpha\varphi A_g f_y \rho_g = P_u - P_{uc}$
$\Rightarrow \alpha\varphi A_g f_y A_g * \frac{A_s}{A_g} = P_u - P_{uc}$
$\therefore A_s = \frac{P - P_c}{\alpha\varphi f_y}$

5. Checking Reinforcemenr ratio, $\rho_g = \frac{A_s}{A_g}$ [Between 0.01 & 0.08; ok]

6. Design of tie
Spacing of tie is the least of –
i) 16 times diameter of longitudinal bar $= 16d_{bL}$
ii) 48 times diameter of tie bar (bar no.) $= 16d_{bT}$
iii) Least column dimension $= b$

Check for extra tie:

$\therefore$ Clear distance between longitudinal bars, S

$$= \frac{b - 2 * cc - 2 * d_{bT} - \text{Bar no. of each side} * d_{bL}}{\text{Bar no. of each side} - 1}$$

$\therefore 1.5d_b < S < 6''$ (ok)
So, no extra tie is required.

7. Reinforcement detailing

1.3.3 Biaxial Bending

1. Load calculation,

Factored load, $P_u = 1.2\ DL + 1.6\ LL$

Nominal load, $P_n = \frac{P_u}{\varphi}$

2. Moment calculation,

$M_{ux} = 1.2\ M_{DLX} + 1.6\ M_{LLX}$

$M_{uy} = 1.2\ M_{DLY} + 1.6\ M_{LLY}$

$\therefore M_{nx} = \frac{M_{ux}}{\varphi}$ and $\therefore M_{ny} = \frac{M_{ux}}{\varphi}$

$\because M_n = M_{nx} + M_{ny}$

$\therefore$ Eccentricity, $e = \frac{M_n}{P_n}$

3. Reinforcement ratio

Here, $A_g = b * t$

$\therefore \frac{e}{h}$ and $\therefore \frac{P_u}{\varphi f_c' A_g}$

Now, $\gamma H = b - cc$

$\because \gamma = \frac{\gamma H}{H}$

Select graph A and from graph, $\rho_g =$

4. Steel calculation

$\therefore A_{st} = \rho_g * A_g$

5. Design of tie

Spacing of tie is the least of –

i) 16 times diameter of longitudinal bar $= 16 d_{bL}$

ii) 48 times diameter of tie bar (bar no.) $= 16 d_{bT}$

iii) Least column dimension $= b$

6. Reinforcement detailing

7. Checking adequacy of the trial design

(a) By the reciprocal load method:

For bending about y – axis, H and γH $\because \gamma = \frac{\gamma H}{H}$

Select graph A

$e = \frac{M_{nx}}{P_n} =$ and $\frac{e}{h} =$

Reinforcement ratio, $\rho = \frac{A_{st}}{bh}$

From graph A, $\frac{P_{ny_o}}{f_c' A_g} =$ and $\because P_{ny_o} =$

For bending about x – axis, H and γH $\because \gamma = \frac{\gamma H}{H}$

Select graph A

$e = \frac{M_{ny}}{P_n} =$ and $\frac{e}{h} =$

Reinforcement ratio, $\rho = \frac{A_{st}}{bh}$

From graph A, $\frac{P_{nx_o}}{f_c' A_g} =$ and $\because P_{nx_o} =$

Axial load, $P_o = 0.85 f_c'(A_g - A_{st}) + f_y A_{st}$

Now from Bresler's Reciprocal Load equation,

$$\frac{1}{P_n} = \frac{1}{P_{nx_o}} + \frac{1}{P_{ny_o}} - \frac{1}{P_o}$$

$\therefore P_n > (P_n)$ (ok)

So, the trial design is adequate.

(b) By the load contour method:

For bending about y – axis, H and γH $\because \gamma = \frac{\gamma H}{H}$

Select graph A

Reinforcement ratio, $\rho =$

From graph A, $\frac{M_{ny_o}}{f_c' A_g h} =$ and $\because M_{ny_o} =$

For bending about x – axis, H and γH $\because \gamma = \frac{\gamma H}{H}$

Select graph A

Reinforcement ratio, $\rho =$

From graph A, $\frac{M_{nx_o}}{f_c' A_g h} =$ and $\because M_{nx_o} =$

Now from Load Contour Method equation,

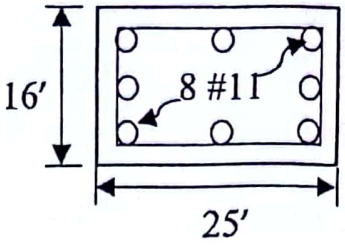
$$\left(\frac{M_{n_x}}{M_{nx_o}}\right)^{\alpha_1} + \left(\frac{M_{n_y}}{M_{ny_o}}\right)^{\alpha_2} \leq 1 \ (ok)$$

So, the trial design is adequate.

1.4 Problem Solution

1.4.1 Axial Compression (Concentric)

Pb-1. A column with the cross-section shown in figure below is loaded concentrically. Calculate the allowable axial load.
Use $f_c' = 4\ ksi\ and\ f_y = 50\ ksi.$



Sol^n:
Here,
$A_g = 16 \times 25 = 400\ in^2$
$A_s = 8 \times 1.56 = 12.5\ in^2$
$\rho_g = \frac{A_s}{A_g} = \frac{12.5}{400} = 0.0313$

For tied column, the allowable axial load is given by
$P = 0.85A_g(0.25f_c' + f_s\rho_g)$
$\Rightarrow P = 0.85 \times 400\ (0.25 \times 4 + 0.4 * 50 \times 0.0313)$
$\therefore P = 553\ kips\ \ (\boldsymbol{Ans})$

Pb-2. Design a circular, spiral-reinforced column for a concentric allowable working load of 480 kips with f_c'= 4 ksi and f_y= 60 ksi for both longitudinal steel and spiral. Also design the necessary spirals. [*CE-2016(similar)*]

Sol^n:
Assume, $\rho_g = 0.02$; [1% ~ 8%]
And, $f_s = 0.4f_y = 24\ ksi$
For spiral column, the allowable axial load is given by
$P = A_g(0.25f_c' + f_s\rho_g)$
$\Rightarrow 480 = A_g(0.25 \times 4 + 24 \times 0.02)$
$\Rightarrow A_g = 324\ in^2$
$\Rightarrow \frac{\pi D^2}{4} = 324\ in^2$
$\therefore D = 20.31\ in.$

Taking, $D = 20\ in$, So, $A_g = \frac{\pi D^2}{4} = 314\ in^2$
The contribution of concrete, $P_c = 0.25A_gf_c' = 0.25 \times 314 \times 4 = 314\ kips$
So, load to carried by steel,
$P_s = 480 - 314 = 166\ kips$
$\Rightarrow f_sA_g\rho_g = 166\ kips$
$\Rightarrow f_sA_g * \frac{A_s}{A_g} = 166\ kips$

$\therefore A_s = \frac{166}{24} = 6.92\ in^2$

Provide 9 – 25 mm φ longitudinal bars.

Check:

$\rho_g = \frac{9 * 0.79}{314} = 0.023$ [Between 0.01 & 0.08; ok]

Spiral design:

Core diameter, $D_c = D - 2 * cc = 20 - 2 * 1.5 = 17\ in.$

Area of core, $A_c = \frac{\pi {D_c}^2}{4} = \frac{\pi * 17^2}{4} = 227\ in^2$

Minimum spiral ratio, $\rho_{s(min)} = 0.45\left(\frac{314}{227} - 1\right)\frac{4}{60} = 0.0115$

Pitch of spiral, $g = \frac{1}{6}D_c = \frac{1}{6} \times 17 = 2.83 \approx 2.75\ in.;$ $so,\ 1'' \le g \le 6''$ (ok)

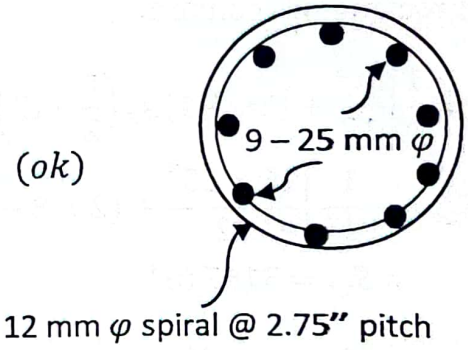
Taking 10 mm φ (#3) bar as spiral,

Spiral ratio, $\rho_s = \frac{4a_s}{gD_c} = \frac{4*0.11}{2.75\times17} = 0.0094 < \rho_{s(min)}$

Taking 12 mm φ (#4) bar as spiral,

Spiral ratio, $\rho_s = \frac{4a_s}{gD_c} = \frac{4*0.20}{2.75\times17} = 0.0171 > \rho_{s(min)}$ (ok)

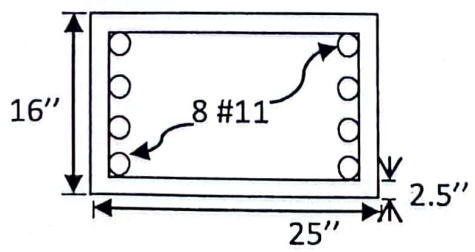
Provide 12 mm φ spiral @ 2.75 in. pitch.



1.4.2 Uniaxial Bending (Compression plus Bending)

I. Method: *WSD*

Pb-3. If the following column carries an axial load of 360 kips, what is the allowable simultaneous moment? Use $f_c' = 4\ ksi\ and\ f_y = 50\ ksi.$



Sol^n:

For symmetrically tied columns,

$$e_b = (0.67\rho_g m + 0.17)d$$

$$\Rightarrow e_b = (0.67 * 0.0312 * 14.7 + 0.17)(25 - 2.5)$$

$$\therefore e_b = 10.74''$$

$$\rho_g = \frac{A_s}{A_g} = \frac{8 * 1.56}{25 \times 16} = 0.0312$$

$$m = \frac{f_y}{0.85 f_c'} = \frac{50}{0.85 \times 4} = 14.7$$

Now, $\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$

For balanced loading, $\frac{P_b}{A_g * F_a} + \frac{M_b}{S_{ut} * F_b} = 1$

Here,

$$A_g = 16 \times 25 = 400\ in^2$$

$$F_a = 0.34 f_c'(1 + m\rho_g) = 0.34 \times 4\ (1 + 14.7 \times 0.0312) = 1.98\ ksi$$

$$M_b = P_b * e_b = 10.74\ P_b$$

$$F_b = 0.45 f_c' = 0.45 * 4 = 1.8\ ksi$$

For rectangular column,

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{f_c'}} = 8$$

$$S_{ut} = \frac{1}{c}\left[\frac{bt^3}{12} + (2n-1)A_s\left(\frac{t}{2} - d'\right)^2\right]$$

$$\Rightarrow S_{ut} = \frac{1}{12.5}\left[\frac{16 \times 25^3}{12} + (2 \times 8 - 1) \times 12 \times \left(\frac{25}{2} - 2.5\right)^2\right]$$

$$\therefore S_{ut} = 3167\ in^3$$

Again, $\frac{P_b}{A_g * F_a} + \frac{M_b}{S_{ut} * F_b} = 1$

$$\Rightarrow \frac{P_b}{400 * 1.98} + \frac{10.74\ P_b}{3167 * 1.8} = 1$$

$$\therefore P_b = 318\ kips < P(= 360\ kips)$$

So, concrete compression governs.

Again, $\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$

$$\Rightarrow \frac{360}{400 * 1.98} + \frac{M}{3167 * 1.8} = 1$$

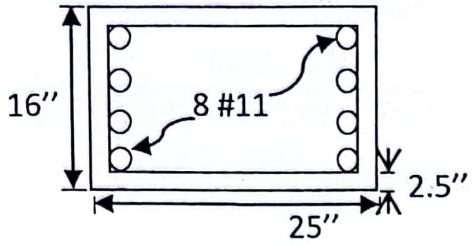
$$\Rightarrow \frac{M}{3167 * 1.8} = 1 - \frac{360}{400 * 1.98}$$

$$\therefore M = 3110\ kip - in.\quad (\textbf{\textit{Ans.}})$$

Check:

$$e = \frac{M}{P} = \frac{3110}{360} = 8.63\ in. < e_b(10.74'')\quad [ok]$$

Pb-4. If the following column carries an axial load of 100 kips, what is the allowable simultaneous moment? Use $f_c' = 4\ ksi\ and\ f_y = 50\ ksi.$



Sol^n:

For symmetrically tied columns,

$$e_b = (0.67\rho_g m + 0.17)d$$

$$\Rightarrow e_b = (0.67 * 0.0312 * 14.7 + 0.17)(25 - 2.5)$$

$$\therefore e_b = 10.74''$$

Now, $\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$

For balanced loading, $\frac{P_b}{A_g * F_a} + \frac{M_b}{S_{ut} * F_b} = 1$

$$\rho_g = \frac{A_s}{A_g} = \frac{8 * 1.56}{25 \times 16} = 0.0312$$

$$m = \frac{f_y}{0.85 f_c'} = \frac{50}{0.85 \times 4} = 14.7$$

Here,

$$A_g = 16 \times 25 = 400\ in^2$$

$$F_a = 0.34 f_c'(1 + m\rho_g) = 0.34 \times 4\ (1 + 14.7 \times 0.0312) = 1.98\ ksi$$

$$M_b = P_b * e_b = 10.74\ P_b$$

$$F_b = 0.45 f_c' = 0.45 * 4 = 1.8\ ksi$$

For rectangular column,

$$S_{ut} = \frac{1}{c}\left[\frac{bt^3}{12} + (2n-1)A_s\left(\frac{t}{2} - d'\right)^2\right]$$

$$\Rightarrow S_{ut} = \frac{1}{12.5}\left[\frac{16 \times 25^3}{12} + (2 \times 8 - 1) \times 12 \times \left(\frac{25}{2} - 2.5\right)^2\right]$$

$$\therefore S_{ut} = 3167\ in^3$$

Again, $\frac{P_b}{A_g * F_a} + \frac{M_b}{S_{ut} * F_b} = 1$

$$\Rightarrow \frac{P_b}{400 * 1.98} + \frac{10.74\ P_b}{3167 * 1.8} = 1$$

$$\therefore P_b = 318\ kips > P(= 100\ kips)$$

So, steel tension governs.

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{f_c'}} = 8$$

Again, $\frac{P}{P_b} = \frac{M - M_o}{M_b - M_o}$

For symmetrically tied columns,

$$M_o = 0.4 A_{st} f_y (d - d') = 0.4 \times \frac{8 * 1.56}{2} \times 50\ (22.5 - 2.5) = 2500\ kip - in.$$

So,

$$\frac{P}{P_b} = \frac{M - M_o}{M_b - M_o}$$

$$\Rightarrow \frac{100}{318} + \frac{M - 2500}{(318 * 10.74) - 2500} = 1$$

$$\therefore M = 2790\ kip - in. \quad (\boldsymbol{Ans.})$$

Check:

$$e = \frac{M}{P} = \frac{2790}{100} = 27.9\ in. > e_b(10.74'') \quad [ok]$$

Note: $e > e_b$ → Tension zone
$e < e_b$ → Compression zone

Note: $P > P_b$ → Compression zone
$P < P_b$ → Tension zone

Pb-5. Design a spiral-reinforced circular column to carry an axial load of 300 kips with an eccentricity 6″. Use $f_c' = 3\ ksi\ and\ f_y = 40\ ksi$.

Sol^n:

Assume, $\rho_g = 0.02$

For spiral column,

$P = A_g(0.25f_c' + f_s\rho_g)$

$\Rightarrow 300 = A_g(0.25 \times 3 + 0.4 * 40 \times 0.02)$

$\Rightarrow A_g = 280\ in^2$

$\Rightarrow \frac{\pi D^2}{4} = 280\ in^2$

$\therefore D = 18.88 \approx 19\ in.$

So, $A_g = \frac{\pi D^2}{4} = 284\ in^2$

Now, $\rho_g = \frac{A_s}{A_g} \Rightarrow 0.02 = \frac{A_s}{284} \therefore A_s = 5.68\ in^2$

Provide 6 – 29 mm φ (#9) longitudinal bars.

So, $A_s = 6.0\ in^2$

We know, for uniaxial eccentric loading,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1$$

$$\therefore \frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{P}{A_g * F_a} + \frac{M}{S_{ut} * F_b}$$

Here,

$$m = \frac{f_y}{0.85f_c'} = \frac{40}{0.85 \times 3} = 15.69$$

$$g_t = D - 5'' = 19 - 5 = 14''$$

$F_a = 0.34f_c'(1 + m\rho_g) = 0.34 \times 3\ (1 + 15.69 \times 0.02) = 1.34\ ksi$

$M = P * e = 300 * 6 = 1800\ k - in.$

$F_b = 0.45f_c' = 0.45 * 3 = 1.35\ ksi$

For circular column,

$$S_{ut} = \frac{1}{c}\left[\frac{\pi D^4}{64} + (2n-1)\frac{A_s}{8}(g_t)^2\right]$$

$$\Rightarrow S_{ut} = \frac{1}{9.5}\left[\frac{\pi * 19^4}{64} + (2 * 9 - 1) \times \frac{6.0}{8}(14)^2\right]$$

$\therefore S_{ut} = 936\ in^3$

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{f_c'}} = 9$$

Again,

$$\frac{P}{A_g * F_a} + \frac{M}{S_{ut} * F_b} = \frac{300}{284 * 1.34} + \frac{1800}{936 * 1.35} = 2.21 > 1 \quad (Not\ ok)$$

Trial No.	D (in.)	A_g (in^2)	Steel No.	A_s (in^2)	$\rho_g = \frac{A_s}{A_g}$	S_{ut} (in^3)	$\frac{f_a}{F_a} + \frac{f_b}{F_b}$	Remarks
1	19	284	6 – 29 mm φ	6.0	0.02	936	2.21	Not ok
2	24	452	8 – 29 mm φ	8.0	0.018	1869	1.22	Not ok
3	25	491	10 – 29 mm φ	10.0	0.02	2214	1.06	Not ok
4	25	491	10 – 32 mm φ	12.7	0.026	2398	0.98≈1.0	***Ok***

Provide 10 – 32 mm φ bars.

$\therefore$ Clear distance between longitudinal bars

$= \frac{\pi D_c - A_s}{Bar\ No.} = \frac{\pi * 22 - 12.7}{10} = 5.64\ in. > 1.5d_b$

$or\ 1.5 * 1.27\ or\ 1.905\ in.$ (Ok)

Core dia, $D_c = D - 2 * cc$

$= 25 - 2 * 1.5 = 22\ in.$

Spiral perimeter $= \pi D_c$

= Linear length of spiral

Spiral design:

Core diameter, $D_c = 22\ in.$ & Area of core, $A_c = \frac{\pi D_c^2}{4} = \frac{\pi * 22^2}{4} = 380\ in^2$

Minimum spiral ratio, $\rho_{s(min)} = 0.45\left(\frac{491}{380} - 1\right)\frac{3}{40} = 0.00986$

Pitch of spiral, $g = \frac{1}{6}D_c = \frac{1}{6} \times 22 = 3.6 \approx 3\ in.;$ $so,\ 1'' \le g \le 6''$ (ok)

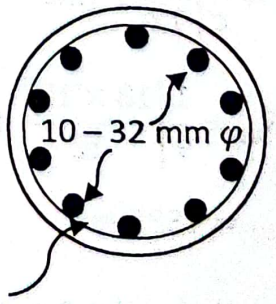
Taking 10 mm φ (#3) bar as spiral,

Spiral ratio, $\rho_s = \frac{4a_s}{gD_c} = \frac{4*0.11}{3 \times 22} = 0.0067 < \rho_{s(min)}$ $(Not\ ok)$

Taking 12 mm φ (#4) bar as spiral,

Spiral ratio, $\rho_s = \frac{4a_s}{gD_c} = \frac{4*0.20}{3 \times 22} = 0.012 > \rho_{s(min)}$ (ok)

Provide 12 mm φ (#4) spiral @ 3 in. pitch.



12 mm φ (#4) spiral @ 3 in. pitch

Pb-6. Design a square tied column to carry a working dead load of 240 kips and a working live load of 160 kips with a uniaxial moment of 300 k-ft. Use $f_c' = 4\ ksi$ *and* $f_y = 60\ ksi$. Follow WSD method. [*CE-2019*]

Sol^n:

Assume, $\rho_g = 0.02$

Here, Total axial load, $P = 240 + 160 = 400\ kips$

For square column,

$$P = 0.85A_g(0.25f_c' + f_s\rho_g)$$
$$\Rightarrow 400 = 0.85A_g(0.25 \times 4 + 0.4 * 60 \times 0.02)$$
$$\Rightarrow A_g = 318\ in^2$$
$$\Rightarrow b^2 = 318\ in^2$$
$$\therefore b = \sqrt{318} = 17.83 \approx 18\ in.$$

So, $A_g = b^2 = 324\ in^2$

Now, $\rho_g = \frac{A_s}{A_g} \Rightarrow 0.02 = \frac{A_s}{324} \quad \therefore A_s = 6.48\ in^2$

Provide 8 – 29 mm φ (#9) longitudinal bars.

So, $A_s = 8.0\ in^2$

For only square column, bar number is multiple of four.

We know, for uniaxial eccentric loading,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1$$

$$\therefore \frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{P}{A_g * F_a} + \frac{M}{S_{ut} * F_b}$$

Here,

$F_a = 0.34f_c'(1 + m\rho_g) = 0.34 \times 4\ (1 + 17.65 \times 0.02) = 1.84\ ksi$

$M = 300\ k - ft. = 3600\ k - in.$

$F_b = 0.45f_c' = 0.45 * 4 = 1.8\ ksi$

$$m = \frac{f_y}{0.85f_c'} = \frac{60}{0.85 \times 4} = 17.65$$

For square column,

$$S_{ut} = \frac{1}{c}\left[\frac{bt^3}{12} + (2n-1)A_s\left(\frac{t}{2} - d'\right)^2\right]$$

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{f_c'}} = 8$$

$$\Rightarrow S_{ut} = \frac{1}{9}\left[\frac{18 \times 18^3}{12} + (2 \times 8 - 1) \times 8 \times \left(\frac{18}{2} - 2.5\right)^2\right]$$

$$\therefore S_{ut} = 1535\ in^3$$

Again, $\frac{P}{A_g * F_a} + \frac{M}{S_{ut} * F_b} = \frac{400}{324 * 1.84} + \frac{3600}{1535 * 1.8} = 1.97 > 1 \quad (Not\ ok)$

Trial No.	b (in.)	A_g (in²)	Steel No.	A_s (in²)	$\rho_g = \frac{A_s}{A_g}$	S_{ut} (in³)	$\frac{f_a}{F_a} + \frac{f_b}{F_b}$	Remarks
1	18	324	8 – 29 mm φ	8.0	0.02	1535	1.97	Not ok
2	22	484	12 – 29 mm φ	12.0	0.025	2957	1.09	Not ok
3	22	484	12 – 32 mm φ	15.24	0.031	3276	1.003	Not ok
4	23	529	12 – 32 mm φ	15.24	0.029	3638	0.92≈1.0	***Ok***

Provide 12 – 32 mm φ bars.

Design of tie:

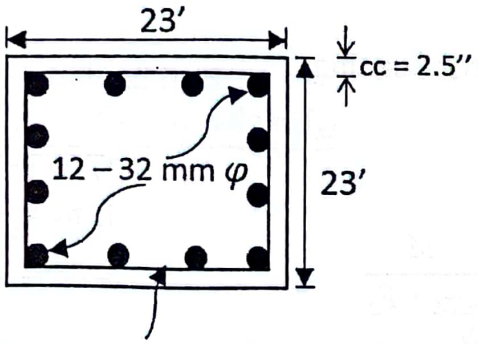
Spacing of tie is the least of –

i) 16 times diameter of longitudinal bar = $16d_{bL} = 16 * 1.27 = 20.32\ in.$

ii) 48 times diameter of tie bar (#3) = $16d_{bT} = 16 * 0.375 = 18\ in.$

iii) Least column dimension = $23\ in.$

Provide 10 mm φ (#3) tie bar @ 18 in. c/c.



10 mm φ (#3) tie bar @ 18 in. c/c

Check for extra tie:

∴ Clear distance between longitudinal bars

$= \frac{b - 2 * cc - 2 * d_{bT} - \text{Bar no. of each side} * d_{bL}}{\text{Bar no. of each side} - 1}$

$= \frac{23 - 2 * 2.5 - 2 * 0.375 - 4 * 1.27}{4 - 1}$

$= 4.06'' > 1.5d_b\ (1.905'')$

$\therefore 1.905'' < 4.06'' < 6''$

So, no extra tie is required.

Pb-7. Design a rectangular tied column to carry an axial load of 400 kips with a uniaxial moment of 200 k-ft. Use $f_c' = 4\ ksi$ *and* $f_y = 60\ ksi$. Follow WSD method. [*CE-2018*]

Sol^n:

Assume, $\rho_g = 0.02$

Here, Axial load, $P = 400\ kips$

For rectangular column,

$$P = 0.85A_g(0.25f_c' + f_s\rho_g)$$

$$\Rightarrow 400 = 0.85A_g(0.25 \times 4 + 0.4 * 60 \times 0.02)$$

$$\Rightarrow A_g = 318\ in^2$$

$$\Rightarrow 1.5b * b = 318\ in^2$$

$$\therefore b = \sqrt{\frac{318}{1.5}} = 14.56 \approx 15\ in.\ \&\ t = 23\ in.$$

So, $A_g = t * b = 345\ in^2$

Now, $\rho_g = \frac{A_s}{A_g} \Rightarrow 0.02 = \frac{A_s}{345} \quad \therefore A_s = 6.9\ in^2$

Width = b
Length = t = 1.5b

Provide 8 – 29 mm φ (#9) longitudinal bars.

So, $A_s = 8.0\ in^2$

We know, for uniaxial eccentric loading,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1$$

$$\therefore \frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{P}{A_g * F_a} + \frac{M}{S_{ut} * F_b}$$

$$m = \frac{f_y}{0.85f_c'} = \frac{60}{0.85 \times 4} = 17.65$$

Here,

$F_a = 0.34f_c'(1 + m\rho_g) = 0.34 \times 4\ (1 + 17.65 \times 0.02) = 1.84\ ksi$

$M = 200\ k - ft. = 2400\ k - in.$

$F_b = 0.45f_c' = 0.45 * 4 = 1.8\ ksi$

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{f_c'}} = 8$$

For rectangular column,

$$S_{ut} = \frac{1}{c}\left[\frac{bt^3}{12} + (2n-1)A_s\left(\frac{t}{2} - d'\right)^2\right]$$

$$\Rightarrow S_{ut} = \frac{1}{11.5}\left[\frac{15 \times 23^3}{12} + (2 \times 8 - 1) \times 8 \times \left(\frac{23}{2} - 1.5\right)^2\right]$$

$$\therefore S_{ut} = 2366\ in^3$$

Again,

$$\frac{P}{A_g * F_a} + \frac{M}{S_{ut} * F_b} = \frac{400}{345 * 1.84} + \frac{2400}{2168 * 1.8} = 1.19 > 1 \quad (Not\ ok)$$

Trial No.	b (in.)	t (in.)	A_g (in^2)	Steel No.	A_s (in^2)	$\rho_g = \frac{A_s}{A_g}$	S_{ut} (in^3)	$\frac{f_a}{F_a} + \frac{f_b}{F_b}$	Remarks
1	15	23	345	8 – #9	8.0	0.02	2366	1.19	Not ok
2	16	24	384	8 – #10	10.2	0.027	2942	0.97≈1.0	Ok

Provide 12 – 32 mm φ bars.

Design of tie:

Select #3 bar as tie,

Spacing of tie is the least of –

i) 16 times diameter of longitudinal bar $= 16d_{bL} = 16 * 1.27 = 20.32\ in.$

ii) 48 times diameter of tie bar (#3) $= 16d_{bT} = 16 * 0.375 = 18\ in.$

iii) Least column dimension $= 16\ in.$

Provide 10 mm φ (#3) tie bar @ 16 in. c/c.

Check for extra tie:

∴ Clear distance between longitudinal bars

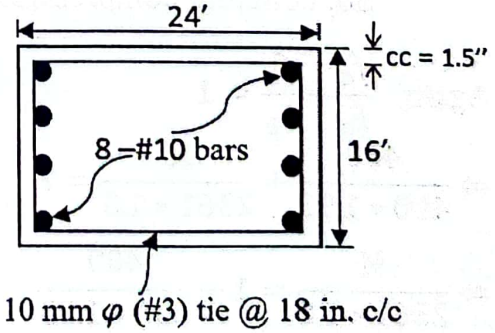
$$= \frac{b - 2 * cc - 2 * d_{bT} - Bar\ no.\ of\ each\ side * d_{bL}}{Bar\ no.\ of\ each\ side - 1}$$

$$= \frac{16 - 2 * 1.5 - 2 * 0.375 - 4 * 1.27}{4 - 1}$$

$= 2.39'' > 1.5d_b\ (1.875'')$

$\therefore 1.875'' < 2.39'' < 6''$

So, no extra tie is required.



Pb-8. A 20 in. square tied column was reinforced with 12 – #8 bars and #3 lateral ties were placed at 18 in. c/c vertically. The clear cover was 1.5 in. on all sides. The column was loaded with an allowable axial load of 400 kips at a certain eccentricity. Determine the value of eccentricity to which it was loaded. Use $f_c' = 4\ ksi\ and\ f_y = 60\ ksi$. [*CE-2017*]

Sol^n:

For symmetrically tied columns,

$$e_b = (0.67\rho_g m + 0.17)d$$

$$\Rightarrow e_b = (0.67 * 0.0237 * 17.65 + 0.17)(20 - 1.5)$$

$$\rho_g = \frac{A_s}{A_g} = \frac{12 * 0.79}{20^2} = 0.0237$$

$$m = \frac{f_y}{0.85 f_c'} = \frac{60}{0.85 \times 4} = 17.65$$

$\therefore e_b = 8.33''$

Now, $\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$

For balanced loading, $\frac{P_b}{A_g * F_a} + \frac{M_b}{S_{ut} * F_b} = 1$

Here,

$A_g = 20^2 = 400\ in^2$

$F_a = 0.34 f_c'(1 + m\rho_g) = 0.34 \times 4\ (1 + 17.65 \times 0.0237) = 1.93\ ksi$

$M_b = P_b * e_b = 8.33\ P_b$

$F_b = 0.45 f_c' = 0.45 * 4 = 1.8\ ksi$

For rectangular column,

$$S_{ut} = \frac{1}{c}\left[\frac{bt^3}{12} + (2n-1)A_s\left(\frac{t}{2} - d'\right)^2\right]$$

$$\Rightarrow S_{ut} = \frac{1}{10}\left[\frac{20 \times 20^3}{12} + (2 \times 8 - 1) \times 9.48 \times \left(\frac{20}{2} - 1.5\right)^2\right]$$

$\therefore S_{ut} = 2361\ in^3$

Again, $\frac{P_b}{A_g * F_a} + \frac{M_b}{S_{ut} * F_b} = 1$

$$\Rightarrow \frac{P_b}{400 * 1.93} + \frac{8.33\ P_b}{2361 * 1.8} = 1$$

$\therefore P_b = 307\ kips < P(= 400\ kips)$

So, concrete compression governs

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{f_c'}} = 8$$

Again, $\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$

$$\Rightarrow \frac{400}{400 * 1.93} + \frac{M}{2361 * 1.8} = 1$$

$$\Rightarrow \frac{M}{2361 * 1.8} = 1 - \frac{400}{400 * 1.93}$$

$\therefore M = 2048\ kip - in.$

$\therefore$ Eccentricity, $e = \frac{M}{P} = \frac{2048}{400} = 5.12\ in.$

II. Method: *Strength Design Method or USD*

Pb-9. Design a square tied column to carry an axial service load of 320 kips dead load and 190 kips live load. Use $f_c' = 4\ ksi\ and\ f_y = 60\ ksi.$

Sol^n:

Ultimate axial load, $P_u = 1.2DL + 1.6LL = 1.2 * 320 + 1.6 * 190 = 688\ kips$

Assume, $\rho_g = 0.03$

Now, Ultimate axial load, $P_u = \alpha\varphi P_n$

$$\Rightarrow P_u = \alpha\varphi A_g[0.85f_c'(1-\rho_g) + f_y\rho_g]$$

$$\Rightarrow 688 = 0.80 * 0.65 * A_g[0.85 * 4(1-0.03) + 60 * 0.03]$$

$$\Rightarrow A_g = 259.53\ in^2$$

$$\therefore b = \sqrt{259.53} = 16.11\ in.$$

Taking, $b = 16\ in.$

So, $A_g = 256\ in^2$

The contribution of concrete, $P_{uc} = \alpha\varphi A_g[0.85f_c'(1-\rho_g)]$

$$\therefore P_{uc} = 0.80 * 0.65 * 256\ [0.85 * 4\ (1-0.03)] = 439\ kips$$

So, load to carried by steel, $P_{us} = 688 - 439 = 249\ kips$

$$\Rightarrow \alpha\varphi A_g f_y\rho_g = 249\ kips$$

$$\Rightarrow 0.80 * 0.65 * f_y A_g * \frac{A_s}{A_g} = 249\ kips$$

$$\therefore A_s = \frac{249}{0.80 * 0.65 * 60} = 7.98\ in^2$$

Provide 8 – #9 longitudinal bars.

Check:

$$\rho_g = \frac{8 * 1.0}{256} = 0.0313 \quad \text{[Between 0.01 \& 0.08; ok]}$$

Design of tie:

Select #3 bar as tie,

Spacing of tie is the least of –

i) 16 times diameter of longitudinal bar $= 16d_{bL} = 16 * 1.128 = 18\ in.$

ii) 48 times diameter of tie bar (#3) $= 48d_{bT} = 48 * 0.375 = 18\ in.$

ii) Least column dimension $= 16\ in.$

Provide #3 tie bar @ 16 in. c/c

Check for extra tie:

$\therefore$ Clear distance between longitudinal bars

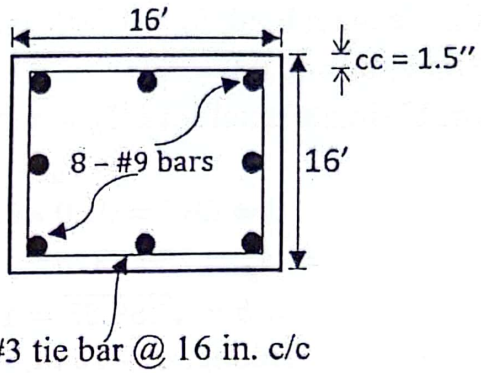
$$= \frac{b - 2 * cc - 2 * d_{bT} - \text{Bar no. of each side} * d_{bL}}{\text{Bar no. of each side} - 1}$$

$$= \frac{16 - 2 * 1.5 - 2 * 0.375 - 3 * 1.128}{3 - 1}$$

$= 4.43'' > 1.5d_b\ (1.692'')$

$\therefore 1.692'' < 4.43'' < 6''$

So, no extra tie is required.



1.4.3 Biaxial Bending

Pb-10. Design a short tied square column to carry a factored load axially $P_u = 640$ kips and a factored moment $M_u = 330$ k-ft. Use $f_c' = 4\ ksi$ *and* $f_y = 60\ ksi$.

Sol^n:

Eccentricity, $e = \frac{M_u}{P_u} = \frac{330 \times 12}{640} = 6.19\ in.$

Now, Ultimate axial load, $P_u = \alpha\varphi P_n$

$$\Rightarrow P_u = \alpha\varphi A_g[0.85f_c'(1 - \rho_g) + f_y\rho_g]$$

$$\Rightarrow 640 = 0.80 * 0.65 * A_g[0.85 * 4(1 - 0) + 60 * 0]$$

$$\Rightarrow A_g = 362\ in^2$$

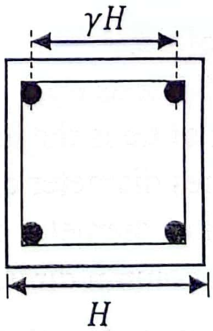
$$\therefore h = \sqrt{362} = 19\ in.$$

Assume, $\rho_g = 0$

Finding ρ_g,

$$\therefore h = \frac{e}{h} = \frac{6.19}{19} = 0.33$$

$$\therefore \frac{P_u}{\varphi f_c' A_g} = \frac{640}{0.65 \times 4 \times 361} = 0.68$$



Now, $\gamma H = H - 2 * cc\ to\ the\ bar\ centers = 19 - 2 * 2.5 = 14''$

$$\because \gamma = \frac{\gamma H}{H} = \frac{14}{19} = 0.74$$

Maximum usable $\rho_g = 1 \sim 4\%$

Select graph A.6 and from graph, $\rho_g = 0.05$

So, revised. Trying 21 in. × 21 in. column with $A_g = 441\ in^2$

$$\therefore \frac{e}{h} = \frac{6.19}{21} = 0.29$$

$$\therefore \frac{P_u}{\varphi f_c' A_g} = \frac{640}{0.65 \times 4 \times 441} = 0.56$$

Now, $\gamma H = 21'' - 5'' = 16''$

$$\because \gamma = \frac{\gamma H}{H} = \frac{16}{21} = 0.76;\quad close\ to\ 0.80$$

Select graph A.7 and from graph, $\rho_g = 0.02$

$\therefore A_{st} = \rho_g * A_g = 0.02 * 441 = 8.82\ in^2$

Provide 12 – #8 bars with $A_{st} = 9.48\ in^2$

Design of tie:

Select #3 bar as tie,

Spacing of tie is the least of –

i) 16 times diameter of longitudinal bar $= 16 d_{bL} = 16 * 1'' = 16\ in.$

ii) 48 times diameter of tie bar (#3) $= 48 d_{bT} = 48 * 0.375'' = 18\ in.$

ii) Least column dimension $= 21\ in.$

Provide #3 tie bar @ 16 in. c/c

Check for extra tie:

$\therefore$ Clear distance between longitudinal bars

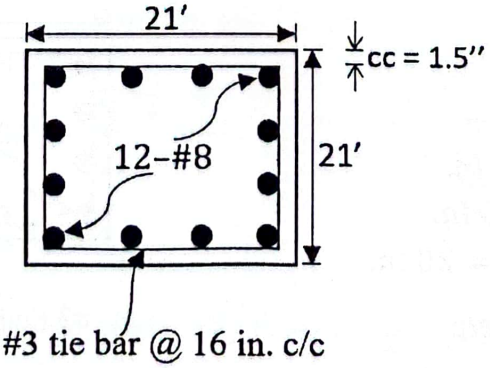
$$= \frac{b - 2 * cc - 2 * d_{bT} - \text{Bar no. of each side} * d_{bL}}{\text{Bar no. of each side} - 1}$$

$$= \frac{21 - 2 * 1.5 - 2 * 0.375 - 4 * 1}{4 - 1}$$

$= 4.42'' > 1.5 d_b\ (1.5'')$

$\therefore 1.5'' < 4.43'' < 6''$

So, no extra tie is required.



N.B: The graphs are provided in the Appendix – A at the end of this book.

Note: If column section is rectangle, stiffness along which dimension is greater that is H. Generally larger dimension = H.

Pb-11. In a three-story structure, an exterior column is to be designed for a service dead load of 222 kips, maximum live load of 297 kips, dead load moment of 136 k-ft. and live load moment of 194 k-ft. Architectural considerations require that a rectangular column be used, with dimensions $b = 20$ in. and $h = 25$ in. Find the required column reinforcement for the condition Use $f_c' = 4\ ksi\ and\ f_y = 60\ ksi.$

Sol^n:

$P_u = 1.2 * 222 + 1.6 * 297 = 742\ kips$

$M_u = 1.2 * 136 + 1.6 * 194 = 474\ k - ft.$

Here, $A_g = 20 * 25 = 500\ in^2$

Eccentricity, $e = \frac{M_u}{P_u} = \frac{474 \times 12}{742} = 7.67\ in.$

$\therefore \frac{e}{h} = \frac{7.67}{25} = 0.31$

$\therefore \frac{P_u}{\varphi f_c' A_g} = \frac{742}{0.65 \times 4 \times 500} = 0.57$

Now, $\gamma H = 25'' - 5'' = 20''$

$\because \gamma = \frac{\gamma H}{H} = \frac{20}{25} = 0.80$

Select graph A.7 and from graph, $\rho_g = 0.024$

$\therefore A_{st} = \rho_g * A_g = 0.024 * 500 = 12\ in^2$

Provide 12 – #9 bars.

<u>Design of tie:</u>

Select #3 bar as tie,

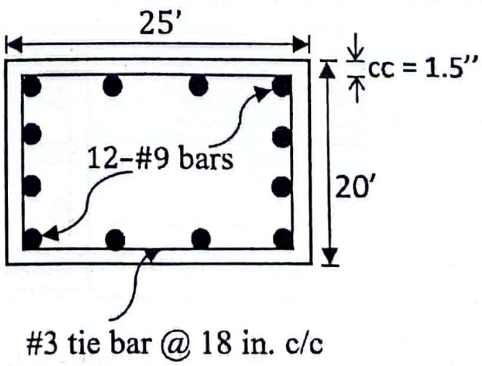
Spacing of tie is the least of –

i) $16 d_{bL} = 16 * 1.128'' = 18\ in.$

ii) $48 d_{bT} = 48 * 0.375'' = 18\ in.$

ii) Least column dimension $= 20\ in.$

Provide #3 tie bar @ 18 in. c/c



Check for extra tie:

∴ Clear distance between longitudinal bars

$$= \frac{b - 2 * cc - 2 * d_{bT} - \text{Bar no. of each side} * d_{bL}}{\text{Bar no. of each side} - 1}$$

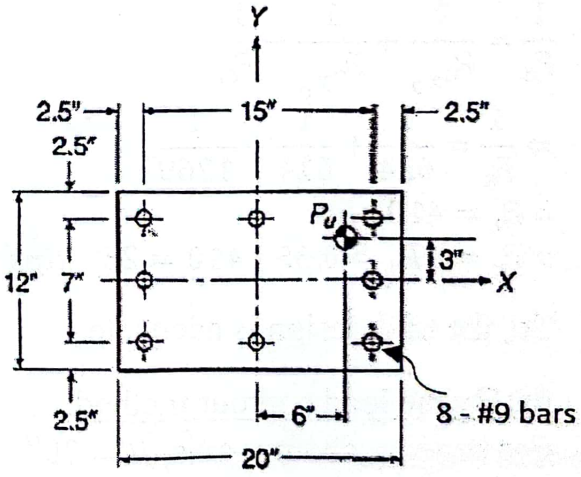
$$\therefore S_b = \frac{20-2*1.5-2*0.375-4*1.128}{4-1}$$
$$= 3.91'' > 1.5d_b\ (1.692'')$$
$$\therefore 1.692'' < 3.91'' < 6''$$

$$\therefore S_h = \frac{25-2*1.5-2*0.375-4*1.128}{4-1}$$
$$= 5.58'' > 1.5d_b\ (1.692'')$$
$$\therefore 1.692'' < 5.58'' < 6''$$

So, no extra tie is required.

Pb-12. The 12×20 in. column shown in figure below is reinforced with $8 - \#9$ bars arranged around the column perimeter, providing an area of $A_{st} = 8.0\ in^2$. A factored load P_u of 255 kips is to be applied with eccentricities e_y= 3 in. and e_x= 6 in. Material strengths are $f_c' = 4\ ksi\ and\ f_y = 60\ ksi$.

Check the adequacy of the trial design.
(a) Using the reciprocal load method and
(b) Using the load contour method.



Sol^n:

(a) By the reciprocal load method:

For bending about y-axis, $H = 20''$

$\gamma H = 20'' - 5'' = 15''$

$$\because \gamma = \frac{\gamma H}{H} = \frac{15}{20} = 0.75$$

Select graph A.6 & A.7

$$and,\ \frac{e}{h} = \frac{6}{20} = 0.30$$

$$\text{Reinforcement ratio, } \rho = \frac{A_{st}}{bh} = \frac{8.0}{12 \times 20} = 0.033$$

From graph A.6,

$$\frac{P_{ny_o}}{f_c' A_g} = 0.62$$

From graph A.7,

$$\frac{P_{ny_o}}{f_c' A_g} = 0.66$$

$$\therefore \frac{P_{ny_o}}{f_c' A_g} = \frac{0.62 + 0.66}{2} = 0.64$$

$$\because P_{ny_o} = 0.64 * f_c' A_g = 0.64 * 4 * (12 * 20) = 614\ kips$$

For bending about x-axis, $H = 12''$

$\gamma H = 12'' - 5'' = 7''$

$\because \gamma = \frac{\gamma H}{H} = \frac{7}{12} = 0.58 \approx 0.60$

Select graph A.5

and, $\frac{e}{h} = \frac{6}{20} = 0.30$

Reinforcement ratio, $\rho = \frac{A_{st}}{bh} = \frac{8.0}{12 \times 20} = 0.033$

From graph A.5, $\frac{P_{nx_o}}{f_c' A_g} = 0.65$

$\because P_{nx_o} = 0.65 * f_c' A_g = 0.64 * 4 * (12 * 20) = 624\ kips$

Axial load, $P_o = 0.85 f_c' (A_g - A_{st}) + f_y A_{st}$

$\therefore P_o = 0.85 * 4\,(240 - 8) + 60 * 8 = 1269\ kips$

Now from Bresler's Reciprocal Load equation,

$\frac{1}{P_n} = \frac{1}{P_{nx_o}} + \frac{1}{P_{ny_o}} - \frac{1}{P_o}$

$\Rightarrow \frac{1}{P_n} = \frac{1}{624} + \frac{1}{614} - \frac{1}{1269}$

$\therefore P_n = 410\ kips$

$\because P_u = \varphi P_n = 0.65 * 410 = 267\ kips > 255\ kips$

So, the trial design is adequate.

(b) By the load contour method:

For bending about y-axis, $H = 20''$

$\gamma H = 20'' - 5'' = 15''$

$\because \gamma = \frac{\gamma H}{H} = \frac{15}{20} = 0.75$

Select graph A.6 & A.7

Reinforcement ratio, $\rho = \frac{A_{st}}{bh} = \frac{8.0}{12 \times 20} = 0.033$

From graph A.6,

$\frac{M_{ny_o}}{f_c' A_g h} = 0.215$

From graph A.7,

$\frac{M_{ny_o}}{f_c' A_g h} = 0.235$

$\therefore \frac{M_{ny_o}}{f_c' A_g h} = \frac{0.215 + 0.235}{2} = 0.225$

$\because M_{ny_o} = 0.225 * f_c' A_g h = 0.64 * 4 * 240 * 20 = 4320\ k - in.$

For bending about x-axis, $H = 12''$

$\gamma H = 12'' - 5'' = 7''$

$\because \gamma = \dfrac{\gamma H}{H} = \dfrac{7}{12} = 0.58 \approx 0.60$

Select graph A.5

Reinforcement ratio, $\rho = \dfrac{A_{st}}{bh} = \dfrac{8.0}{12 \times 20} = 0.033$

From graph A. 5, $\dfrac{M_{nx_o}}{f_c' A_g h} = 0.185$

$\because M_{nx_o} = 0.185 * f_c' A_g h = 0.184 * 4 * 240 * 12 = 2130\ k - in.$

Now, Nominal load, $P_n = \dfrac{P_u}{\varphi} = \dfrac{255}{0.65} = 392\ kips$

$\because$ Nominal moments about x and y-axes are,

$M_{n_x} = P_n * e_y = 392 * 3 = 1176\ k - in.$

$M_{n_y} = P_n * e_x = 392 * 6 = 2352\ k - in.$

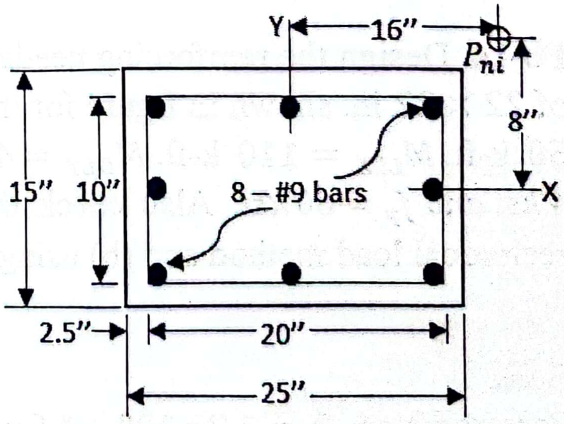
Now from Load Contour Method equation,

$$\left(\frac{M_{n_x}}{M_{n_{x_o}}}\right)^{\alpha_1} + \left(\frac{M_{n_y}}{M_{n_{y_o}}}\right)^{\alpha_2} \leq 1$$

$$\therefore \left(\frac{1176}{2130}\right)^{1.15} + \left(\frac{2352}{4320}\right)^{1.15} = 0.505 + 0.497 = 1.002 \approx 1$$

So, the trial design is adequate.

Pb-13. Determine the design capacity, P_{ni} of the short tied column shown in figure below which is subjected to biaxial bending. The column is reinforced with 8 nos. of #9 bars. Material strengths are $f_c' = 4\ ksi\ and\ f_y = 60\ ksi$. *[CE-2018]*



Sol^n:

For bending about y-axis, $H = 25''$

$\gamma H = 25'' - 5'' = 20''$

$\because \gamma = \dfrac{\gamma H}{H} = \dfrac{20}{25} = 0.80$

Select graph A.7

and, $\dfrac{e}{h} = \dfrac{16}{25} = 0.64$

Reinforcement ratio, $\rho = \dfrac{A_{st}}{bh} = \dfrac{8 * 1.0}{25 \times 15} = 0.0213$

From graph A. 7, $\frac{P_{nx_o}}{f_c' A_g} = 0.29$

$\because P_{nx_o} = 0.29 * f_c' A_g = 0.29 * 4 * (25 * 15) = 435\ kips$

For bending about x-axis, $H = 15''$

$\gamma H = 15'' - 5'' = 10''$

$\because \gamma = \frac{\gamma H}{H} = \frac{10}{15} = 0.67 \approx 0.70$

Select graph A.6

and, $\frac{e}{h} = \frac{8}{15} = 0.53$

Reinforcement ratio, $\rho = \frac{A_{st}}{bh} = \frac{8 * 1.0}{12 \times 20} = 0.0213$

From graph A. 6, $\frac{P_{nx_o}}{f_c' A_g} = 0.33$

$\because P_{nx_o} = 0.33 * f_c' A_g = 0.33 * 4 * (25 * 15) = 495\ kips$

Axial load, $P_o = 0.85 f_c' (A_g - A_{st}) + f_y A_{st}$

$\therefore P_o = 0.85 * 4\ (375 - 8) + 60 * 8 = 1728\ kips$

Now from Bresler's Reciprocal Load equation,

$$\frac{1}{P_{ni}} = \frac{1}{P_{nx_o}} + \frac{1}{P_{ny_o}} - \frac{1}{P_o}$$

$$\Rightarrow \frac{1}{P_{ni}} = \frac{1}{495} + \frac{1}{435} - \frac{1}{1728}$$

$\therefore P_{ni} = 268\ kips$ ***(Ans.)***

Pb-14. Design the reinforcing needed for a short square tied column having cross-section of 22×22 in. shown in figure for the following $DL = 100$ kips, $LL = 200$ kips, $M_{DLX} = 50$ k-ft, $M_{LLX} = 110$ k-ft, $M_{DLY} = 40$ k-ft, $M_{LLY} = 90$ k-ft. Material strengths are $f_c' = 4\ ksi$ *and* $f_y = 60\ ksi$. Also check the adequacy of the trial design, using (a) using the reciprocal load method and (b) using the load contour method. [*CE-2017*]

Sol^n:

Factored load, $P_u = 1.2 * 100 + 1.6 * 200 = 440\ kips$

$\therefore P_n = \frac{440}{0.65} = 677\ kips$

$M_{ux} = 1.2 * 50 + 1.6 * 110 = 236\ k - ft.$

$M_{uy} = 1.2 * 40 + 1.6 * 90 = 192\ k - ft.$

$\therefore M_{nx} = \frac{236}{0.65} = 363\ k-ft.$ & $\therefore M_{ny} = \frac{192}{0.65} = 295\ k-ft.$

$\because M_n = 363 + 295 = 658\ k-ft.$

$\therefore$ Eccentricity, $e = \frac{M_n}{P_n} = \frac{658 * 12}{677} = 11.66''$

Here, $A_g = 22 * 22 = 484\ in^2$

$\therefore \frac{e}{h} = \frac{11.66}{22} = 0.53$

$\therefore \frac{P_u}{\varphi f_c' A_g} = \frac{440}{0.65 \times 4 \times 484} = 0.35$

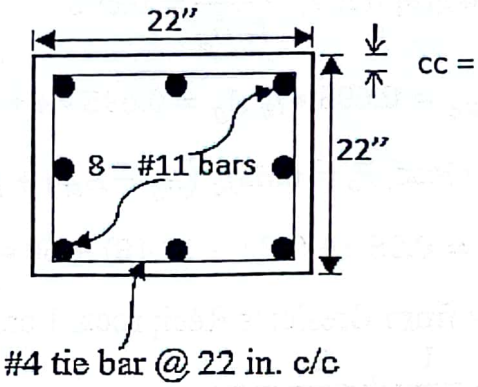
Now, $\gamma H = 22'' - 5'' = 17''$

$\because \gamma = \frac{\gamma H}{H} = \frac{17}{22} = 0.77 \approx 0.80$

Select graph A.7 and from graph, $\rho_g = 0.021$

$\therefore A_{st} = \rho_g * A_g = 0.021 * 484 = 10.16\ in^2$

Provide $8 - \#11$ bars with $A_{st} = 12.48\ in^2$



Design of tie:

Select #4 bar as tie,

Spacing of tie is the least of –

i) $16 d_{bL} = 16 * 1.41'' = 22.5\ in.$

ii) $48 d_{bT} = 48 * 0.5'' = 24\ in.$

ii) Least column dimension $= 22\ in.$

Provide #4 tie bar @ 22 in. c/c.

N.B: Use #4 bar as tie for above #10 longitudinal bar. [See basic concepts – 4]

Checking adequacy of the trial design:

(a) By the reciprocal load method:

For bending about y-axis, $H = 22''$

$\gamma H = 22'' - 5'' = 17''$

$\because \gamma = \frac{\gamma H}{H} = \frac{17}{22} = 0.77 \approx 0.80$

Select graph A.7

$e = \frac{M_{nx}}{P_n} = \frac{363 * 12}{677} = 6.43''$ *and,* $\frac{e}{h} = \frac{6.43}{22} = 0.29$

Reinforcement ratio, $\rho = \frac{A_{st}}{bh} = \frac{8 * 1.56}{22 \times 22} = 0.026$

From graph A. 7, $\frac{P_{ny_o}}{f_c' A_g} = 0.62$

$\because P_{ny_o} = 0.62 * f_c' A_g = 0.64 * 4 * 484 = 1200\ kips$

For bending about x-axis, $H = 22''$

$\gamma H = 22'' - 5'' = 17''$

$\because \gamma = \frac{\gamma H}{H} = \frac{17}{22} = 0.77 \approx 0.80$

Select graph A.7

$e = \frac{M_{ny}}{P_n} = \frac{295 * 12}{677} = 5.23''$ *and,* $\frac{e}{h} = \frac{5.23}{22} = 0.24$

Reinforcement ratio, $\rho = 0.026$

From graph A. 7, $\frac{P_{nx_o}}{f_c' A_g} = 0.695$

$\because P_{nx_o} = 0.695 * f_c' A_g = 0.695 * 4 * 484 = 1346\ kips$

Axial load, $P_o = 0.85 f_c' (A_g - A_{st}) + f_y A_{st}$

$\therefore P_o = 0.85 * 4\ (484 - 12.48) + 60 * 12.48 = 2352\ kips$

Now from Bresler's Reciprocal Load equation,

$\frac{1}{P_n} = \frac{1}{P_{nx_o}} + \frac{1}{P_{ny_o}} - \frac{1}{P_o}$

$\Rightarrow \frac{1}{P_n} = \frac{1}{1346} + \frac{1}{1200} - \frac{1}{2352}$

$\therefore P_n = 869\ kips > 677\ kips (P_n)$

So, the trial design is adequate.

(b) By the load contour method:

For bending about y-axis, $H = 22''$

$\gamma H = 22'' - 5'' = 17''$

$\because \gamma = \frac{\gamma H}{H} = \frac{17}{22} = 0.77 \approx 0.80$

Select graph A.7

Reinforcement ratio, $\rho = 0.026$

From graph A. 7, $\frac{M_{ny_o}}{f_c' A_g h} = 0.21$

$\because M_{ny_o} = 0.21 * f_c' A_g h = 0.21 * 4 * 484 * 22 = 8944\ k - in.$

For bending about x-axis,

Since square column, $\because M_{nx_o} = 8944\ k - in.$

Now from Load Contour Method equation,

$$\left(\frac{M_{n_x}}{M_{n_{x_o}}}\right)^{\alpha_1} + \left(\frac{M_{n_y}}{M_{n_{y_o}}}\right)^{\alpha_2} \leq 1$$

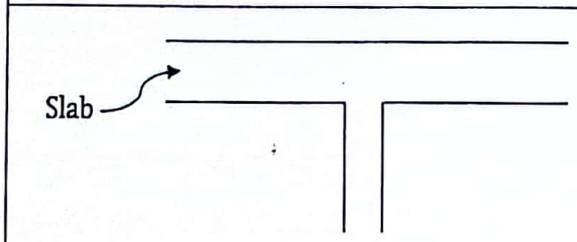
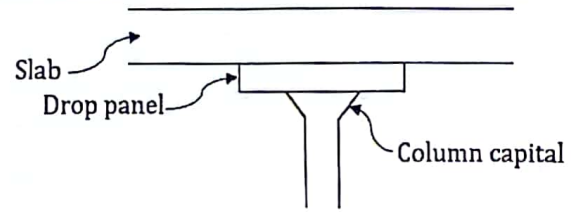
$$\therefore \left(\frac{363 * 12}{8944}\right)^{1.15} + \left(\frac{295 * 12}{8944}\right)^{1.15} = 0.437 + 0.344 = 0.78 < 1 \quad (ok)$$

So, the trial design is adequate.

Chapter - 2
Flat Plate and Flat Slabs

2.1 Theory Analysis

Q-1. What are the differences between flat plate and flat slab?
Ans: The differences between flat plate and flat slab are described below:

Flat plate	Flat slab
(i) It has uniform slab that rests directly on columns.	(i) It rests directly on columns.
(ii) It has no beams, drop panel or column capital.	(ii) It has no beams but has drop panel and column capital.
(iii) Flat plate can join only smaller spans.	(iii) Flat slabs can join larger spans when compared to flat plates.
(iv) Flat plates are most economical for spans from 15 to 20ft.	(iv) Flat slabs are used for spans of 20 to 30 ft.
	

Q-2. What are the design methods of flat plate and flat slab?
Ans: The flat plate and flat slabs can be analyzed using several methods of structural analysis. Some of this methods are given below:
(a) Direct Design Method (DDM): For the moment coefficients determined by the direct design method to be applicable. This method has such limitations:
(i) ≥ 3 spans
(ii) $\frac{l_l}{l_s} \leq 2$
(iii) $(l_{l_1} - l_{l_2}) \leq \frac{1}{3} l_{l_1}$
(v) Column offset $\leq 10\% L$
(vi) $0.2 \leq \alpha \leq 5.0$

(b) Equivalent Frame Method (EFM): The only difference between the direct design method and the equivalent frame method is in the determination of the longitudinal moments in the spans of the equivalent rigid frame. Whereas the direct design method involves a one-cycle moment distribution, the equivalent frame method involves a normal

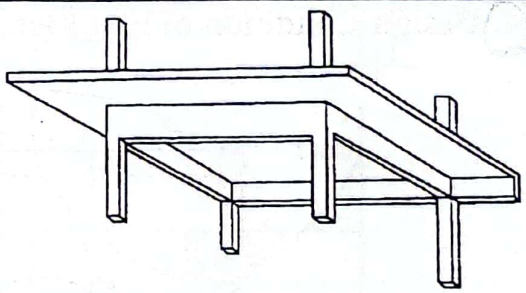
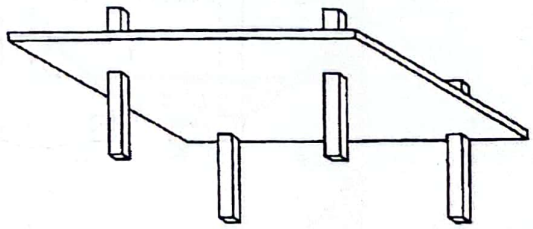
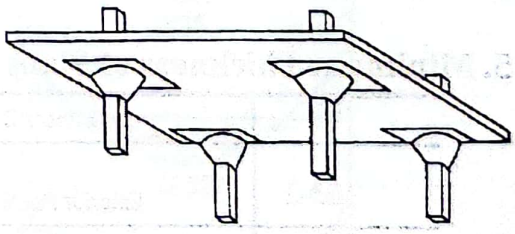
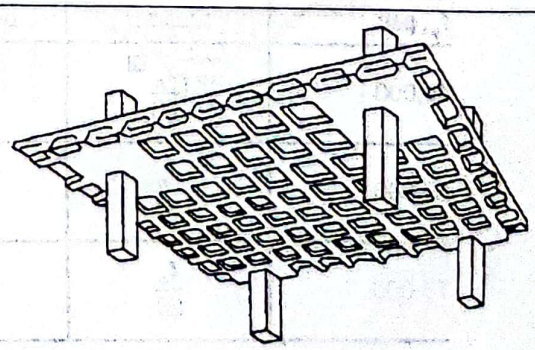
moment distribution of several cycles. The design moments obtained by either method are distributed to the column and middle strips in the same fashion.

And finally, this is a very important matter because so many floor systems do not meet the limitations specified for the direct design method.

For such cases, it is simpler to use the *direct design method (DDM).*

2.2 Basic Concepts

1. Types of Two Way Slabs or Floor Systems:

Types of two way slabs	Description	Economical choice	Figure
1. Two way slabs on beams	- Supported by beams on all four sides. - Beams transfer the loads to the column.	- Spans = 20~30 ft. - Live loads = 60~120 psf.	
2. Flat plates	- Uniform slab that rests directly on columns. - It has no beams or column capital.	- Spans = 15~20 ft. - Live loads = 60~100 psf.	
3. Flat slabs	- It has no beams or girders and columns carry the loads. - It has drop panel and column capital.	- Spans = 20~30 ft. - Live loads = 80~150 psf.	
4. Waffle slabs	- Consists of voids between the ribs with the ribs continuing in both directions. - Without supporting beams and resting directly on columns.	- Spans = 20~25 ft. - Live loads = 60~100 psf.	

2. Relative Stiffness (α_f):

Here, α_f = Ratio of flexural stiffness of beam to the slab

I_b = Gross moment of inertia of beam

I_s = Gross moment of inertia slab

So, $\alpha_f = \dfrac{\text{beam stiffness}}{\text{slab stiffness}} = \dfrac{E_b I_b}{E_s I_s}$

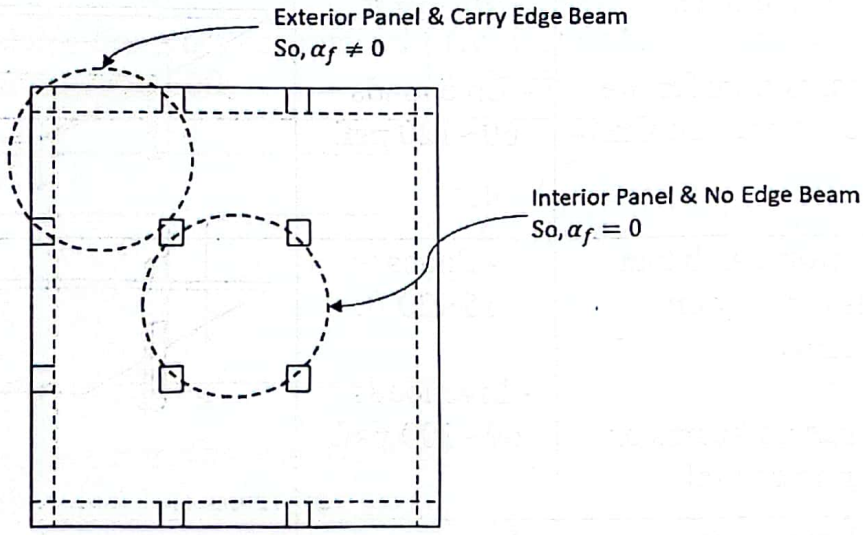
3. Relative Restraint (β_t):

$$\beta_t = \frac{\text{beam torsional stiffness}}{\text{slab flexural stiffness}} = \frac{E_b C}{2E_s I_s}$$

where, C = torsional constant $= \sum \left(1 - \dfrac{0.63 l_s}{l_l}\right)\left(\dfrac{{l_s}^3 l_l}{3}\right)$

l_s = shorter span & l_l = longer span

4. Design Condition of Flat Plate according to Panel:



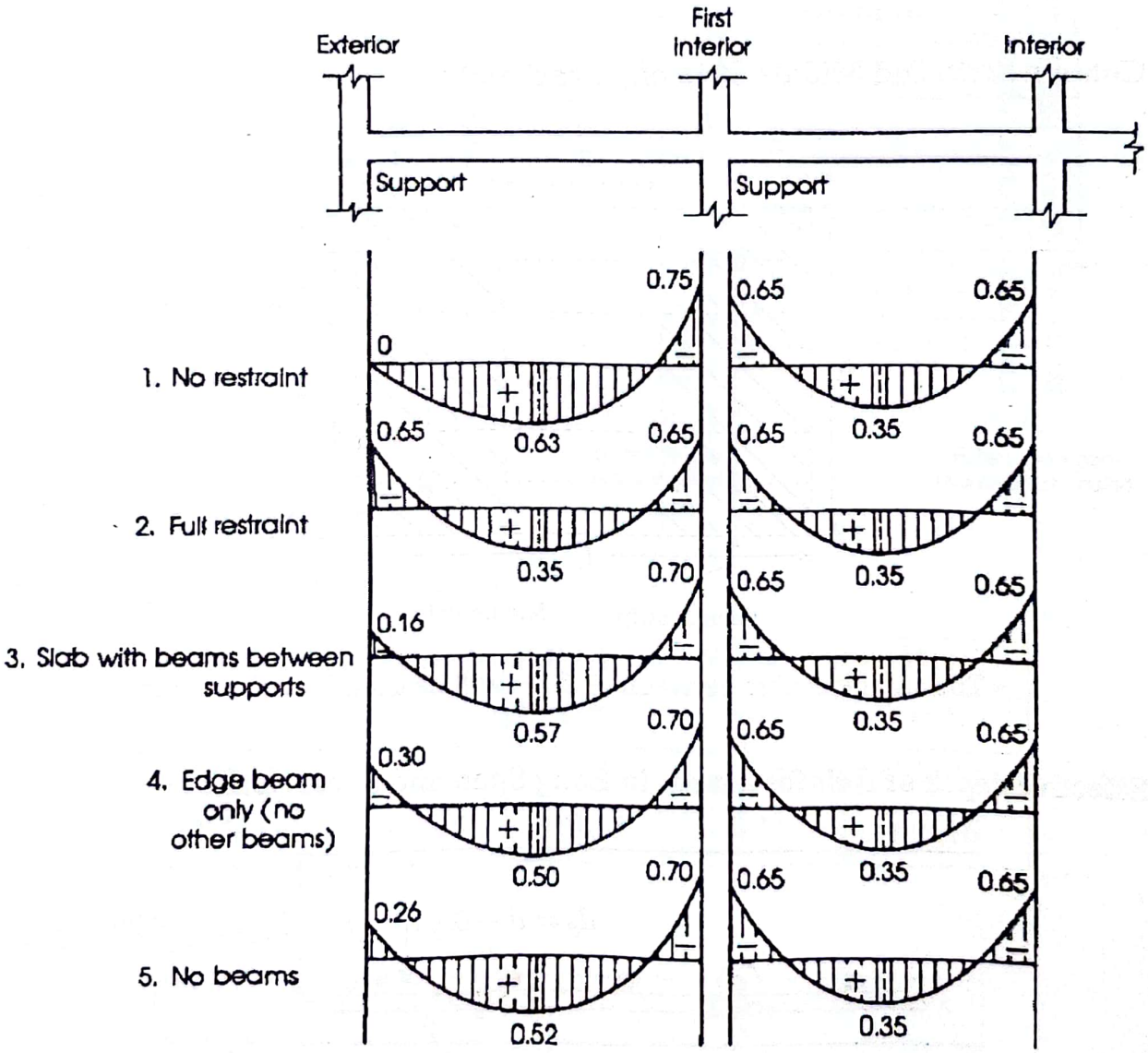
5. Minimum Thickness of Slabs without Interior Beams:

	Without Drop Panels†			With Drop Panels†		
	Exterior Panels		Interior Panels	Exterior Panels		Interior Panels
Yield strength, f_y, psi*	Without edge beams	With edge beams‡		Without edge beams	With edge beams‡	
40,000	$\frac{\ell_n}{33}$ [§]	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{40}$	$\frac{\ell_n}{40}$
60,000	$\frac{\ell_n}{30}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$
75,000	$\frac{\ell_n}{28}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{34}$	$\frac{\ell_n}{34}$

* The value of α_f shall not be less than 0.8.

6. Distribution of Total Span Moment in an End Span:

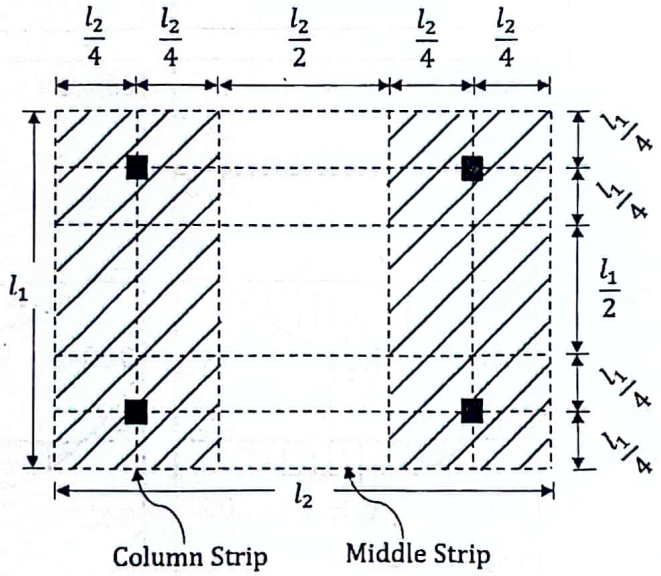
	Exterior Edge		Slab with Beams Between All Supports	Slab Without Beams Between Interior Supports	
	Unrestrained (1)	Fully Restrained (2)	(3)	With Edge Beam (4)	Without Edge Beam (5)
Exterior negative factored moment	0	0.65	0.16	0.30	0.26
Positive factored moment	0.63	0.35	0.57	0.50	0.52
Interior negative factored moment	0.75	0.65	0.70	0.70	0.70



7. Percentages of Moments in Column Strips:

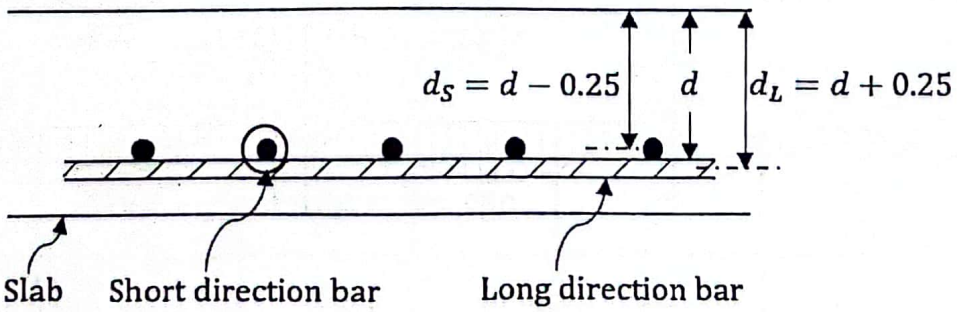
	$\alpha_{f_1} l_2/l_1$	β_t	Aspect Ratio l_2/l_1 0.5	1.0	2.0
Negative moment at exterior support	0	0	100	100	100
		≥2.5	75	75	75
	≥1.0	0	100	100	100
		≥2.5	90	75	45
Positive moment near midspan	0		60	60	60
	≥1.0		90	75	45
Negative moment at interior support	0		75	75	75
	≥1.0		90	75	45

8. Column Strip and Middle Strip of a Panel Slab:

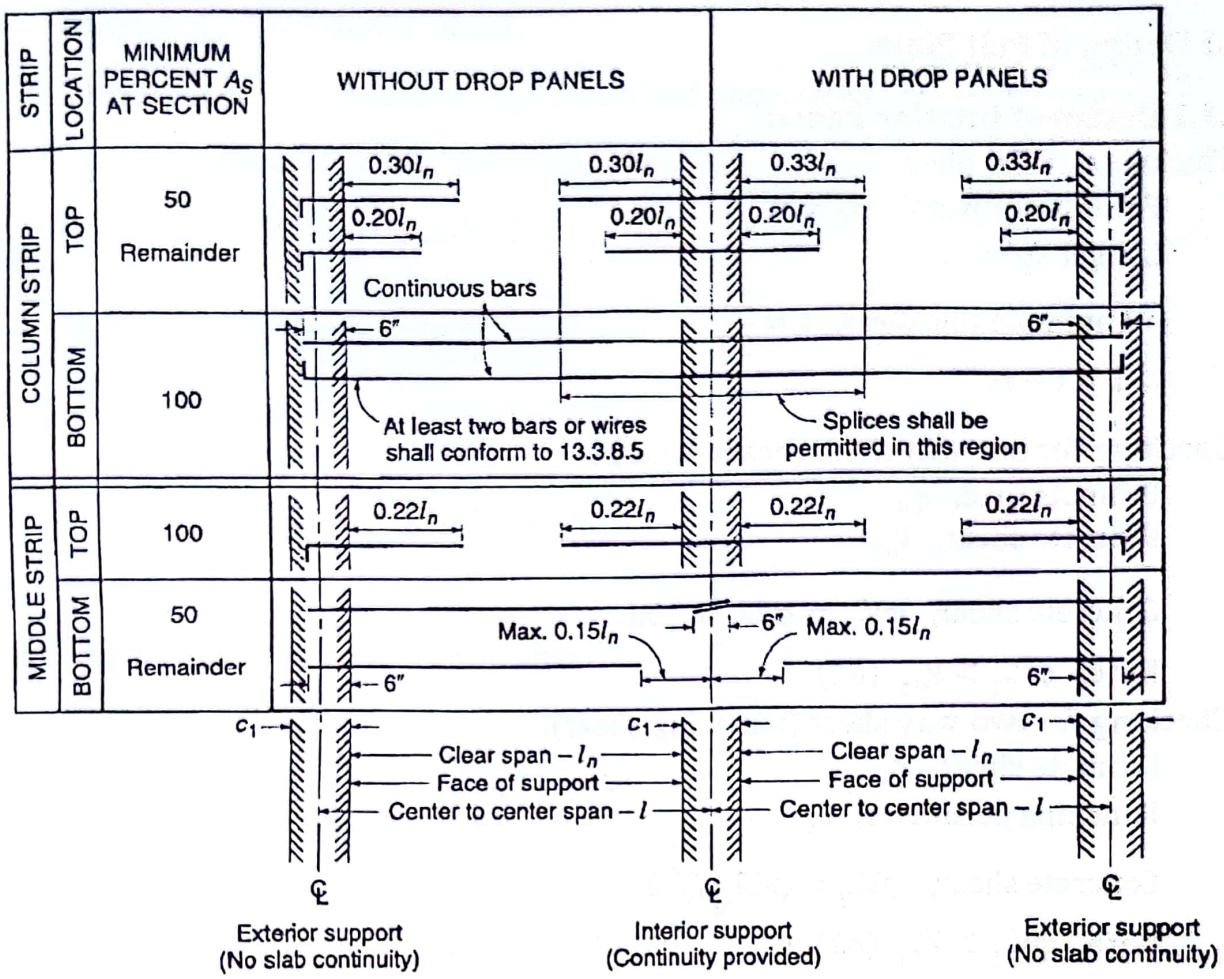


* The smaller value between $\frac{l_1}{4}$ & $\frac{l_2}{4}$ will be used for column strip.

9. Effective depth of Reinforcement in Long Span and Short Span:



10. Reinforcement of Slabs without Beams:



11. Moment Coefficient for Flat Slab:

Strip	With Drop Panel		Without Drop Panel	
	(-)	(+)	(-)	(+)
Column strip	- 0.50	+ 0.20	- 0.46	+ 0.22
Middle strip	- 0.15	+ 0.15	- 0.16	+ 0.16

12. Concrete Aggregate Factor, λ:

Aggregate type	Aggregate factor, λ
Normal-weight concrete	1.0
Sand, CA light-weight concrete	0.85
All light-weight concrete	0.75

2.3 Solution Techniques

2.3.1 Design of Flat Plate:

2.3.1.1 Design of Interior Panel:

1. Thickness of flat plate:

For interior panel, $\alpha_f = 0$

Longer span, l_n

$\therefore$ Minimum thickness, $t = \frac{l_n}{33}$

$\therefore d = t - cc$

2. Checking for one way shear (beam shear):

Ultimate load, q_u

Ultimate shear, V_{u_1}

Concrete shear, $\varphi V_{c_1} = \varphi 2\lambda \sqrt{f_c'}\, bd$

Here, $\varphi V_{c_1} > V_{u_1}$ (ok)

3. Checking for two way shear (punching shear):

Ultimate shear, V_{u_2}

Punching perimeter, b_o

Concrete shear, $\varphi V_{c_2} = \varphi 4\lambda \sqrt{f_c'}\, b_o d$

Here, $\varphi V_{c_1} > V_{u_1}$ (ok)

4. Static moments:

$$M_o = \frac{q_u L {l_n}^2}{8}$$

$$\therefore M_{oL} = \frac{q_u L_2 {l_n}^2}{8}$$

$$\therefore M_{oS} = \frac{q_u L_1 {l_n}^2}{8}$$

5. Summary of moments and steel calculation:

Long span, M_{oL}; $d = d_{provided} + 0.25$

Short span, M_{oS}; $d = d_{provided} - 0.25$

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}; \quad a = \frac{A_s f_y}{0.85 f_c' b}$$

Strip	Moment coefficients	M_u (k – ft.)	$A_s(in^2)$	$A_s(min) = 0.0018bt$ (in^2)	Reinforcement

6. Reinforcement detailing:

2.3.1.2 Design of exterior panel:

1. Calculation of α_f for exterior panel:

Centroid of cross – hatched edge beam and slab section $= \dfrac{A_s \times y_s + A_b \times y_b}{A_s + A_b}$

Gross moment of inertia of beam and slab, I_b

Gross moment of inertia slab, I_s

For exerior panel, $\alpha_f = \dfrac{E_b I_b}{E_s I_s}$

2. Thickness of flat plate:

Longer span, l_n

$\therefore$ Minimum thickness, $t = \dfrac{l_n}{33} \geq 5''$

$\therefore$ d = t – cc

3. Checking for one way shear (beam shear):

Ultimate load, q_u

Ultimate shear, V_{u_1}

Concrete shear, $\varphi V_{c_1} = \varphi 2\lambda \sqrt{f_c'}\, bd$

Here, $\varphi V_{c_1} > V_{u_1}$ (ok)

4. Checking for two way shear (punching shear):

Ultimate shear, V_{u_2}

Punching perimeter, b_o

Concrete shear, $\varphi V_{c_2} = \varphi 4\lambda \sqrt{f_c'}\, b_o d$

Here, $\varphi V_{c_1} > V_{u_1}$ (ok)

5. Static moments:

$$M_o = \frac{q_u L {l_n}^2}{8}$$

$$\therefore M_{oL} = \frac{q_u L_2 {l_n}^2}{8}$$

$$\therefore M_{oS} = \frac{q_u L_1 {l_n}^2}{8}$$

6. Moment coefficients:

Interior negative factored moment with edge beam = 0.70

Positive factored moment with edge beam = 0.50

Exterior negative factored moment with edge beam = 0.30

For, $\alpha_f = 1.47$ & $\dfrac{l_2}{l_1}$ and from interpolation,

Percentages of interior negative moments taken by column strip =
Percentages of exterior negative moments taken by column strip =
Percentages of positive moments taken by column strip =

7. Summary of moments and steel calculation:

Long span, M_{oL}; $d = d_{provided} + 0.25$

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}; \quad a = \frac{A_s f_y}{0.85 f_c' b}$$

Strip	Moment coefficients	M_u (k – ft.)	$A_s(in^2)$	$A_s(min) = 0.0018bt$ (in^2)	Reinforcement

8. Reinforcement detailing:

2.3.2 Design of Flat Slabs – WSD Method:

1. Load calculation:

Assume, t_2

∴ Load, $w' = LL + DL$

2. Thickness of flat slab (t_2):

(i) $t_2 = 4\ in.$

(ii) $t_2 = \frac{L}{40}$

(iii) $t_2 = 0.024L\left(1 - \frac{2C}{3L}\right)\sqrt{\frac{w'}{f_c'/2000}} + 1$

Let, t_2 =

And, thickness of drop = $3\ in.$

∴ $t_1 = t_2 + 3 \leq 1.5\, t_2$

3. Moment calculation:

Total load on panel including drop, W =

$$F' = \left(1.15 - \frac{C}{L}\right) \geq 1$$

Moment, $M_o = 0.09WLF'\left(1 - \frac{2C}{3L}\right)^2$

Column strip:

(–)ve moment = $-0.50M_o$

(+)ve moment = $+0.20M_o$

Middle strip:

(−)ve moment $= -0.15M_o$
(+)ve moment $= +0.15M_o$

4. Check for depth by moment:

(a) For d_1 with max. (-)ve moment:

Design constants:

$$n = \frac{E_s}{E_c};\ r = \frac{f_s}{f_c};\ k = \frac{n}{n+r};\ j = 1 - \frac{k}{3}$$

Here, $b_1 = \frac{3}{4}$th of the drop panel

Now, $M = \frac{1}{2} f_c k j b_1 {d_1}^2$

$\therefore d_1 =$

$d_1(\text{provided}) = t_1 - cc \geq d_1\ \ (ok)$

(b) For d_2 with max. (+)ve moment:

Here, $b_2 = \frac{3}{4}$th of the middle strip $= \frac{3}{4} \times \frac{l_2}{2}$

Now, $M = \frac{1}{2} f_c k j b_2 {d_2}^2$

$\therefore d_2 =$

$d_2(\text{provided}) = t_2 - cc \geq d_2\ \ (ok)$

5. Check for depth by shear:

(a) Punching shear:

(i) For 1st critical section: The 1st critical section is attained at a distance $\frac{d_1}{2}$ from the face of the column capital.

Dia. of critical section = (Dia. of column capital + d_1)

Developed shear, V_{dev}
Punching perimeter, b_o

Allowable shear, $V_{all} = 2\sqrt{f_c'}\, b_o d_1$

Here, $V_{all} > V_{dev}\ \ (ok)$

(ii) For 2nd critical section: The 2nd critical section is attained at a distance $\frac{d_2}{2}$ from the face of the drop panel.

Side of critical section = (Side of drop + d_2)

Developed shear, V_{dev}
Punching perimeter, b_o

Allowable shear, $V_{all} = 2\sqrt{f_c'}\, b_o d_1$

Here, $V_{all} > V_{dev}$ (ok)

(b) Beam shear:

Side of equivalent square, $a = \sqrt{\frac{\pi d^2}{4}}$

Side of critical section $= \left((11 - d_2 - \frac{a}{2}\right)$

Developed shear, V_{dev}

Allowable shear, $V_{all} = 2\sqrt{f_c'}\, bd_2$

Here, $V_{all} > V_{dev}$ (ok)

5. Steel calculation:

Strip	Location	Moment, M (k – ft.)	$A_s = \frac{M}{f_s jd}$ (in^2)	Reinforcement

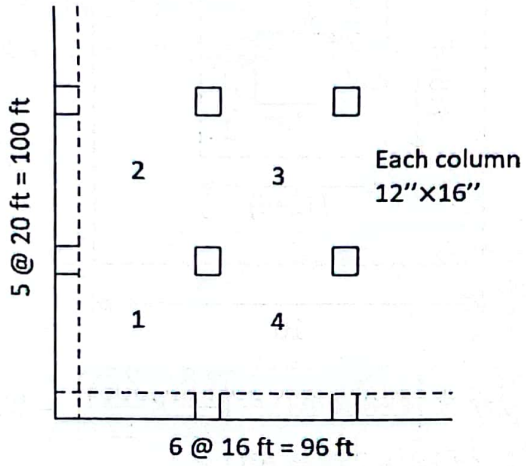
6. Reinforcement detailing:

2.4 Problem Solution

2.4.1 Design Flat Plate

2.4.1.1 Design of Interior Panel

Pb-1. Design an interior flat plate (panel 3) for the structure shown in figure below. Assume a service live load of 80 psf, a service dead load of 110 psf including self-weight. Use materials of $f_c' = 4000$ psi and $f_y = 60{,}000$ psi, and normal weight concrete.



Sol^n:

1. Thickness of flat plate:

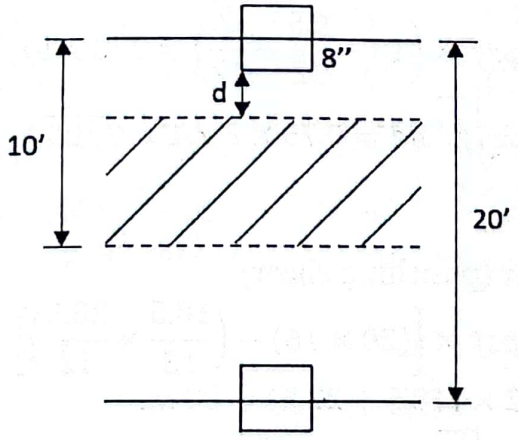
For interior panel, $\alpha_f = 0$

Longer span, $l_n = 20 - \frac{16}{12} = 18.67'$

$\therefore$ Minimum thickness, $t = \frac{l_n}{33} = \frac{18.67 \times 12}{33} = 6.79'' \approx 7'' \geq 5''$

$\therefore d = t - cc = 7 - 1 = 6''$

2. Checking for one way shear (beam shear):



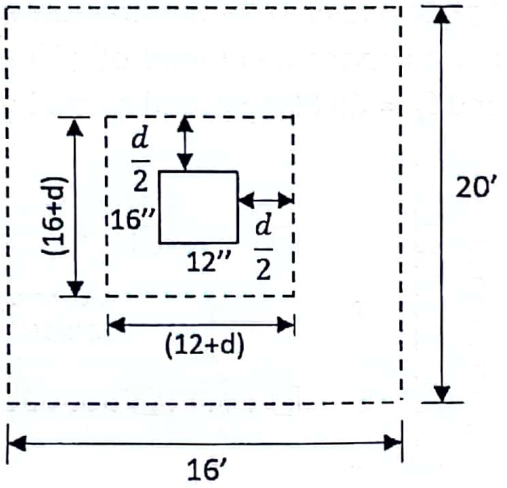
Ultimate load, $q_u = 1.2 * 110 + 1.6 * 80 = 260\ psf$

Ultimate shear, $V_{u_1} = 260\ psf \times \left(10 - \frac{6}{12} - \frac{8}{12}\right) = 2297\ lb/ft$

Concrete shear, $\varphi V_{c_1} = \varphi 2\lambda \sqrt{f_c'}\, bd = 0.75 \times 2 \times 1 \times \sqrt{3000} \times 12 \times 6 = 5915\ lb/ft$

Here, $\varphi V_{c_1} > V_{u_1}$ (ok)

3. Checking for two way shear (punching shear):



Ultimate shear, $V_{u_2} = 260\ psf \times \left[(20 \times 16) - \left(\frac{18}{12} \times \frac{22}{12}\right)\right] = 82485\ lb$

Punching perimeter, $b_o = 2 \times (18 + 22) = 80\ in.$

Concrete shear, $\varphi V_{c_2} = \varphi 4\lambda \sqrt{f_c'}\, b_o d = 0.75 \times 4 \times 1 \times \sqrt{3000} \times 80 \times 6 = 78872\ lb/ft$

Here, $\varphi V_{c_1} < V_{u_1}$ $(Not\ ok)$

4. Revised thickness:

$\therefore$ Minimum thickness, $t = 7.5''$

$\therefore$ d = t − cc = 7.5 − 1 = 6.5″

Checking for one way shear (beam shear):

Ultimate load, $q_u = 1.2 * 110 + 1.6 * 80 = 260\ psf$

Ultimate shear, $V_{u_1} = 260\ psf \times \left(10 - \frac{6.5}{12} - \frac{8}{12}\right) = 2286\ lb/ft$

Concrete shear, $\varphi V_{c_1} = \varphi 2\lambda \sqrt{f_c'}\, bd = 0.75 \times 2 \times 1 \times \sqrt{3000} \times 12 \times 6.5 = 6408\ lb/ft$

Here, $\varphi V_{c_1} > V_{u_1}$ (ok)

Checking for two way shear (punching shear):

Ultimate shear, $V_{u_2} = 260\ psf \times \left[(20 \times 16) - \left(\frac{18.5}{12} \times \frac{22.5}{12}\right)\right] = 82448\ lb$

Punching perimeter, $b_o = 2 \times (18.5 + 22.5) = 82\ in.$

Concrete shear, $\varphi V_{c_2} = \varphi 4\lambda \sqrt{f_c'}\, b_o d = 0.75 \times 4 \times 1 \times \sqrt{3000} \times 82 \times 6.5 = 87581\ lb/ft$

Here, $\varphi V_{c_1} > V_{u_1}$ (ok)

5. Static moments:

$$M_o = \frac{q_u\, L\, {l_n}^2}{8}$$

$$\therefore M_{oL} = \frac{q_u\, L_2\, {l_n}^2}{8} = \frac{260 \times 16 \times \left(20 - \frac{16}{12}\right)^2}{8} = 181.2\ k-ft$$

$$\therefore M_{oS} = \frac{q_u\, L_1\, {l_n}^2}{8} = \frac{260 \times 20 \times \left(16 - \frac{12}{12}\right)^2}{8} = 146.2\ k-ft$$

6. Summary of moments and steel calculation:

Long span, $M_{oL} = 181.2\ k-ft;\ d = 6.5 + 0.25 = 6.75\ in.$

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)};\quad a = \frac{A_s f_y}{0.85 f_c{}' b}$$

Strip	Moment coefficients		M_u (k – ft.)	$A_s(in^2)$	$A_s(min) = 0.0018bt$ (in^2)	Reinforcement
Column strip (b = 8 ft.)	(-)	(0.65) (0.75)	- 88.4	***2.92***	1.30	15 - #4 bars
	(+)	(0.35) (0.60)	+ 38.1	***1.33***	1.30	7 - #4 bars
Middle strip (b = 8 ft.)	(-)	(0.65) (0.25)	- 29.4	0.992	***1.30***	7 - #4 bars
	(+)	(0.35) (0.40)	+ 25.4	0.851	***1.30***	7 - #4 bars

Short span, $M_{oS} = 146.2\ k-ft;\ d = 6.5 - 0.25 = 6.25\ in.$

Strip	Moment coefficients		M_u (k – ft.)	$A_s(in^2)$	$A_s(min) = 0.0018bt$ (in^2)	Reinforcement
Column strip (b = 8 ft.)	(-)	(0.65) (0.75)	- 71.3	***2.31***	1.30	12 - #4 bars
	(+)	(0.35) (0.60)	+ 30.7	1.15	***1.30***	7 - #4 bars
Middle strip (b = 12 ft.)	(-)	(0.65) (0.25)	- 23.8	0.86	***1.94***	10 - #4 bars
	(+)	(0.35) (0.40)	+ 20.5	0.74	***1.94***	10 - #4 bars

7. Reinforcement detailing:

Long span, $l_n = 18.67'$

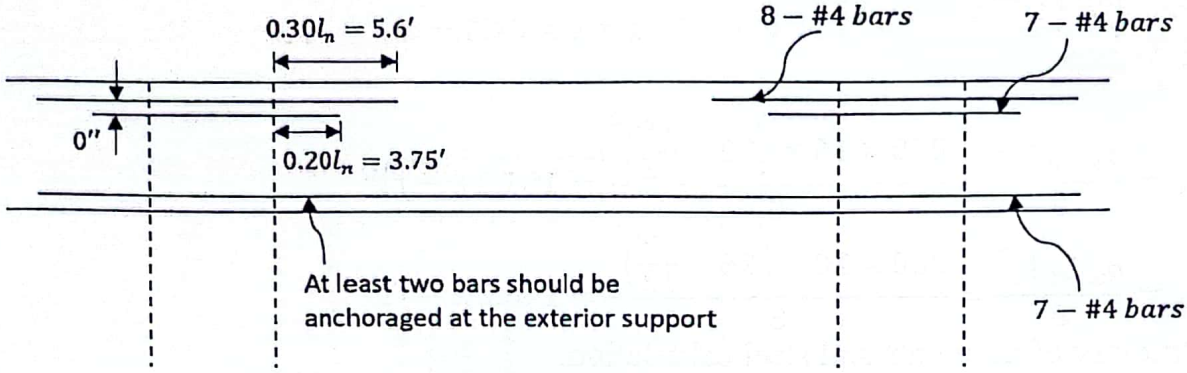


Figure: Column strip

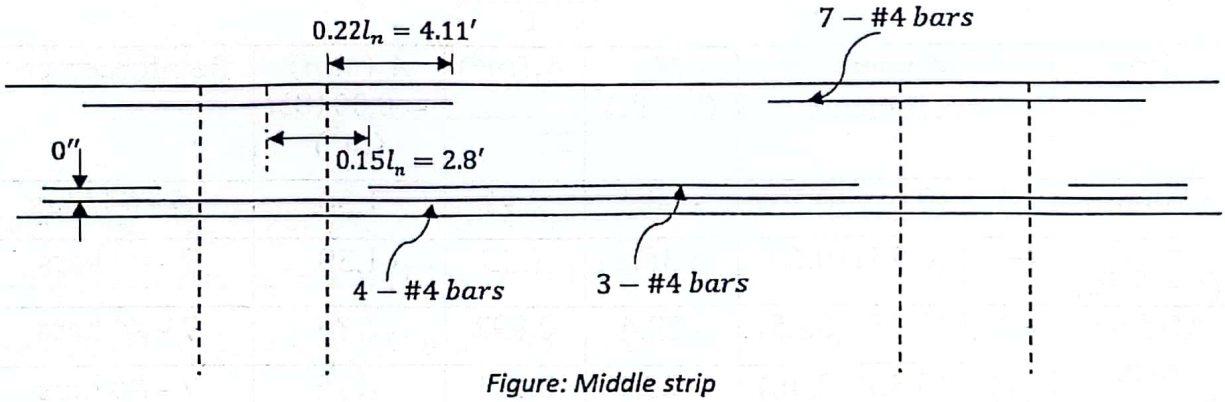


Figure: Middle strip

Pb-2. A flat plate floor system rests on $12'' \times 12''$ square columns spanning 16 ft. on centers in both directions. Design the interior column and middle strip of a typical interior panel. The floor has to carry a service live load of 80 psf, a floor finish of 25 psf and its own weight. Use $f_c' = 4000$ psi, $f_y = 60{,}000$ psi, and consider the followings – (i) Percentage of moment transferred to support = 65%, (ii) Percentage of negative moment taken by column strips = 75%, (ii) Percentage of positive moment taken by column strips = 60%. [*CE-2019, 2017*]

Sol^n:

1. Thickness of flat plate:

For interior panel, $\alpha_f = 0$

Longer span, $l_n = 16 - \frac{12}{12} = 15'$

$\therefore$ Minimum thickness, $t = \frac{l_n}{33} = \frac{15 \times 12}{33} = 5.45'' \approx 6'' \geq 5''$

$\therefore d = t - cc = 6 - 1 = 5''$

2. Load calculation:

Dead load:

Self weight of flat plate $= \frac{6}{12} \times 150 = 75\ psf$

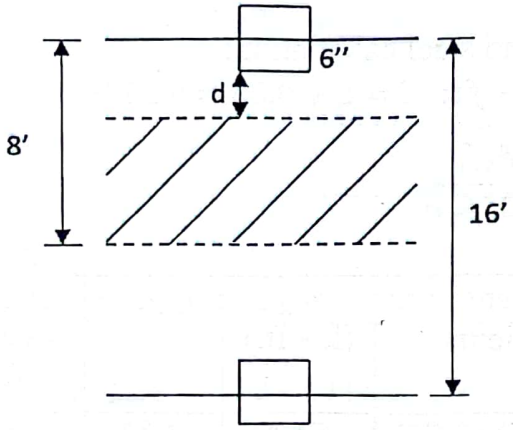
Floor finish $= 25$ psf

Live load:

Live load $= 80$ psf

Ultimate load, $q_u = 1.2 * (75 + 25) + 1.6 * 80 = 248\ psf$

3. Checking for one way shear (beam shear):

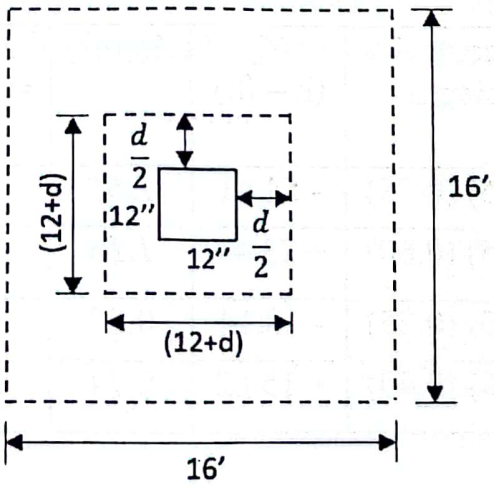


Ultimate shear, $V_{u_1} = 248\ psf \times \left(8 - \frac{5}{12} - \frac{6}{12}\right) = 1757\ lb/ft$

Concrete shear, $\varphi V_{c_1} = \varphi 2\lambda \sqrt{f_c'}\, bd = 0.75 \times 2 \times 1 \times \sqrt{4000} \times 12 \times 5 = 5692\ lb/ft$

Here, $\varphi V_{c_1} > V_{u_1}$ (ok)

4. Checking for two way shear (punching shear):



Ultimate shear, $V_{u_2} = 248\ psf \times \left[(16)^2 - \left(\frac{17}{12}\right)^2\right] = 62990\ lb$

Punching perimeter, $b_o = 2 \times (17 + 17) = 68\ in.$

Concrete shear, $\varphi V_{c_2} = \varphi 4\lambda \sqrt{f_c'}\, b_o d = 0.75 \times 4 \times 1 \times \sqrt{4000} \times 68 \times 5 = 64510\ lb/ft$

Here, $\varphi V_{c_2} > V_{u_2}\ (ok)$

5. Static moments:

$$M_o = \frac{q_u L\, l_n^{\,2}}{8}$$

$$\therefore M_{oL} = M_{oS} = \frac{q_u L_2\, l_n^{\,2}}{8} = \frac{248 \times 16 \times (15)^2}{8} = 111.6\ k-ft$$

6. Summary of moments and steel calculation:

Long span, $M_{oL} = 111.6\ k-ft;\ d = 5 + 0.25 = 5.25\ in.$

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)};\quad a = \frac{A_s f_y}{0.85 f_c' b}$$

Strip	Moment coefficients		M_u (k – ft.)	$A_s(in^2)$	$A_s(min) = 0.0018bt$ (in^2)	Reinforcement
Column strip (b = 8 ft.)	(-)	(0.65) (0.75)	- 54.41	***2.30***	1.04	12 - #4 bars
	(+)	(0.35) (0.60)	+ 23.44	***1.04***	1.04	6 - #4 bars
Middle strip (b = 8 ft.)	(-)	(0.65) (0.25)	- 18.14	0.78	***1.04***	6 - #4 bars
	(+)	(0.35) (0.40)	+ 15.62	0.67	***1.04***	6 - #4 bars

Short span, $M_{oS} = 111.6\ k-ft;\ d = 5 - 0.25 = 4.75\ in.$

Strip	Moment coefficients		M_u (k – ft.)	$A_s(in^2)$	$A_s(min) = 0.0018bt$ (in^2)	Reinforcement
Column strip (b = 8 ft.)	(-)	(0.65) (0.75)	- 54.41	***2.55***	1.04	13 - #4 bars
	(+)	(0.35) (0.60)	+ 23.44	***1.16***	1.04	6 - #4 bars
Middle strip (b = 8 ft.)	(-)	(0.65) (0.25)	- 18.14	0.87	***1.04***	6 - #4 bars
	(+)	(0.35) (0.40)	+ 15.62	0.74	***1.04***	6 - #4 bars

7. Reinforcement detailing:

Long span, $l_n = 15'$

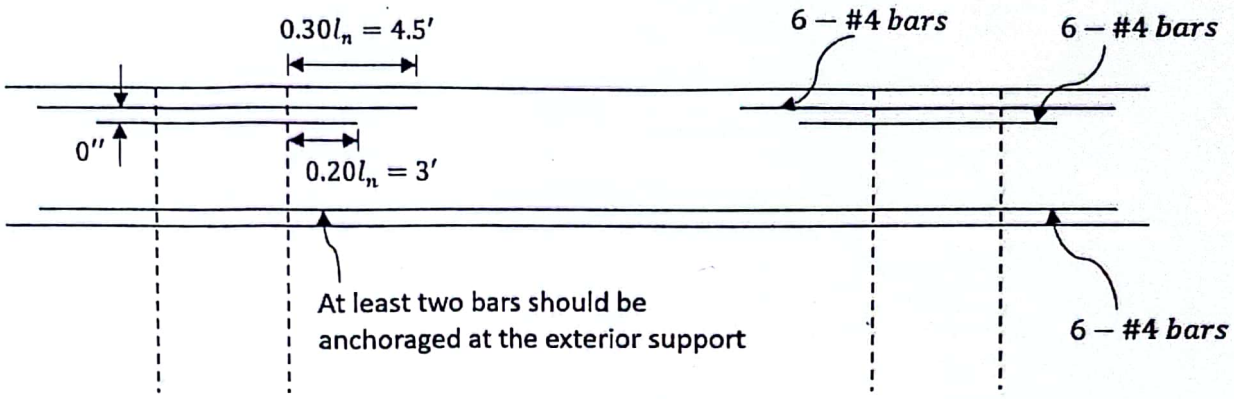


Figure: Column strip

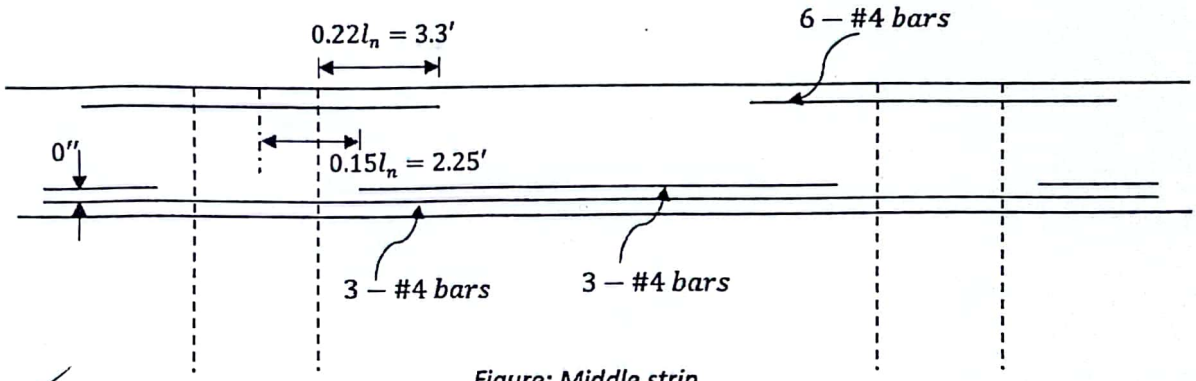


Figure: Middle strip

Pb-3. A flat plate floor system rests on $18'' \times 18''$ square columns spanning at 20 ft. on centers in both directions. Design the interior column and middle strip of a typical interior panel. The floor has to carry a service live load of 100 psf, a floor finish of 30 psf and its own weight. Use $f_c' = 3000$ psi and $f_y = 60{,}000$ psi. Consider – (i) Percentage of moment transferred to support = 65%, (ii) Percentage of negative moments taken by column strips = 75%, (ii) Percentage of positive moments taken by column strips = 60%. [***CE-2018***]

Sol^n:

1. Thickness of flat plate:

For interior panel, $\alpha_f = 0$

Longer span, $l_n = 20 - \frac{18}{12} = 18.5'$

$\therefore$ Minimum thickness, $t = \frac{l_n}{33} = \frac{18.5 \times 12}{33} = 7'' \approx 8'' \geq 5''$

$\therefore d = t - cc = 8 - 1 = 7''$

2. Load calculation:

Dead load:

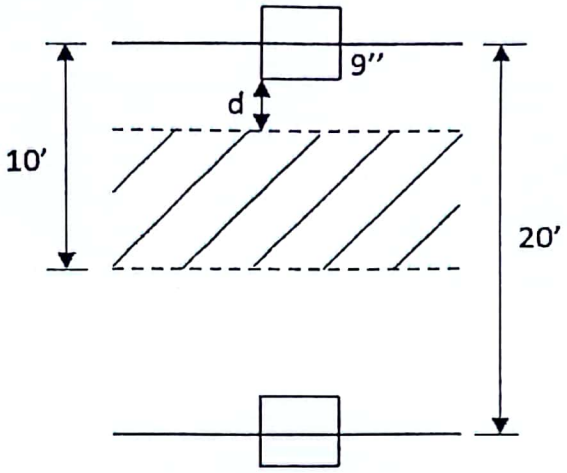
Self weight of flat plate $= \frac{8}{12} \times 150 = 100\ psf$

Floor finish = 30 psf

Live load:
Live load = 100 psf

Ultimate load, $q_u = 1.2 * (100 + 30) + 1.6 * 100 = 316\, psf$

3. Checking for one way shear (beam shear):

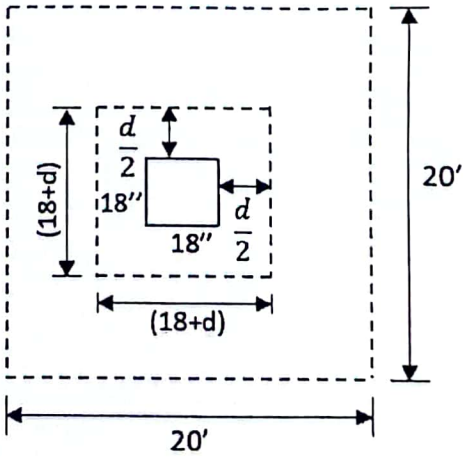


Ultimate shear, $V_{u_1} = 316\, psf \times \left(10 - \frac{7}{12} - \frac{9}{12}\right) = 2739\, lb/ft$

Concrete shear, $\varphi V_{c_1} = \varphi 2\lambda \sqrt{f_c'}\, bd = 0.75 \times 2 \times 1 \times \sqrt{3000} \times 12 \times 7 = 6901\, lb/ft$

Here, $\varphi V_{c_1} > V_{u_1}$ (ok)

4. Checking for two way shear (punching shear):



Ultimate shear, $V_{u_2} = 316\, psf \times \left[(20)^2 - \left(\frac{25}{12}\right)^2\right] = 125028\, lb$

Punching perimeter, $b_o = 2 \times (25 + 25) = 100\, in.$

Concrete shear, $\varphi V_{c_2} = \varphi 4\lambda \sqrt{f_c'}\, b_o d = 0.75 \times 4 \times 1 \times \sqrt{3000} \times 100 \times 7 = 115022\, lb/ft$

Here, $\varphi V_{c_1} < V_{u_1}$ $(Not\ ok)$

5. Revised thickness:
∴ Minimum thickness, $t = 9''$
∴ d = t − cc = 9 − 1 = 8″

Load calculation:
Dead load:

Self weight of flat plate $= \frac{9}{12} \times 150 = 112.5\ psf$

Floor finish = 30 psf
Live load:
Live load = 100 psf

Ultimate load, $q_u = 1.2 * (112.5 + 30) + 1.6 * 100 = 331\ psf$

Checking for one way shear (beam shear):

Ultimate shear, $V_{u_1} = 331\ psf \times \left(10 - \frac{8}{12} - \frac{9}{12}\right) = 2841\ lb/ft$

Concrete shear, $\varphi V_{c_1} = \varphi 2\lambda \sqrt{f_c'}\, bd = 0.75 \times 2 \times 1 \times \sqrt{3000} \times 12 \times 8 = 7887\ lb/ft$

Here, $\varphi V_{c_1} > V_{u_1}\ (ok)$

Checking for two way shear (punching shear):

Ultimate shear, $V_{u_2} = 331\ psf \times \left[(20)^2 - \left(\frac{26}{12}\right)^2\right] = 1{,}30{,}846\ lb$

Punching perimeter, $b_o = 2 \times (26 + 26) = 104\ in.$

Concrete shear, $\varphi V_{c_2} = \varphi 4\lambda \sqrt{f_c'}\, b_o d = 0.75 \times 4 \times 1 \times \sqrt{3000} \times 104 \times 8 = 1{,}36{,}712\ lb/ft$

Here, $\varphi V_{c_1} > V_{u_1}\ (ok)$

6. Static moments:

$$M_o = \frac{q_u L\, {l_n}^2}{8}$$

$$\therefore M_{oL} = M_{oS} = \frac{q_u L_2\, {l_n}^2}{8} = \frac{331 \times 20 \times (18.5)^2}{8} = 283.21\ k - ft$$

7. Summary of moments and steel calculation:
Long span, $M_{oL} = 283.21\ k - ft;\ d = 8 + 0.25 = 8.25\ in.$

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)};\quad a = \frac{A_s f_y}{0.85 f_c' b}$$

Strip	Moment coefficients		M_u (k – ft.)	$A_s(in^2)$	$A_s(min) = 0.0018bt$ (in^2)	Reinforcement
Column strip (b = 10 ft.)	(-)	(0.65) (0.75)	- 138.06	***3.72***	1.94	12 - #5 bars
	(+)	(0.35) (0.60)	+ 59.47	1.68	***1.94***	7 - #5 bars
Middle strip (b = 10 ft.)	(-)	(0.65) (0.25)	- 46.02	1.26	***1.94***	7 - #5 bars
	(+)	(0.35) (0.40)	+ 39.65	1.08	***1.94***	7 - #5 bars

Short span, $M_{os} = 111.6\,k-ft;\ d = 8 - 0.25 = 7.75\ in.$

Strip	Moment coefficients		M_u (k – ft.)	$A_s(in^2)$	$A_s(min) = 0.0018bt$ (in^2)	Reinforcement
Column strip (b = 10 ft.)	(-)	(0.65) (0.75)	- 138.06	***3.96***	1.94	13 - #5 bars
	(+)	(0.35) (0.60)	+ 59.47	1.80	***1.94***	7 - #5 bars
Middle strip (b = 10 ft.)	(-)	(0.65) (0.25)	- 46.02	1.35	***1.94***	7 - #5 bars
	(+)	(0.35) (0.40)	+ 39.65	1.16	***1.94***	7 - #5 bars

8. Reinforcement detailing:

Long span, $l_n = 18.5'$

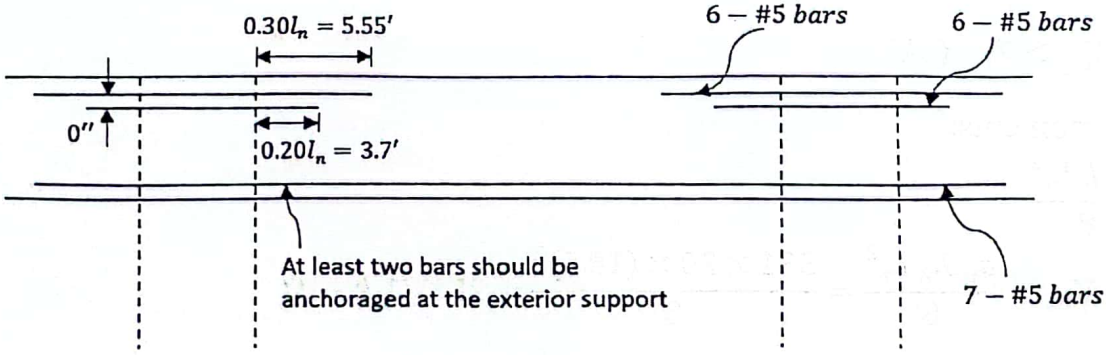


Figure: Column strip

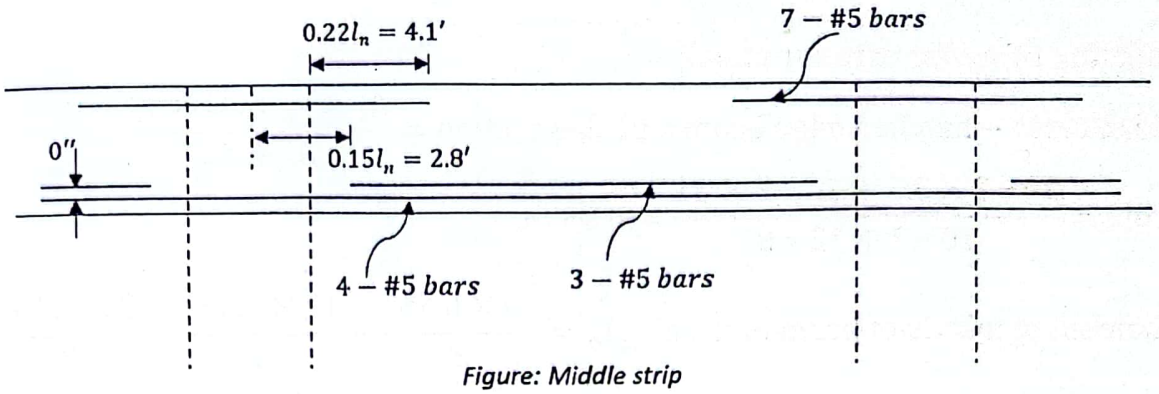
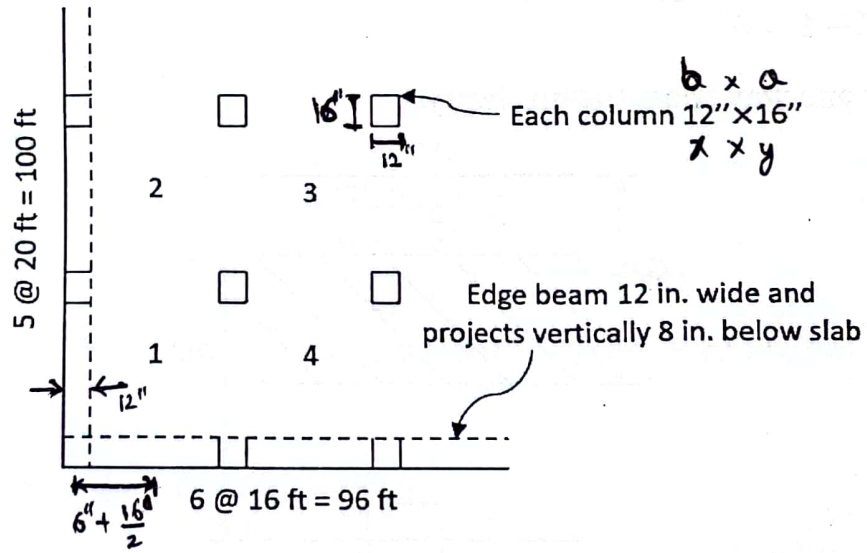


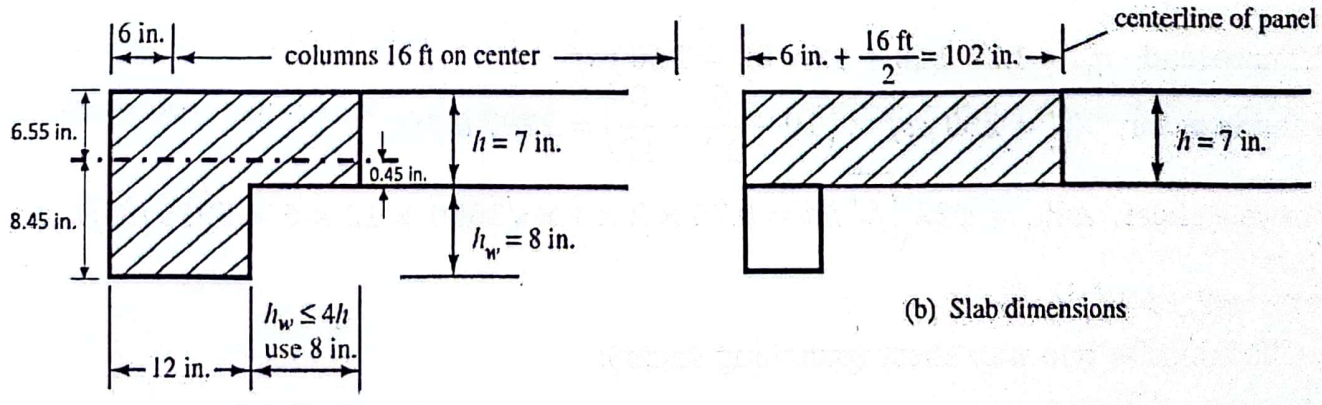
Figure: Middle strip

2.4.1.2 Design of Exterior Panel

Pb-4. Design an exterior flat plate (panel 1) for the structure shown in figure below. Edge beams are used around the building perimeter, and they are 12 in. wide and extend vertically for 8 in. below the plate. Assume a service live load of 80 psf, a service dead load of 110 psf including self-weight. Use materials of $f_c' = 4000$ psi and $f_y = 60{,}000$ psi, and normal weight concrete.



Sol^n:



(a) Edge beam dimensions

(b) Slab dimensions

1. Calculation of α_f for exterior panel:

Centroid of cross – hatched edge beam and slab section $= \dfrac{A_s \times y_s + A_b \times y_b}{A_s + A_b}$

$\therefore$ Centroid $= \dfrac{20 * 7 \times 3.5 + 12 * 8 \times 11}{20 * 7 + 12 * 8} = 6.55\ in.$

Gross moment of inertia of beam and slab, $I_b = \dfrac{20 \times 6.55^3}{3} + \dfrac{12 \times 8.45^3}{3} + \dfrac{8 \times 0.45^3}{3}$

$\therefore I_b = 4287\ in^4$

Gross moment of inertia slab, $I_s = \dfrac{102 \times 7^3}{3} = 2915.5\ in^4$

For exerior panel, $\alpha_f = \dfrac{E_b I_b}{E_s I_s} = \dfrac{E \times 4287}{E \times 2915.5} = 1.47 > 0.8$

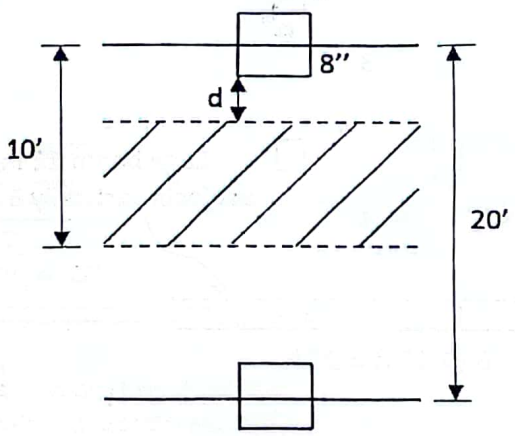
2. Thickness of flat plate:

Longer span, $l_n = 20 - \dfrac{16}{12} = 18.67'$

$\therefore$ Minimum thickness, $t = \dfrac{l_n}{33} = \dfrac{18.67 \times 12}{33} = 6.79'' \approx 7'' \geq 5''$

$\therefore$ d = t – cc = 7 – 1 = 6″

3. Checking for one way shear (beam shear):



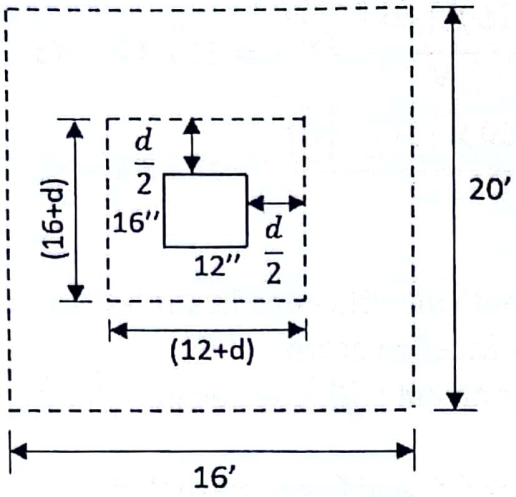
Ultimate load, $q_u = 1.2 * 110 + 1.6 * 80 = 260\ psf$

Ultimate shear, $V_{u_1} = 260\ psf \times \left(10 - \dfrac{6}{12} - \dfrac{8}{12}\right) = 2297\ lb/ft$

Concrete shear, $\varphi V_{c_1} = \varphi 2\lambda \sqrt{f_c'}\, bd = 0.75 \times 2 \times 1 \times \sqrt{3000} \times 12 \times 6 = 5915\ lb/ft$

Here, $\varphi V_{c_1} > V_{u_1}$ (ok)

4. Checking for two way shear (punching shear):



Ultimate shear, $V_{u_2} = 260\,psf \times \left[(20 \times 16) - \left(\frac{18}{12} \times \frac{22}{12}\right)\right] = 82485\,lb$

Punching perimeter, $b_o = 2 \times (18 + 22) = 80\,in.$

Concrete shear, $\varphi V_{c_2} = \varphi 4\lambda \sqrt{f_c'}\, b_o d = 0.75 \times 4 \times 1 \times \sqrt{3000} \times 80 \times 6 = 78872\,lb/ft$

Here, $\varphi V_{c_1} < V_{u_1}$ $(Not\ ok)$

5. Revised thickness:

$\therefore$ Minimum thickness, $t = 7.5''$

$\therefore$ d = t − cc = 7.5 − 1 = 6.5″

Checking for one way shear (beam shear):

Ultimate load, $q_u = 1.2 * 110 + 1.6 * 80 = 260\,psf$

Ultimate shear, $V_{u_1} = 260\,psf \times \left(10 - \frac{6.5}{12} - \frac{8}{12}\right) = 2286\,lb/ft$

Concrete shear, $\varphi V_{c_1} = \varphi 2\lambda \sqrt{f_c'}\, bd = 0.75 \times 2 \times 1 \times \sqrt{3000} \times 12 \times 6.5 = 6408\,lb/ft$

Here, $\varphi V_{c_1} > V_{u_1}$ (ok)

Checking for two way shear (punching shear):

Ultimate shear, $V_{u_2} = 260\,psf \times \left[(20 \times 16) - \left(\frac{18.5}{12} \times \frac{22.5}{12}\right)\right] = 82448\,lb$

Punching perimeter, $b_o = 2 \times (18.5 + 22.5) = 82\,in.$

Concrete shear, $\varphi V_{c_2} = \varphi 4\lambda \sqrt{f_c'}\, b_o d = 0.75 \times 4 \times 1 \times \sqrt{3000} \times 82 \times 6.5 = 87581\,lb/ft$

Here, $\varphi V_{c_1} > V_{u_1}$ (ok)

6. Static moments:

$$M_o = \frac{q_u L {l_n}^2}{8}$$

$$\therefore M_{oL} = \frac{q_u L_2 {l_n}^2}{8} = \frac{260 \times 16 \times \left(20 - \frac{16}{12}\right)^2}{8} = 181.2\ k-ft$$

$$\therefore M_{oS} = \frac{q_u L_1 {l_n}^2}{8} = \frac{260 \times 20 \times \left(16 - \frac{12}{12}\right)^2}{8} = 146.2\ k-ft$$

7. Moment coefficients:

Interior negative factored moment with edge beam = 0.70

Positive factored moment with edge beam = 0.50

Exterior negative factored moment with edge beam = 0.30

For, $\alpha_f = 1.47$ & $\frac{l_2}{l_1} = \frac{16}{20} = 0.8$ and from interpolation,

Percentages of interior negative moments taken by column strip = 81%

Percentages of exterior negative moments taken by column strip = 81%

Percentages of positive moments taken by column strip = 81%

8. Summary of moments and steel calculation:

Long span, $M_{oL} = 181.2\ k-ft;\ d = 6.5 + 0.25 = 6.75\ in.$

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)};\quad a = \frac{A_s f_y}{0.85 f_c' b}$$

Strip	Moment coefficients		M_u (k – ft.)	$A_s(in^2)$	$A_s(min) = 0.0018bt$ (in^2)	Reinforcement
Interior column strip (b = 4 ft.)	(-)	(0.70) (0.81)	- 102.7	***3.53***	0.65	12 - #5 bars
	(+)	(0.50) (0.81)	+ 73.4	***2.77***	0.65	9 - #5 bars
Exterior column strip (b = 4 ft.)	(-)	(0.30) (0.81)	- 44.0	***1.61***	0.65	6 - #5 bars
	(+)	(0.50) (0.81)	+ 73.4	***2.77***	0.65	9 - #5 bars
Middle strip (b = 8 ft.)	(-)	(0.70) (0.19)	- 24.1	0.83	***1.30***	5 - #5 bars
	(+)	(0.50) (0.19)	+ 17.2	0.57	***1.30***	5 - #5 bars

9. Reinforcement detailing:

Long span, $l_n = 18.67'$

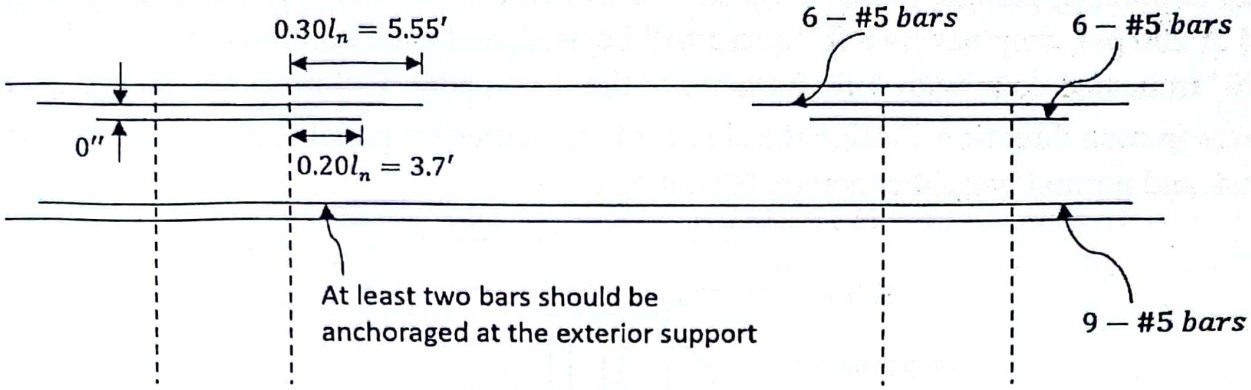


Figure: Interior Column strip

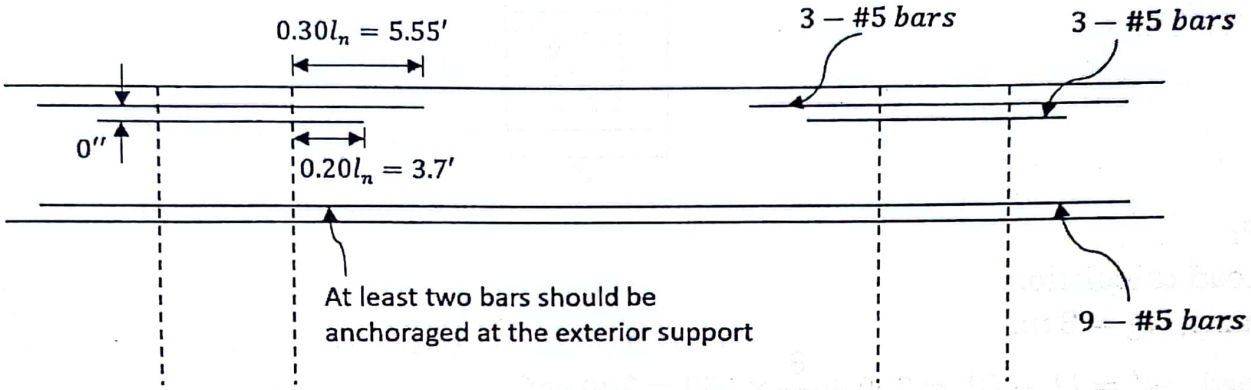


Figure: Exterior Column strip

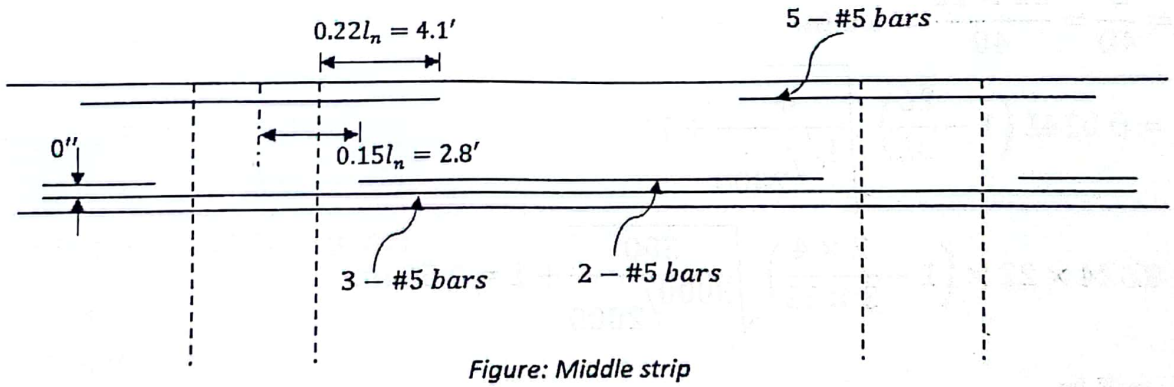
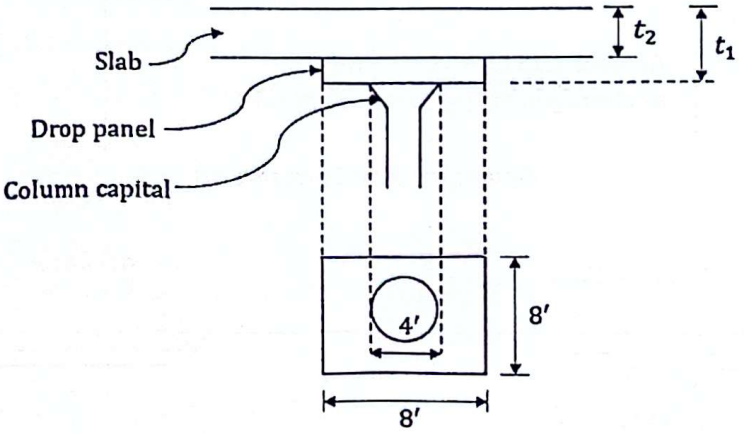


Figure: Middle strip

2.4.2 Design of Flat Slab – WSD Method

Pb-5. A parking garage, to be designed as a flat slab structure, is to carry a working live load of 200 psf, drop panels 8 ft. square will be used, and each column capital will consists of 90° truncated cone with 4 ft. diameter at the drop panel. Columns are spaced 22 ft. on centers in each direction. Design the slab for typical interior panel. Use $f_c' = 3$ ksi and $f_s = 20$ ksi, and normal weight concrete. [*CE-2018*]



Sol^n:

1. Load calculation:

Assume, $t_2 = 8\ in.$

$\therefore$ Load, $w' = LL + DL = 200 + \frac{8}{12} \times 150 = 300\ psf$

2. Thickness of flat slab (t_2):

(i) $t_2 = 4\ in.$

(ii) $t_2 = \frac{L}{40} = \frac{22 \times 12}{40} = 6.6\ in.$

(iii) $t_2 = 0.024L\left(1 - \frac{2C}{3L}\right)\sqrt{\frac{w'}{f_c'/2000}} + 1$

$\therefore t_2 = 0.024 \times 22 \times \left(1 - \frac{2 \times 4}{3 \times 22}\right)\sqrt{\frac{300}{3000/2000}} + 1 = 7.56\ in.$

Let, $t_2 = 8\ in.$

And, thickness of drop $= 3\ in.$

$\therefore t_1 = 8 + 3 = 11\ in. \leq 1.5\ t_2$

3. Moment calculation:

Total load on panel including drop, $W = \left(\frac{3}{12} \times 8 \times 8\right) * 150 + (22 \times 22) * 300$

$\therefore W = 147{,}600\ lb$

$$F' = \left(1.15 - \frac{C}{L}\right) \geq 1$$

$$\therefore F' = \left(1.15 - \frac{4}{22}\right) = 0.97 \approx 1$$

Moment, $M_o = 0.09WLF'\left(1 - \frac{2C}{3L}\right)^2$

$$\therefore M_o = 0.09 \times 147{,}600 \times 22 \times 1 \times \left(1 - \frac{2 \times 4}{3 \times 22}\right)^2 = 225{,}694\ lb - ft$$

Column strip:
(−)ve moment $= -0.50M_o = -112{,}847\ lb - ft$
(+)ve moment $= +0.20M_o = +45{,}139\ lb - ft$

Middle strip:
(−)ve moment $= -0.15M_o = -33{,}854\ lb - ft$
(+)ve moment $= +0.15M_o = +33{,}854\ lb - ft$

4. Check for depth by moment:

(a) For d_1 with max. (-)ve moment:

Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \approx 9$$

$$r = \frac{f_s}{f_c} = \frac{20}{0.45 \times 3} = \frac{20}{1.35} = 14.81$$

$$k = \frac{n}{n + r} = 0.378$$

$$j = 1 - \frac{k}{3} = 0.874$$

Here, $b_1 = \frac{3}{4}$th of the drop panel

Now, $M = \frac{1}{2} f_c k j b_1 {d_1}^2$

$$\Rightarrow 112.847 \times 12 = \frac{1}{2} \times 1.35 \times 0.378 \times 0.874 \times \left(\frac{3}{4} * 8 * 12\right) \times {d_1}^2$$

$$\therefore d_1 = 9.2\ in.$$

d_1(provided) $= t_1 - cc = 11 - 1.5 = 9.5\ in. \geq d_1$ (ok)

(b) For d_2 with max. (+)ve moment:

Here, $b_2 = \frac{3}{4}$th of the middle strip $= \frac{3}{4} \times \frac{l_2}{2}$

Now, $M = \frac{1}{2} f_c k j b_2 {d_2}^2$

$\Rightarrow 45.139 \times 12 = \frac{1}{2} \times 1.35 \times 0.378 \times 0.874 \times \left(\frac{3}{4} * 11 * 12\right) \times {d_2}^2$

$\therefore d_2 = 4.95\ in.$

$d_2(\text{provided}) = t_2 - cc = 8 - 1.5 = 6.5\ in. \geq d_2\ (ok)$

5. Check for depth by shear:

(a) Punching shear:

(i) For 1st critical section: The 1st critical section is attained at a distance $\frac{d_1}{2}$ from the face of the column capital.

Dia. of critical section = (Dia. of column capital + d_1) $= \left(4 + \frac{9.5}{12}\right) = 4.79\ ft$

Developed shear, $V_{dev} = 300\ psf \times \left[(22)^2 - \left(\frac{\pi \times 4.79^2}{4}\right)\right] = 139{,}800\ lb$

Punching perimeter, $b_o = \pi \times 4.79 \times 12 = 180.6\ in.$

Allowable shear, $V_{all} = 2\sqrt{f_c'}\, b_o d_1 = 2 \times \sqrt{3000} \times 180.6 \times 9.5 = 187{,}946\ lb$

Here, $V_{all} > V_{dev}\ (ok)$

(ii) For 2nd critical section: The 2nd critical section is attained at a distance $\frac{d_2}{2}$ from the face of the drop panel.

Side of critical section = (Side of drop + d_2) $= \left(8 + \frac{6.5}{12}\right) = 8.54\ ft$

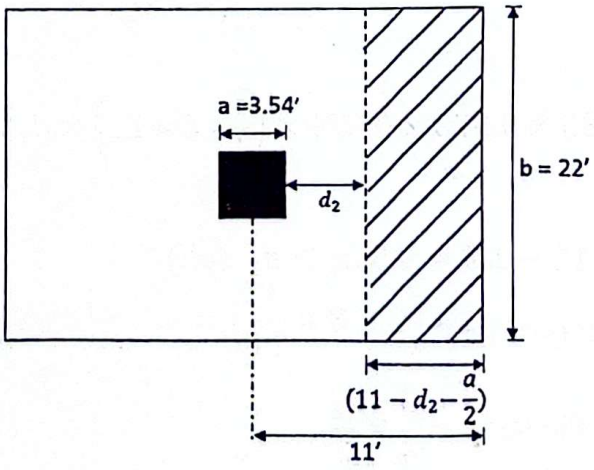
Developed shear, $V_{dev} = 300\ psf \times [(22)^2 - (8.54)^2] = 123{,}320\ lb$

Punching perimeter, $b_o = 4 \times 8.54 \times 12 = 409.92\ in.$

Allowable shear, $V_{all} = 2\sqrt{f_c'}\, b_o d_1 = 2 \times \sqrt{3000} \times 409.92 \times 6.5 = 291{,}879\ lb$

Here, $V_{all} > V_{dev}\ (ok)$

(b) Beam shear:



Side of equivalent square, $a = \sqrt{\frac{\pi d^2}{4}} = \sqrt{\frac{\pi \times 4^2}{4}} = 3.54\, ft$

Side of critical section $= \left((11 - d_2 - \frac{a}{2}\right) = \left(11 - \frac{6.5}{12} - \frac{3.54}{2}\right) = 8.69\, ft$

Developed shear, $V_{dev} = 300\, psf \times (22 \times 8.69) = 57{,}354\, lb$

Allowable shear, $V_{all} = 2\sqrt{f_c'}\, bd_2 = 2 \times \sqrt{3000} \times (22 * 12) \times 6.5 = 103{,}388\, lb$

Here, $V_{all} > V_{dev}$ (ok)

5. Steel calculation:

Strip	Location	Moment, M (k – ft.)	$A_s = \frac{M}{f_s jd}$ (in^2)	Reinforcement
Column Strip (d_1)	Top (-)ve	- 112.847	8.15	19 - #6 bars
	Bottom (+)ve	+ 45.139	3.26	11 - #5 bars
Middle strip (d_2)	Top (-)ve	- 33.854	3.58	9 - #6 bars
	Bottom (+)ve	+ 33.854	3.58	12 - #5 bars

6. Reinforcement detailing:

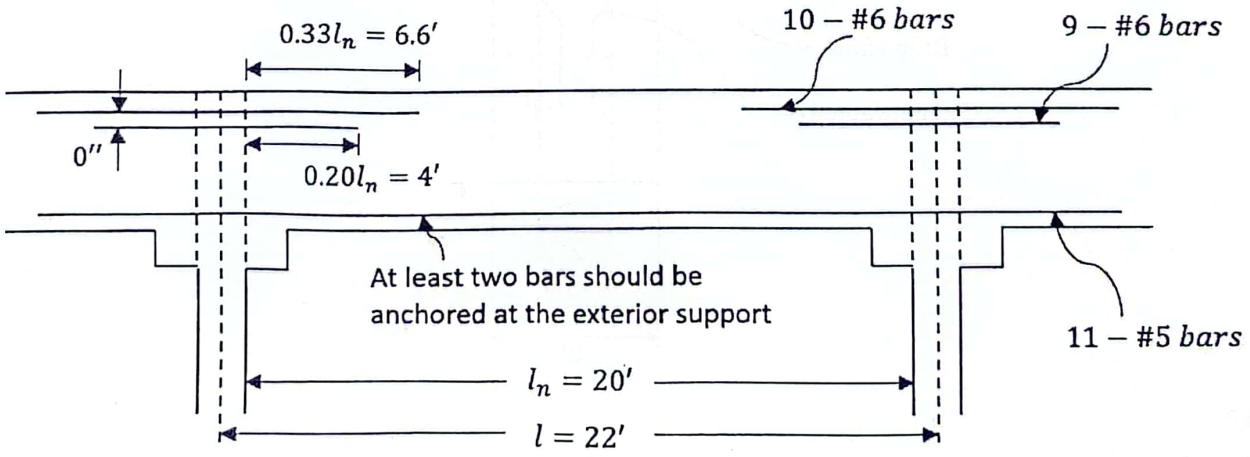


Figure: Column strip

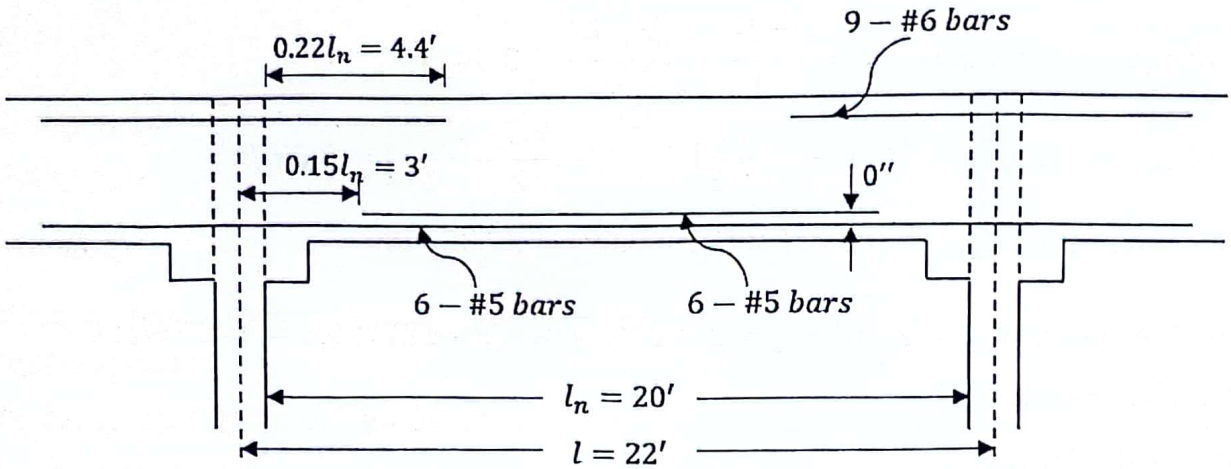
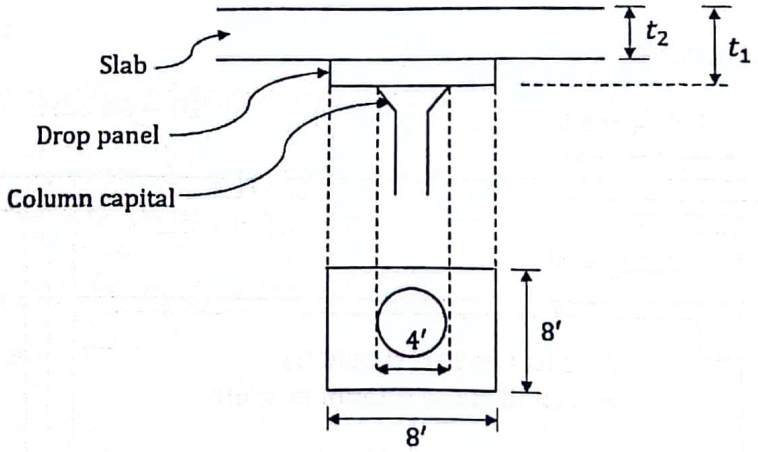


Figure: Middle strip

Pb-6. Design the column strip of a typical interior panel of a flat slab floor system by WSD method, which has to carry a service live load of 100 psf in addition to its own weight and typical floor finish on a 20 ft. span in either direction. Drop panels of 8 ft. square will be used in conjunction with column capitals in the form of 90° truncated cone. Diameter of column capital at the junction of drop panel is 4 ft. Use $f_c' = 4000$ psi and $f_y = 60{,}000$ psi. [*CE-2019*]



Sol^n:

1. Load calculation:

Assume, $t_2 = 8\ in.$

$\therefore$ Load, $w' = LL + DL = 100 + \frac{8}{12} \times 150 = 200\ psf$

2. Thickness of flat slab (t_2):

(i) $t_2 = 4\ in.$

(ii) $t_2 = \frac{L}{40} = \frac{20 \times 12}{40} = 6\ in.$

(iii) $t_2 = 0.024L\left(1 - \frac{2C}{3L}\right)\sqrt{\frac{w'}{f_c'/2000}} + 1$

$\therefore t_2 = 0.024 \times 20 \times \left(1 - \frac{2 \times 4}{3 \times 20}\right)\sqrt{\frac{200}{4000/2000}} + 1 = 5.16\ in.$

Let, $t_2 = 7\ in.$
And, thickness of drop $= 3\ in.$
$\therefore t_1 = 7 + 3 = 10\ in. \leq 1.5\ t_2$

3. Moment calculation:

Total load on panel including drop, $W = \left(\frac{3}{12} \times 8 \times 8\right) * 150 + (20 \times 20) * 300$

$\therefore W = 122{,}400\ lb$

$F' = \left(1.15 - \frac{C}{L}\right) \geq 1$

$\therefore F' = \left(1.15 - \frac{4}{20}\right) = 0.95 \approx 1$

Moment, $M_o = 0.09WLF'\left(1 - \frac{2C}{3L}\right)^2$

$\therefore M_o = 0.09 \times 122{,}400 \times 20 \times 1 \times \left(1 - \frac{2 \times 4}{3 \times 20}\right)^2 = 165{,}485\ lb - ft$

Column strip:
(−)ve moment $= -0.50M_o = -82{,}742\ lb - ft$
(+)ve moment $= +0.20M_o = +33{,}097\ lb - ft$

Middle strip:
(−)ve moment $= -0.15M_o = -24{,}823\ lb - ft$
(+)ve moment $= +0.15M_o = +24{,}823\ lb - ft$

4. Check for depth by moment:

(a) For d_1 with max. (-)ve moment:

Design constants:

$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8.04 \approx 8$

$r = \frac{f_s}{f_c} = \frac{0.4 \times 60}{0.45 \times 4} = \frac{24}{1.8} = 13.33$

$k = \frac{n}{n + r} = 0.375$

$$j = 1 - \frac{k}{3} = 0.875$$

Here, $b_1 = \frac{3}{4}$th of the drop panel

Now, $M = \frac{1}{2} f_c k j b_1 {d_1}^2$

$\Rightarrow 82.742 \times 12 = \frac{1}{2} \times 1.8 \times 0.375 \times 0.875 \times \left(\frac{3}{4} * 8 * 12\right) \times {d_1}^2$

$\therefore d_1 = 6.83\ in.$

$d_1(\text{provided}) = t_1 - cc = 10 - 1.5 = 8.5\ in. \geq d_1\ \ (ok)$

(b) For d_2 with max. (+)ve moment:

Here, $b_2 = \frac{3}{4}$th of the middle strip $= \frac{3}{4} \times \frac{l_2}{2}$

Now, $M = \frac{1}{2} f_c k j b_2 {d_2}^2$

$\Rightarrow 33.097 \times 12 = \frac{1}{2} \times 1.8 \times 0.375 \times 0.875 \times \left(\frac{3}{4} * 10 * 12\right) \times {d_2}^2$

$\therefore d_2 = 3.87\ in.$

$d_2(\text{provided}) = t_2 - cc = 7 - 1.5 = 5.5\ in. \geq d_2\ \ (ok)$

5. Check for depth by shear:

(a) Punching shear:

(i) For 1st critical section: The 1st critical section is attained at a distance $\frac{d_1}{2}$ from the face of the column capital.

Dia. of critical section = (Dia. of column capital + d_1) $= \left(4 + \frac{8.5}{12}\right) = 4.71\ ft$

Developed shear, $V_{dev} = 200\ psf \times \left[(20)^2 - \left(\frac{\pi \times 4.71^2}{4}\right)\right] = 76{,}515\ lb$

Punching perimeter, $b_o = \pi \times 4.71 \times 12 = 177.6\ in.$

Allowable shear, $V_{all} = 2\sqrt{f_c'}\, b_o d_1 = 2 \times \sqrt{4000} \times 177.6 \times 8.5 = 190{,}951\ lb$

Here, $V_{all} > V_{dev}\ \ (ok)$

(ii) For 2nd critical section: The 2nd critical section is attained at a distance $\frac{d_2}{2}$ from the face of the drop panel.

Side of critical section = (Side of drop + d_2) $= \left(8 + \frac{5.5}{12}\right) = 8.46\ ft$

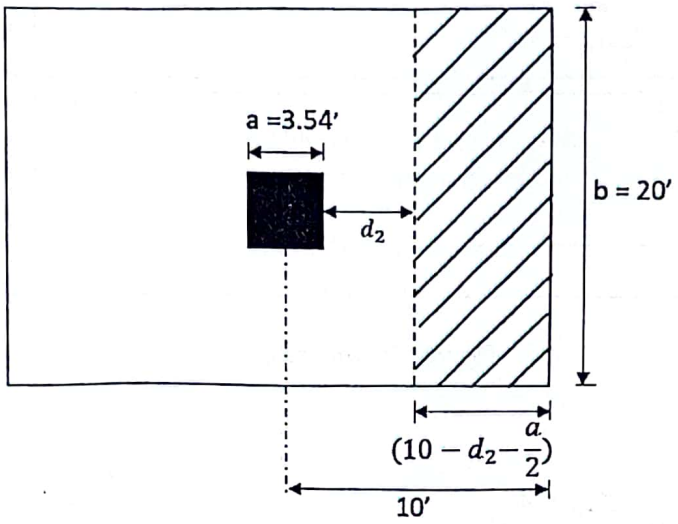
Developed shear, $V_{dev} = 200\ psf \times [(20)^2 - (8.46)^2] = 65{,}686\ lb$

Punching perimeter, $b_o = 4 \times 8.46 \times 12 = 406.08\ in.$

Allowable shear, $V_{all} = 2\sqrt{f_c'}\, b_o d_1 = 2 \times \sqrt{4000} \times 406.08 \times 5.5 = 282{,}510\ lb$

Here, $V_{all} > V_{dev}\ (ok)$

(b) Beam shear:



Side of equivalent square, $a = \sqrt{\frac{\pi d^2}{4}} = \sqrt{\frac{\pi \times 4^2}{4}} = 3.54\ ft$

Side of critical section $= \left((10 - d_2 - \frac{a}{2}\right) = \left(10 - \frac{5.5}{12} - \frac{3.54}{2}\right) = 5.48\ ft$

Developed shear, $V_{dev} = 200\ psf \times (20 \times 5.48) = 21{,}920\ lb$

Allowable shear, $V_{all} = 2\sqrt{f_c'}\, b d_2 = 2 \times \sqrt{4000} \times (20 * 12) \times 5.5 = 166{,}968\ lb$

Here, $V_{all} > V_{dev}\ (ok)$

5. Steel calculation:

Strip	Location	Moment, M (k – ft.)	$A_s = \frac{M}{f_s j d}$ (in^2)	Reinforcement
Column Strip (d_1)	Top (-)ve	- 82.742	5.56	18 - #5 bars
	Bottom (+)ve	+ 33.097	2.23	12 - #4 bars
Middle strip (d_2)	Top (-)ve	- 24.823	2.58	9 - #5 bars
	Bottom (+)ve	+ 24.823	2.58	13 - #4 bars

6. Reinforcement detailing:

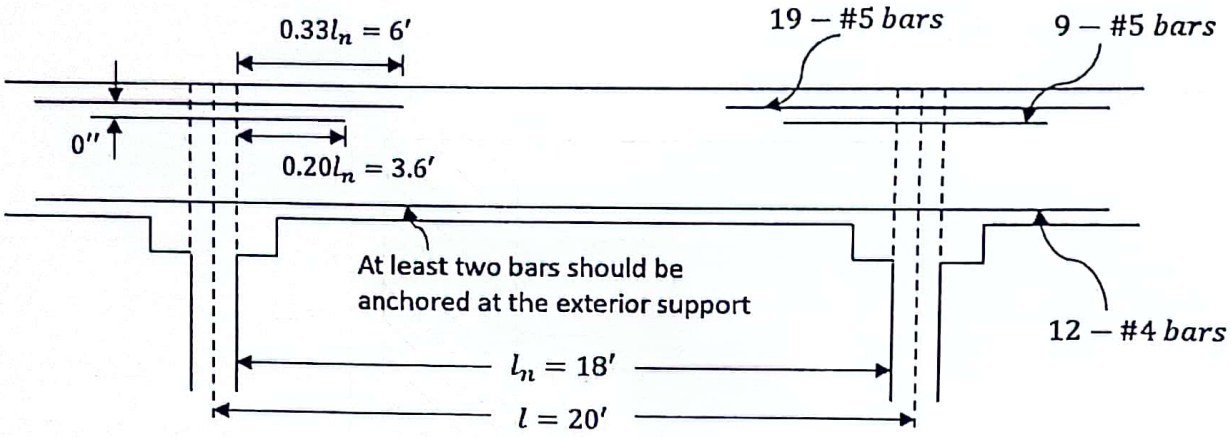


Figure: Column strip

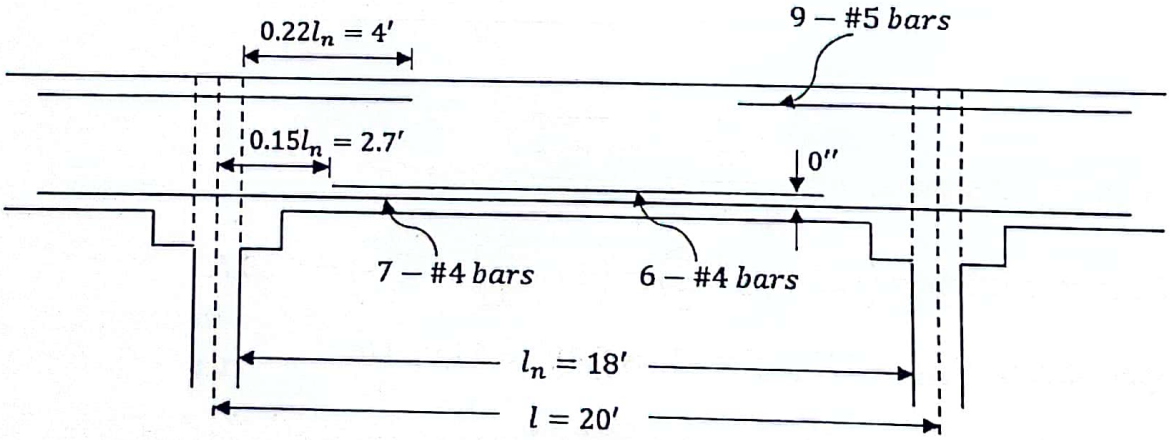


Figure: Middle strip

Chapter - 3
Footings and Foundations

3.1 Theory Analysis

Q-1. What are the basic differences between footings and foundations?

Ans: Foundation is a structure which transfers the loads from the superstructure to the ground, while footing is the foundation which is in contact with the earth. A foundation can be shallow and deep, while a footing is a type of a shallow foundation. *So, all footings are foundations but all foundations cannot be footings.*

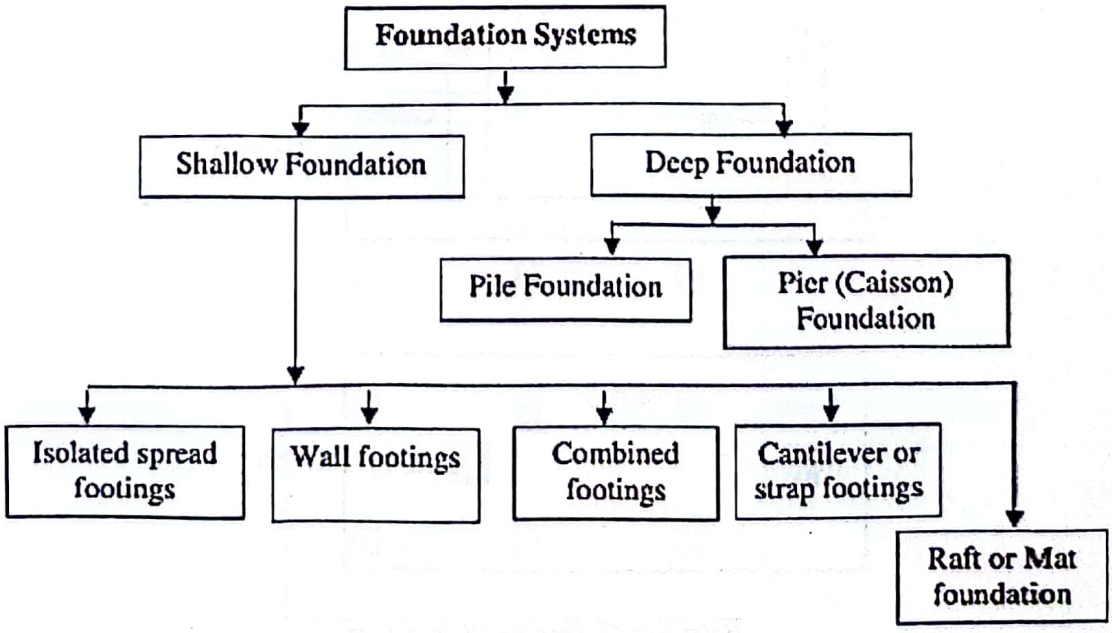
Bangla Terminology: Ground level থেকে উপরের building structure কে super structure এবং নিচের building structure কে sub structure বলা হয়। এই sub structure এর মধ্যে থাকে grade beams, columns, piles, footings। পুরো sub structure কে-ই এক কথায় foundation বলা হয়। আর footings হলো যে অংশটুকু সরাসরি soil এর contact-এ লেগে থাকে।

Q-2. What are the types of foundations based on depth? Describes the all types of foundation with neat sketches. [*CE-2019, 2018*]

Ans: There are two types of foundations based on the depth such as:

(a) Shallow foundation

(b) Deep foundation



(a) Shallow foundation: The shallow foundation is also known as the footings. Foundation which is placed near the surface of the earth or transfers the loads at shallow depth is called the shallow foundation. The ratio of depth to width is less than 1. The most common shallow foundation or footings are described below:

(i) Wall footings: It is an enlargement of the bottom of a wall that will sufficiently distribute the load to the foundation soil. Wall footings are normally used around the perimeter of a building and perhaps for some of the interior walls.

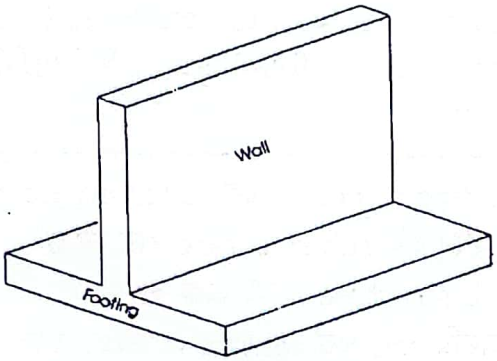


Figure: Wall footing

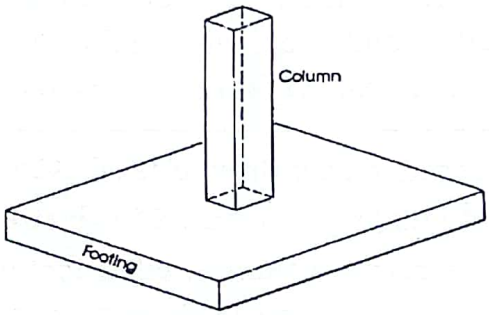


Figure: Single column footing

(ii) Isolated or single column footings: It is used to support the load of a single column. These are the most commonly used footings, particularly where the loads are relatively light and the columns are not closely spaced. They may be square, rectangular, or circular.

(iii) Combined footings: It usually supports two or three columns loads. Combined footings are used when two columns are so close that single footings cannot be used or when one column is located at or near a property line.

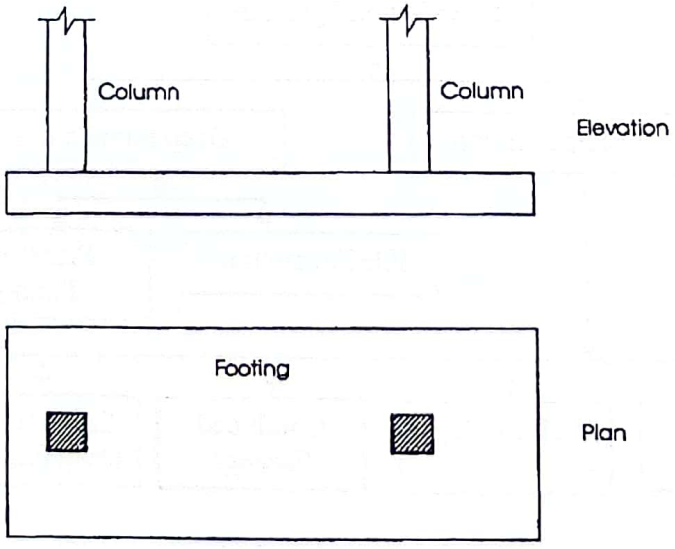


Figure: Combined footing

(iv) Cantilever or strap footings: It consists of two single column footings connected with a single beam or a strap and support two single columns. This types replaces combined footing and is sometimes more economical.

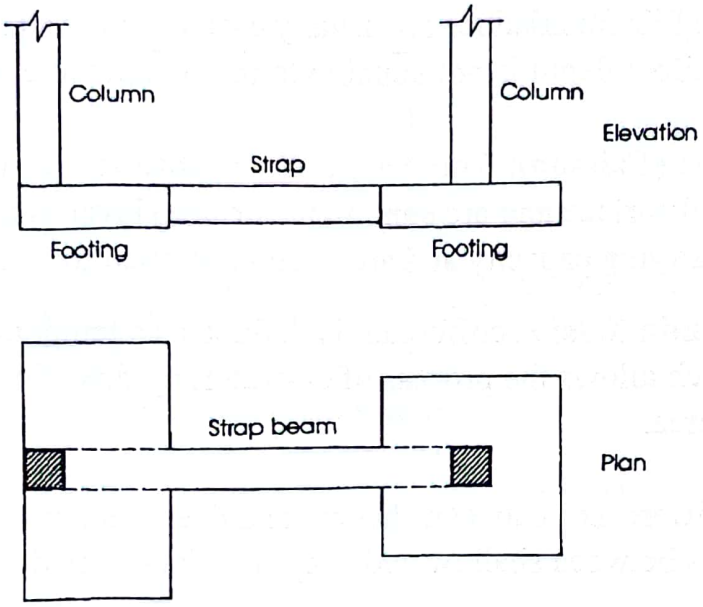


Figure: Cantilever or strap footing

(v) Mat or Raft foundations: It consists of one footing, usually placed under the entire building area and support the columns of the building. They are used when – the soil bearing capacity is low, column loads are very heavy, single footing cannot be used, piles are not used, differential settlement must be reduced through the entire footing.

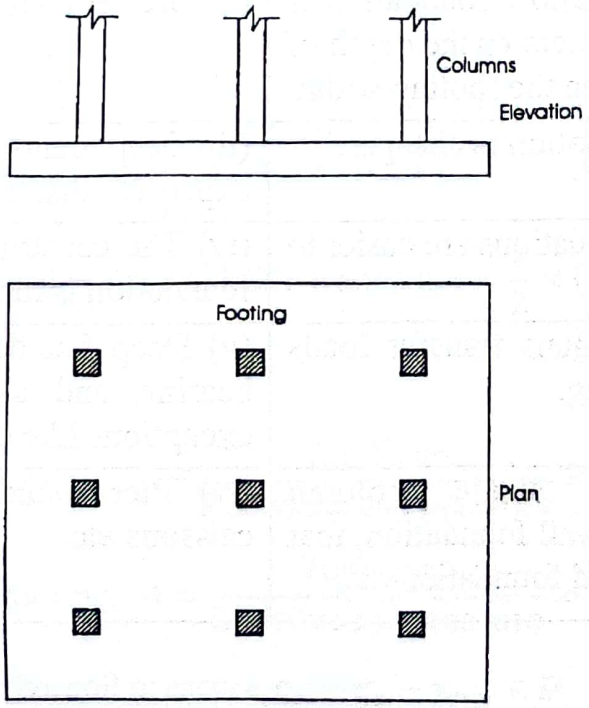


Figure: Mat or Raft foundation

(b) Deep foundation: Foundation which is placed at a greater depth or transfers the loads to deep strata is called the deep foundation. The ratio of depth to width is greater than 4. The most common deep foundations are described below:

(i) Pile foundations: Pile foundations are usually used for large structures and in situations where the soil at shallow depth is not suitable to resist excessive settlement, resist uplift, etc.

(ii) Pier foundations (Caisson): These are hollow substructures that can be constructed near or on the ground surface and are sunk to the desired level as a single unit. They have an enormous load-carrying capacity and are commonly used for bridges.

(iii) Cofferdam foundations: A cofferdam is defined as a temporary barrier in or around a body of water which allows the process of de-watering, diversion, or damming of water within an enclosed area.

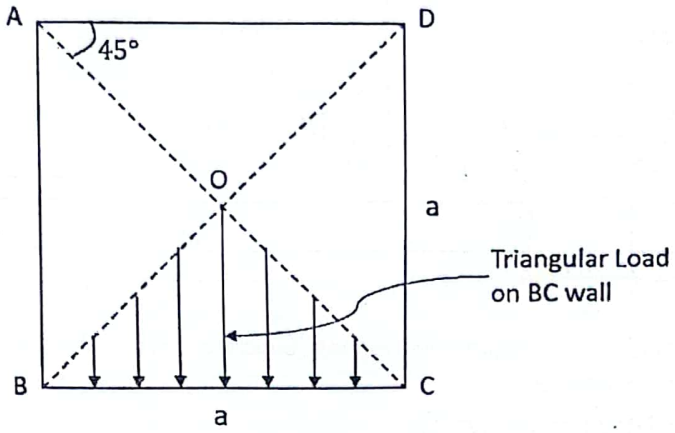
Q-3. What are the differences between shallow and deep foundation? [*CE-2019*]
Ans: The differences between shallow and deep foundation are described below:

Shallow Foundation	Deep Foundation
(i) Foundation which is placed near the surface of the earth or transfers the loads at shallow depth is called the shallow foundation.	(i) Foundation which is placed at a greater depth or transfers the loads to deep strata is called the deep foundation.
(ii) The depth of shallow foundation is generally about 3 meters or the depth of foundation is less than the footing width.	(ii) Greater than the shallow foundation.
(iii) A shallow foundation is cheaper.	(iii) Deep foundations are generally more expensive than shallow foundations.
(iv) Shallow foundations are easier to construct.	(iv) The construction process of a deep foundation is more complex.
(v) Shallow foundations transfer loads mostly by end bearing.	(v) Deep foundations rely both on end bearing and skin friction, with few exceptions like end-bearing pile.
(iv) Isolated or single column foundation, strip or wall foundation, mat foundation, combined foundation etc.	(iv) Pier foundation, pile foundation, caissons etc.

3.2 Basic Concepts

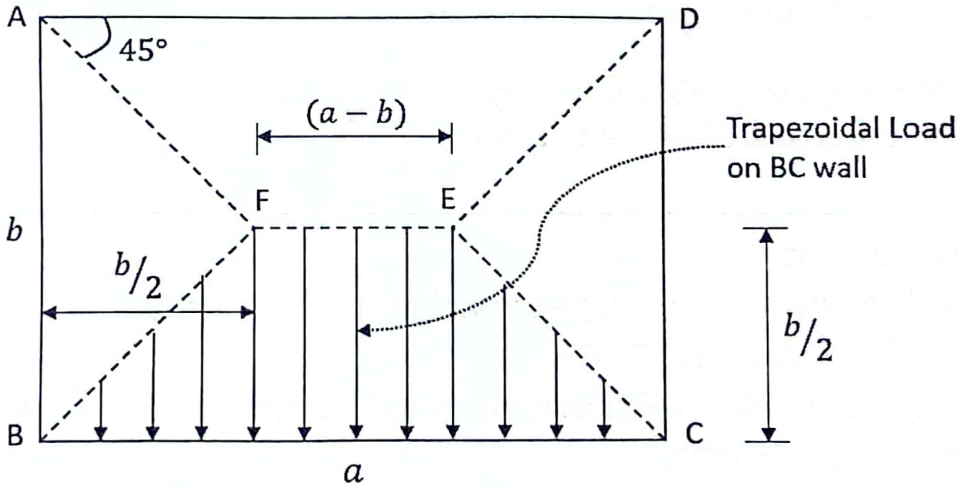
3.2.1 Load Area of Slab:

(i) For square slab:



Load area of slab on a side wall = Area of OBC triangle = $\left(\frac{a \times a}{4}\right)$ *or*, $\left(\frac{1}{2} \times a \times \frac{a}{2}\right)$

(ii) For rectangular slab:



Load area of slab on a side wall = Area of BCEF trapezium = $\frac{1}{2} \times \{a + (a - b)\} \times \frac{b}{2}$

3.2.2 Size of Footings:

(i) Area of footing, $A = \dfrac{\text{Total load}}{\text{Allowable soil pressure}} = \dfrac{P}{q_a}$

(ii) Area of footing, $A = \dfrac{\text{Total load}}{\text{Effective soil pressure}} = \dfrac{P}{q_e}$

Here, Effective soil pressure, $q_e = q_a - \gamma_{avg} \times D$

3.2.3 Critical Section of Moment:

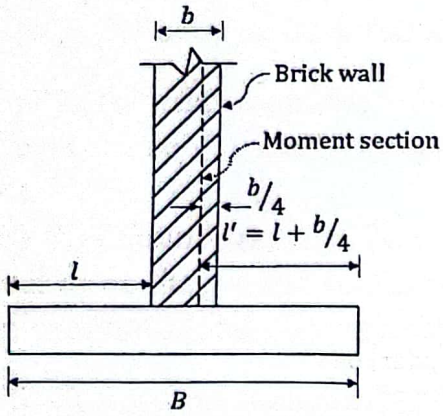


Figure: Wall footing (Brick wall)

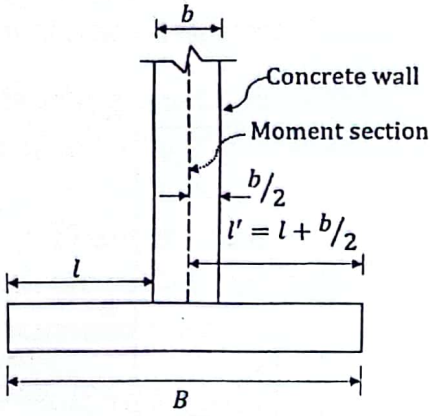


Figure: Wall footing (Concrete wall)

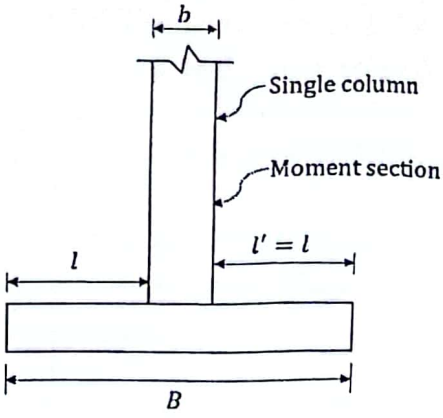


Figure: Single or Combined footing

(i) For wall footing (brick wall):

Lever arm of moment, $l' = l + \frac{b}{4}$

(ii) For wall footing (concrete wall):

Lever arm of moment, $l' = l + \frac{b}{2}$

(iii) For single or combined footing:

Lever arm of moment, $l' = l$

Bangla Terminology:

(i) Brick wall এর ক্ষেত্রে wall footing-এ critical moment* section, wall এর face থেকে b/4 distance ভিতরে consider করা হয়।

(ii) Concrete wall এর ক্ষেত্রে wall footing-এ critical moment section, wall এর mid-section বরাবর consider করা হয়।

(iii) Single column, combined এবং অন্যান্য footing এর ক্ষেত্রে critical moment section, column এর face বরাবর consider করা হয়।

* Critical moment হলো সেই moment যার কারনে footing fail করে।

3.2.4 One Way Shear (Wide Beam Shear):

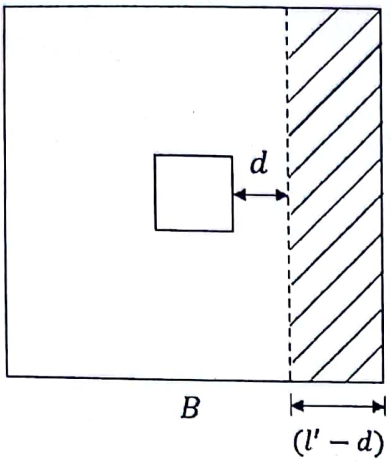


Figure: Plan of one way shear

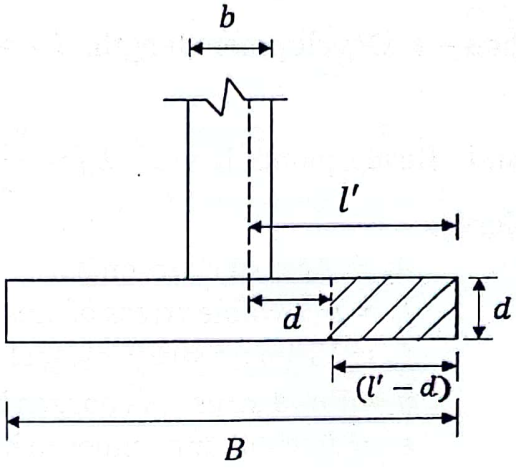


Figure: Cross – section

For one way shear (wide beam shear):
Developed shear, $V_{dev} = q_{dev} \times b \times (l' - d) = q_{dev} \times bd'$

3.2.5 Two Way Shear (Punching Shear):

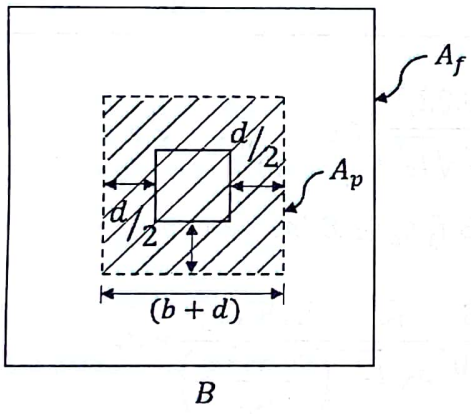


Figure: Plan of two way shear

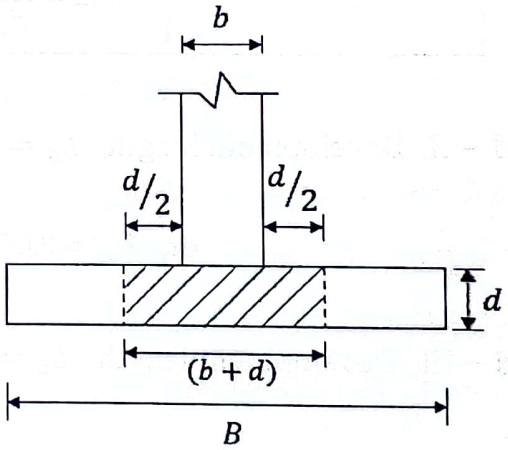


Figure: Cross – section

For two way shear (punching shear):
Developed shear, $V_{dev} = q_{dev} \times [A_f - A_p]$

Here, A_f = Area of footing & A_p = Area of punching

3.2.6 Development length of Bars:

(1) Method – I: Development length, $L_d = \frac{A_s f_s}{u\varepsilon_o}$ (WSD)

And, Development length, $L_d = \frac{A_s f_y}{u\varepsilon_o}$ (USD)

Here,

A_s = Area of choosen bar
f_s = Allowable stress of steel
f_y = Ultimate stress of steel
u = Bond stress of choosen bar*
ε_o = Perimeter of choosen bar = πd_b

* Bond stress of different types of bars:

Type of bars	WSD	USD
Bottom bars	$u = \frac{4.8\sqrt{f_c'}}{d_b} \leq 500$	$u = \frac{9.5\sqrt{f_c'}}{d_b} \leq 800$
Top bars	$u = \frac{3.4\sqrt{f_c'}}{d_b} \leq 350$	$u = \frac{6.7\sqrt{f_c'}}{d_b} \leq 560$

(2) Method – II: Development length, $L_d = \left(\frac{0.02 f_y}{\lambda\sqrt{f_c'}}\right) d_b$

or, $L_d = 0.0003\, f_y d_b \geq 8\ in.$

(3) Method – III: Development length, $L_d = \frac{3}{40} \cdot \frac{f_y}{\lambda\sqrt{f_c'}} \cdot \frac{\Psi_t \Psi_e \Psi_s}{\left(\frac{c_b + k_{tr}}{d_b}\right)} * d_b$

3.2.7 Self-weight of Foundation:

It considers that the self-weight of foundation will be **10 – 15%** of the total load on the foundation.

(1) For wall footing, Percentages of self weight = 12%
(2) For single column or combined footing, Percentages of self weight = 15%

Bangla Terminology: Mathematical Analysis **করে দেখা গেছে যে,** Foundation self-weight **এর** percentages, wall footing **এর ক্ষেত্রে** 12% এবং single column footing/combined footing এর ক্ষেত্রে 15% consider করা হলে foundation self-weight check করার সময় তা satisfy করে।

3.3 Solution Techniques

3.3.1 Wall Footing

WSD

1. Load calculation $= P$
2. Foundation area, $A = \frac{P}{q_{all}}$
3. Width of foundation, $B = \frac{A}{1\text{ ft}}$
4. Developed soil pressure, $q_{dev} = \frac{P}{b \times 1}$
5. Developed moment,
$$M_{dev} = \frac{q_{dev} \times l^2}{2}$$
6. Design constants: $n,\ r,\ k,\ j$
7. Depth check

i) Flexure check, $M = \frac{1}{2} f_c k j b d^2$

ii) Wide beam shear check

Developed shear, $V_{dev} = q_{dev} \times bd'$

Concrete shear stress, $v = 1.1\sqrt{f_c'}$

Allowable shear, $V_{all} = vbd$

8. Reinforcement calculation:

i) Longitudinal steel, $A_s = \frac{M}{f_s j d}$

Development length check:

$$u = \frac{4.8\sqrt{f_c'}}{d_b} \leq 500$$

$$L_d = \frac{A_s f_s}{u \varepsilon_o}$$

$\therefore L_d(provided) = l' - cc$

ii) Distribution steel,

$$A_s(\text{min}) = 0.0018\ bt$$

9. Reinforcemnt detailing

USD

1. Load calculation $= P$
2. Foundation area, $A = \frac{P}{q_{all}}$
3. Width of foundation, $B = \frac{A}{1\text{ ft}}$
4. Ultimate soil pressure,
$$q_u = \frac{1.2\ DL + 1.6\ LL}{b \times 1}$$
5. Developed moment,
$$M_{dev} = \frac{q_u \times l^2}{2}$$
6. Depth check

i) Flexure check,

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t}$$

$$M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

ii) Wide beam shear check

Ultimae shear, $V_u = q_u \times bd'$

Concrete shear stress, $v = 2\varphi\sqrt{f_c'}$

Allowable shear, $V_{all} = vbd$

7. Reinforcement calculation:

i) Longitudinal steel, $A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

Development length check:

$$u = \frac{9.5\sqrt{f_c'}}{d_b} \leq 800$$

$$L_d = \frac{A_s f_y}{u \varepsilon_o}$$

$\therefore L_d(provided) = l' - cc$

ii) Distribution steel,

$$A_s(\text{min}) = 0.0018\ bt$$

8. Reinforcemnt detailing

3.3.2 Single Column Footing

WSD

1. Load calculation $= P$
2. Foundation area, $A = \frac{P}{q_{all}}$
3. Side of foundation, $L = \frac{A}{B}$ or $\sqrt{A}$
4. Developed soil pressure, $q_{dev} = \frac{P}{L \times B}$
5. Developed moment,
$$M_{dev} = \frac{q_{dev} \times l^2}{2}$$
6. Design constants: $n,\ r,\ k,\ j$
7. Depth check

i) Flexure check, $M = \frac{1}{2} f_c k j b d^2$

ii) Wide beam shear check
Developed shear, $V_{dev} = q_{dev} \times bd'$
Concrete shear stress, $v = 1.1\sqrt{f_c'}$
Allowable shear, $V_{all} = vbd$

iii) Punching shear check
Developed punching force,
$$V_{dev} = q_{dev} \times [A_f - A_p]$$
Concrete shear stress, $v = 2\sqrt{f_c'}$
Allowable punching force, $V_{all} = vb_o d$

8. Check foundation self weight
9. Reinforcement calculation:

i) Longitudinal steel, $A_s = \frac{M}{f_s j d}$

Development length check:
$$u = \frac{4.8\sqrt{f_c'}}{d_b} \leq 500$$
$$L_d = \frac{A_s f_s}{u \varepsilon_o}$$
$$\therefore L_d(provided) = l' - cc$$

USD

1. Load calculation $= P$
2. Foundation area, $A = \frac{P}{q_{all}}$
3. Side of foundation, $L = \frac{A}{B}$ or $\sqrt{A}$
4. Ultimate soil pressure,
$$q_u = \frac{1.2\,DL + 1.6\,LL}{L \times B}$$
5. Ultimate moment,
$$M_u = \frac{q_{dev} \times l^2}{2}$$
6. Depth check

i) Flexure check,
$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t}$$
$$M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

ii) Wide beam shear check
Ultimae shear, $V_u = q_u \times bd'$
Concrete shear stress, $v = 2\varphi\sqrt{f_c'}$
Allowable shear, $V_{all} = vbd$

iii) Punching shear check
Ultimate punching force,
$$V_u = q_{dev} \times [A_f - A_p]$$
Punching shear stress, $v = 4\varphi\sqrt{f_c'}$
Allowable punching force, $V_{all} = vb_o d$

7. Check foundation self weight
8. Reinforcement calculation:

i) Longitudinal steel, $A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$
$$a = \frac{A_s f_y}{0.85 f_c' b}$$

ii) Distribution steel (short direction)

$$\text{Moment, } M_{dev} = \frac{q_{dev} \times l^2}{2}$$

$$A_s = \frac{M}{f_s j d}$$

$$\text{Here, } A_s(min) = \frac{200}{f_y} bd$$

10. Reinforcemnt detailing

Development length check:

$$u = \frac{9.5\sqrt{f_c'}}{d_b} \leq 800$$

$$L_d = \frac{A_s f_y}{u \varepsilon_o}$$

$$\therefore L_d(provided) = l' - cc$$

ii) Distribution steel (short direction)

$$\text{Moment, } M_u = \frac{q_u \times l^2}{2}$$

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$\text{Here, } A_s(min) = \frac{200}{f_y} bd$$

9. Reinforcemnt detailing

3.3.3 Combined Footing

WSD

1. Load calculation $= P$

2. Foundation area, $A = \dfrac{P}{q_{all}}$

3. Load c.g $= \bar{x}$

4. Foundation size
Foundation length, $L = (a + \bar{x}) \times 2$
Foundation width, $B = A/L$

5. Developed soil pressure, $q_{dev} = \dfrac{P}{L \times B}$

6. Diagram of SFD and BMD

7. Design constants: $n,\ r,\ k,\ j$

8. Depth check

i) Flexure check, $M = \dfrac{1}{2} f_c k j b d^2$

ii) Wide beam shear check
Developed shear,
$$V_{dev} = \text{Max. shear force} - wd$$
Concrete shear stress, $v = 1.1\sqrt{f_c'}$
Allowable shear, $V_{all} = vbd$

iii) Punching shear check
Developed punching force,
$V_{dev} = \text{column load} - (b + d)^2 \times q_{dev}$
Concrete shear stress, $v = 2\sqrt{f_c'}$
Allowable punching force, $V_{all} = vb_o d$

9. Check foundation self weight

10. Reinforcement calculation:

i) Longitudinal steel, $A_s = \dfrac{M}{f_s j d}$

Development length check:
a) For top bars
$$u = \frac{3.4\sqrt{f_c'}}{d_b} \leq 350$$

USD

1. Load calculation $= P$

2. Foundation area, $A = \dfrac{P}{q_{all}}$

3. Load c.g $= \bar{x}$

4. Foundation size
Foundation length, $L = (a + \bar{x}) \times 2$
Foundation width, $B = A/L$

5. Ultimate soil pressure,
$$q_u = \frac{1.2\,DL + 1.6\,LL}{L \times B}$$

6. Diagram of SFD and BMD

7. Depth check
i) Flexure check,
$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t}$$
$$M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

ii) Wide beam shear check
Ultimate shear,
$$V_u = \text{Max. shear force} - wd$$
Concrete shear stress, $v = 2\sqrt{f_c'}$
Allowable shear, $V_{all} = vbd$

iii) Punching shear check
Ultimate punching force,
$V_u = \text{column load} - (b + d)^2 \times q_{dev}$
Concrete shear stress, $v = 4\varphi\sqrt{f_c'}$
Allowable punching force, $V_{all} = vb_o d$

8. Check foundation self weight

9. Reinforcement calculation:

i) Longitudinal steel, $A_s = \dfrac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$L_d = \frac{A_s f_s}{u\varepsilon_o}$$

$\therefore L_d(provided) = x_1 + b - x_2$

b) For bottom bars

$$u = \frac{4.8\sqrt{f_c'}}{d_b} \leq 500$$

$$L_d = \frac{A_s f_s}{u\varepsilon_o}$$

$\therefore L_d(provided) = (a_1 + x_2) - cc$

ii) Distribution steel (short direction)
Design of transverse beam

Load on column $= P_1$
UDL on beam $= P_1/B$

Effective width of beam $= (b + d)$

$$\text{Moment, } M_{dev} = \frac{q_{dev} \times l^2}{2}$$

$$A_s = \frac{M}{f_s jd}$$

$$\text{Here, } A_s(min) = \frac{200}{f_y} bd$$

11. Reinforcemnt detailing

Development length check:
a) For top bars

$$u = \frac{6.7\sqrt{f_c'}}{d_b} \leq 560$$

$$L_d = \frac{A_s f_s}{u\varepsilon_o}$$

$\therefore L_d(provided) = x_1 + b - x_2$

b) For bottom bars

$$u = \frac{9.5\sqrt{f_c'}}{d_b} \leq 800$$

$$L_d = \frac{A_s f_s}{u\varepsilon_o}$$

$\therefore L_d(provided) = (a_1 + x_2) - cc$

ii) Distribution steel (short direction)
Design of transverse beam

Load on column $= P_1$
UDL on beam $= P_1/B$

Effective width of beam $= (b + d)$

$$\text{Moment, } M_u = \frac{q_u \times l^2}{2}$$

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

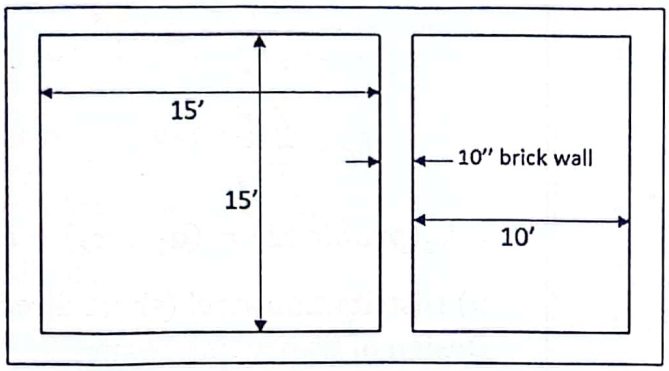
$$\text{Here, } A_s(min) = \frac{200}{f_y} bd$$

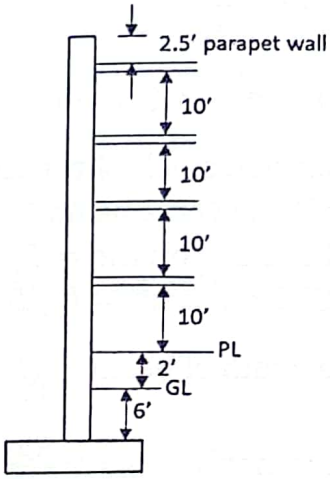
11. Reinforcemnt detailing

3.4 Problem Solution

3.4.1 Wall Footing – WSD

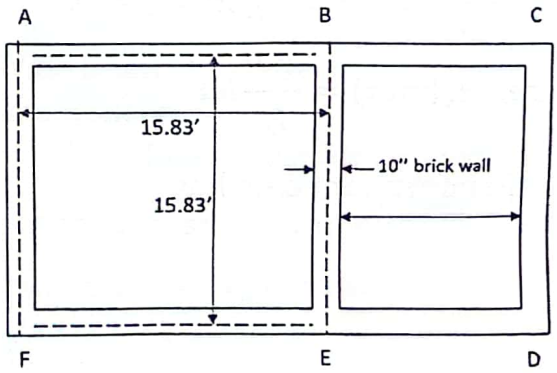
Pb-1. A four storied building is shown in figure given below. The allowable bearing capacity of soil is 3 ksf at 6 ft. below the ground. Slab thickness = 6 in. Floor finish thickness = 2.5 in. of 30 psf dead load. Load bearing wall thickness = 10 in. Live load on the building is 40 psf. Design the wall footing of the building. From the design data, material strengths are $f_c' = 3000$ psi and $f_y = 60{,}000$ psi.





Sol^n:

Wall footing should be designed for larger wall such as AB wall.



Load coming from super structure on footing:

(i) Dead load:

Wall height = $6 + 2 + 4 * 10 + 2.5 = 50.5'$

$$\therefore \text{Weight of wall on footing} = \left(50.5 \times \frac{10}{12}\right) \times 120 = 5050 \text{ lb/ft}$$

$$\text{Weight from slab} = \left(\frac{15 \times 15}{4}\right) \times \frac{6}{12} \times 150 = 4218.75 \text{ lb}$$

$$\therefore \text{Weight from slab on footing} = \frac{4 \times 4218.75}{15.83} = 1066 \text{ lb/ft}$$

$$\text{Weight from floor finish} = \left(\frac{15 \times 15}{4}\right) \times \frac{2.5}{12} \times 30 = 351.56 \text{ lb}$$

$$\therefore \text{Weight from slab on footing} = \frac{4 \times 351.56}{15.83} = 89 \text{ lb/ft}$$

$$\therefore \text{Total dead load on footing} = 5050 + 1066 + 89 = 6205 \text{ lb/ft}$$

(ii) Live load:

$$\text{Live load from slab} = \left(\frac{15 \times 15}{4}\right) \times 40 = 2250\ lb$$

$$\therefore \text{Live load from slab on footing} = \frac{4 \times 2250}{15.83} = 569\ \text{lb/ft}$$

(iii) Self-weight:

$$\text{Foundation self weight} = 0.10 \times (6205 + 569) = 678\ \text{lb/ft}$$

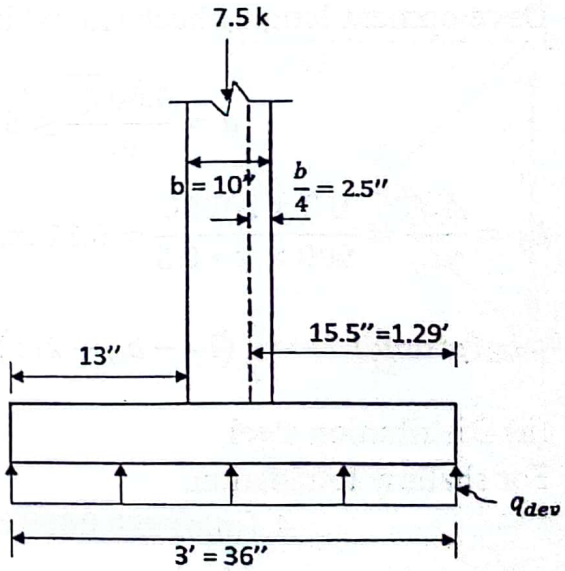
$$\therefore \text{Total load on footing, } P = 6205 + 569 + 678 = 7452 \approx 7500\ \text{lb/ft} = 7.5\ \text{k/ft}$$

$$\text{Now, Foundation area} = \frac{P}{q_{all}} = \frac{7.5}{3} = 2.5\ \text{ft}^2$$

$$\therefore \text{Width of foundation} = \frac{2.5\ \text{ft}^2}{1\ \text{ft}} = 2.5\ \text{ft} \approx 3\ \text{ft}$$

$$\therefore \text{Developed soil pressure, } q_{dev} = \frac{7.5}{3 \times 1} = 2.5\ \text{ksf}$$

$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{2.5 \times 1.29^2}{2} = 2.08\ k - ft$$



Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \approx 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 3} = \frac{24}{1.35} = 17.8$$

$$k = \frac{n}{n+r} = 0.336$$

$$j = 1 - \frac{k}{3} = 0.888$$

Depth check:

(i) Flexure check:

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Rightarrow 2.08 \times 12 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 12 \times d^2$$

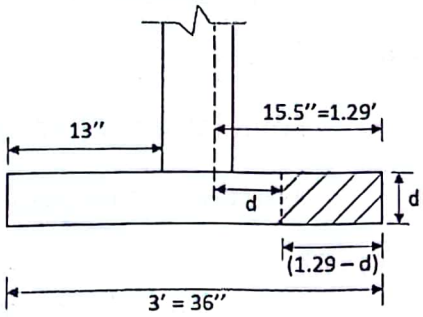
$$\therefore d = 3.21\ in.$$

(ii) Wide beam shear check:

Developed shear, $V_{dev} = q_{dev} \times bd' = 2.5 \times 1' \times (1.29 - d)$

Concrete shear stress, $v = 1.1\sqrt{f_c'} = 1.1\sqrt{3000} = 60.25\ psi = 60.25 \times \frac{144}{1000} = 8.68\ ksf$

Allowable shear, $V_{all} = vbd = 8.68 \times 1' \times d$



Now, $V_{dev} = V_{all}$

$\Rightarrow 2.5 \times 1' \times (1.29 - d) = 8.68 \times 1' \times d$

$\therefore d = 0.29\ ft = 3.46\ in.$

Taking maximum $d = 3.46\ in. \approx 4\ in.$

$\therefore$ Minimum foundation thickness, $t = d + cc = 4 + 3 = 7\ in.$

Reinforcement calculation:

(i) Longitudinal steel

$$\therefore A_s = \frac{M}{f_s jd} = \frac{2.08 \times 12}{24 \times 0.888 \times 4} = 0.293\ in^2/ft$$

Provide #4 bar @ 8 in. c/c

Development length check (for #4 bar):

$$u = \frac{4.8\sqrt{f_c'}}{d_b} \leq 500; \quad \therefore u = \frac{4.8\sqrt{3000}}{0.5} = 526 \approx 500$$

$$L_d = \frac{A_s f_s}{u\varepsilon_o} = \frac{0.2 \times 24000}{500 \times \pi * 0.5} = 6.11\ in.$$

$$\therefore L_d(provided) = \frac{1}{2}(b_f - b_w - 2cc) = \frac{1}{2} \times (36 - 10 - 2 * 3) = 10\ in. > L_d\ (6.11\ in.)\ (ok)$$

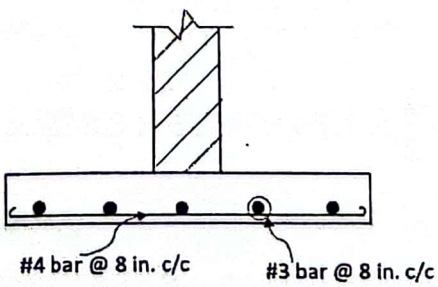
(ii) Distribution steel

For shallow foundation,

$$A_s(\min) = 0.0018\, bt = 0.0018 \times 12 \times 7 = 0.1512\ in^2/ft$$

Provide #3 bar @ 8 in. c/c

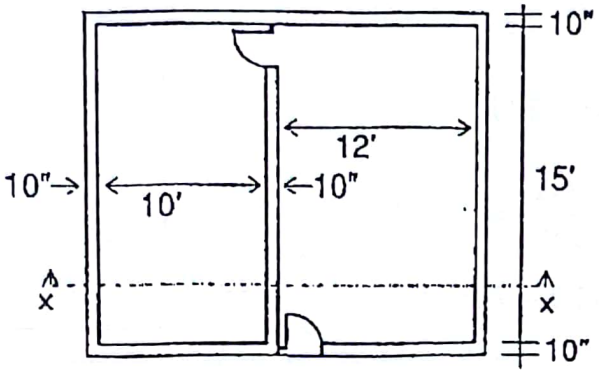
Reinforcement detailing:



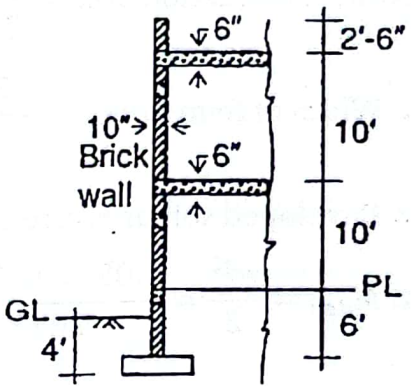
Pb-2. A two storied building is shown in the figure given below. The allowable soil bearing capacity at the building site is 2.6 ksf. Live load on the building is 40 psf. The concrete strength is 3.5 ksi and yield strength of steel is 60 ksi. Slab thickness is 6 inches. Using these data –

(i) Calculate the maximum developed soil pressure under the foundation.

(ii) Find out amount of reinforcement and also draw reinforcement detailing for the foundation. [*CE-2017*]



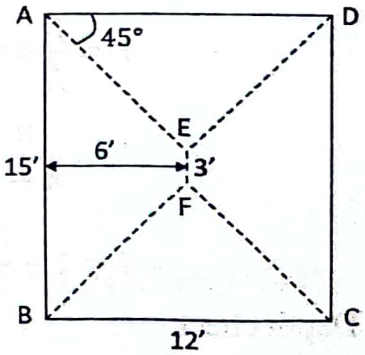
Plan



x-x Section Partial

Sol^{n}:

Wall footing should be designed for larger wall of larger room dimension such as AB wall.



Load coming from super structure on footing:

(i) Dead load:

Wall height $= 6 + 2 * 10 + 2.5 = 28.5'$

$\therefore$ Weight of wall on footing $= \left(28.5 \times \frac{10}{12}\right) \times 120$

$= 2850$ lb/ft

Weight from slab $= \left\{\frac{1}{2} \times (15 + 3) \times 6\right\} \times \frac{6}{12} \times 150 = 4050$ lb

$\therefore$ Weight from slab on footing $= \frac{2 \times 4050}{\left(15 + \frac{10}{12}\right)} = 512$ lb/ft

Weight from floor finish $= \left\{\frac{1}{2} \times (15 + 3) \times 6\right\} \times \frac{2.5}{12} \times 30 = 338$ lb

$\therefore$ Weight from slab on footing $= \frac{2 \times 338}{15.83} = 43$ lb/ft

$\therefore$ Total dead load on footing $= 2850 + 512 + 43 = 3405$ lb/ft

(ii) Live load:

Live load from slab $= \left\{\frac{1}{2} \times (15 + 3) \times 6\right\} \times 40 = 2160\ lb$

$$\therefore \text{Live load from slab on footing} = \frac{2 \times 2160}{15.83} = 273 \text{ lb/ft}$$

(iii) Self-weight:

Foundation self weight $= 0.10 \times (3405 + 273) = 368$ lb/ft

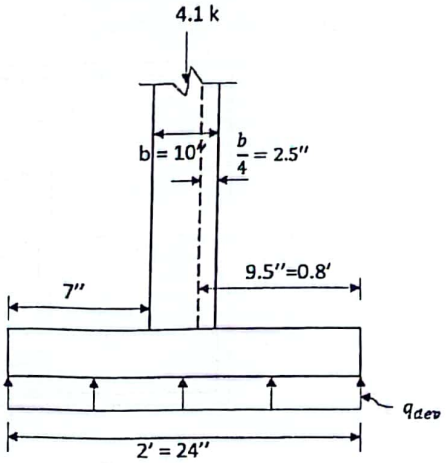
$\therefore$ Total load on footing, $P = 3405 + 273 + 368 = 4046 \approx 4100$ lb/ft $= 4.1$ k/ft

Now, Foundation area $= \frac{P}{q_{all}} = \frac{4.1}{2.6} = 1.58 \text{ ft}^2$

$$\therefore \text{Width of foundation} = \frac{1.58 \text{ ft}^2}{1 \text{ ft}} = 1.58 \text{ ft} \approx 2 \text{ ft}$$

$$\therefore \text{Developed soil pressure, } q_{dev} = \frac{4.1}{2 \times 1} = 2.05 \text{ ksf}$$

$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{2.05 \times 0.8^2}{2} = 0.66\ k - ft$$



Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3500}} = 8.59 \approx 8$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 3.5} = \frac{24}{1.575} = 15.24$$

$$k = \frac{n}{n+r} = 0.344$$

$$j = 1 - \frac{k}{3} = 0.885$$

Depth check:

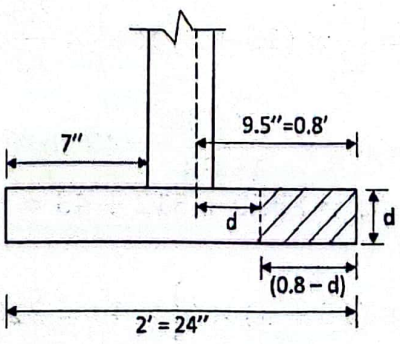
(i) Flexure check:

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Longrightarrow 0.66 \times 12 = \frac{1}{2} \times 1.575 \times 0.344 \times 0.885 \times 12 \times d^2$$

$$\therefore d = 1.66\ in.$$

(ii) Wide beam shear check:



Developed shear, $V_{dev} = q_{dev} \times bd' = 2.05 \times 1' \times (0.8 - d)$

Concrete shear stress, $v = 1.1\sqrt{f_c'} = 1.1\sqrt{3500} = 65.08\ psi = 65.08 \times \frac{144}{1000} = 9.37\ ksf$

Allowable shear, $V_{all} = vbd = 9.37 \times 1' \times d$

Now, $V_{dev} = V_{all}$

$\Rightarrow 2.05 \times 1' \times (0.8 - d) = 9.37 \times 1' \times d$

$\therefore d = 0.144\ ft = 1.73\ in. < 3\ in.$

Taking maximum $d = 1.73\ in. \approx 3\ in.$

$\therefore$ Minimum foundation thickness, $t = d + cc = 3 + 2 = 5\ in.$

Reinforcement calculation:

(i) Longitudinal steel

$$\therefore A_s = \frac{M}{f_s jd} = \frac{0.66 \times 12}{24 \times 0.885 \times 3} = 0.124\ in^2/ft$$

Provide #3 bar @ 10 in. c/c

Development length check (for #3 bar):

$$u = \frac{4.8\sqrt{f_c'}}{d_b} \leq 500; \quad \therefore u = \frac{4.8\sqrt{3500}}{0.375} = 758 \approx 500$$

$$L_d = \frac{A_s f_s}{u\varepsilon_o} = \frac{0.11 \times 24000}{500 \times \pi * 0.375} = 4.5\ in.$$

$$\therefore L_d(provided) = \frac{1}{2}(b_f - b_w - 2cc) = \frac{1}{2} \times (24 - 10 - 2 * 2) = 5\ in. > L_d\ (4.5\ in.)\ \ (ok)$$

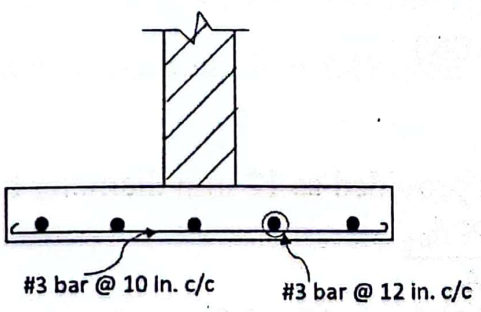
(ii) Distribution steel

For shallow foundation,

$$A_s(\min) = 0.0018\ bt = 0.0018 \times 12 \times 5 = 0.108\ in^2/ft$$

Provide #3 bar @ 12 in. c/c

Reinforcement detailing:



Pb-3. Analysis Problem: An existing foundation under load bearing 15″ wall was 56″ wide and 12″ thick reinforced with 12 mm diameter bar @ 6″ and 8″ c/c parallel to its cross and long directions, respectively. It was also found from recorded data that f_c' is 3 ksi and f_y is 60 ksi.

(i) Calculate the load carrying capacity of the wall foundation.

(ii) If allowable bearing capacity of soil is 1.6 ksf then find out the safe load capacity of the foundation system. [*CE-2019*]

Sol^n:

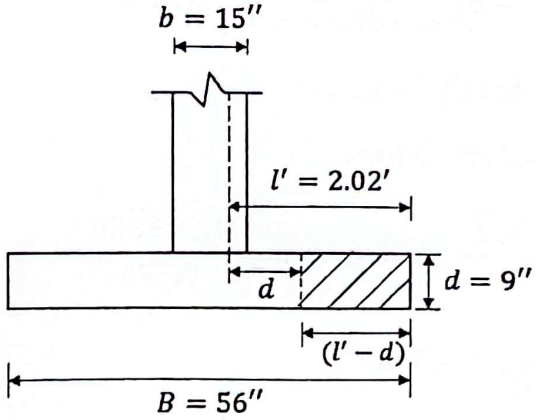
Here, Wall width, $b = 15''$

Width of foundation, $B = 56'' = 4.67'$

Thickness of foundation, $t = 12''$

$\therefore$ Effective depth, $d = t - cc = 12 - 3 = 9'' = 0.75'$

$\therefore$ Lever arm of moment, $l' = l + \frac{b}{4} = 20.5 + \frac{15}{4} = 24.25'' = 2.02'$



Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \approx 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 3} = \frac{24}{1.35} = 17.78$$

$$k = \frac{n}{n + r} = 0.336$$

$$j = 1 - \frac{k}{3} = 0.888$$

Moment calculation:

The main reinforcement was provided as 12 mm diameter bar @ 6″ c/c.

Bar spacing, $S = \frac{12 \times A_{s_1}}{A_s}$

$\therefore$ Steel area, $A_s = \frac{12 \times A_{s_1}}{S} = \frac{12 \times 0.2}{6} = 0.4\ in^2$

If steel controls,

Moment, $M = A_s f_s jd = 0.4 \times 24 \times 0.888 \times 9 = 76.72\ k-in. = 6.39\ k-ft$

If concrete controls,

Moment, $M = \frac{1}{2} f_c kjbd^2 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 12 \times 9^2 = 195.76\ k-in.$
$= 16.31\ k-ft$

So maximum developed moment, $M_{dev} = 6.39\ k-ft$

(i) Check load carrying capacity:

(a) Flexure check

$$M_{dev} = \frac{q_{dev} \times l^2}{2}$$

$$\therefore q_{dev} = \frac{2 \times M_{dev}}{l^2} = \frac{2 \times 6.39}{2.02^2} = 3.13\ ksf$$

(b) Wide beam shear check:

Developed shear, $V_{dev} = q_{dev} \times b \times (l' - d) = q_{dev} \times 1' \times (2.02 - 0.75)$

Concrete shear stress, $v = 1.1\sqrt{f_c'} = 1.1\sqrt{3000} = 60.25\ psi = 8.68\ ksf$

Allowable shear, $V_{all} = vbd = 8.68 \times 1' \times 0.75 = 6.51\ kips$

Now, $V_{dev} = V_{all}$

$\Rightarrow q_{dev} \times 1' \times (2.02 - 0.75) = 6.51$

$\therefore q_{dev} = 5.13\ ksf$

Minimum developed soil pressure, $q_{dev} = 5.13\ ksf$

Now, $q_{dev} = \frac{P}{B \times 1}$

$\therefore$ Load carrying capacity, $P = 5.13 \times 4.67 \times 1 = 23.96 \approx 24\ kips$

(ii) Safe load capacity of foundation:

If allowable bearing capacity is 1.6 ksf,

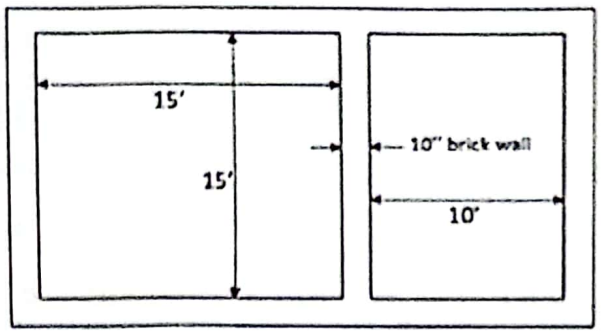
Foundation area, $A = \frac{P}{q_{all}}$

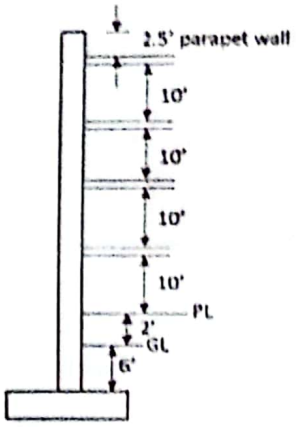
$\therefore$ Load carrying capacity, $P = q_{all} \times (B \times 1) = 1.6 \times (4.67 \times 1) = 7.47\ kips$

$\therefore$ Safe load capacity of the foundation, $P = 7.47\ kips$ **(Ans)**

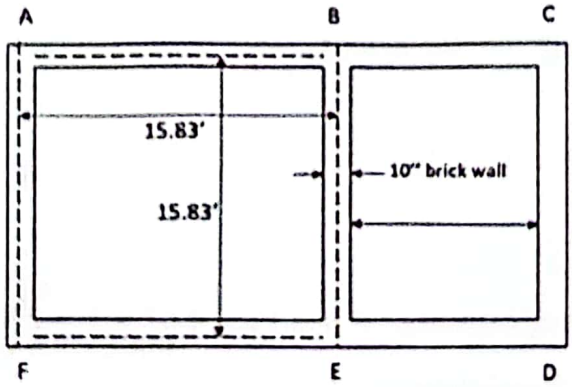
3.4.2 Wall Footing – USD

Pb-4. A four storied building is shown in figure given below. The allowable bearing capacity of soil is 3 ksf at 6 ft. below the ground. Slab thickness = 6 in. Floor finish thickness = 2.5 in. of 30 psf dead load. Load bearing wall thickness = 10 in. Live load on the building is 40 psf. Design the wall footing of the building. From the design data, material strengths are $f_c' = 3000$ psi and $f_y = 60{,}000$ psi.





Sol^n:



Wall footing should be designed for larger wall such as AB wall.

Load coming from super structure on footing:

(i) Dead load:

$$\text{Wall height} = 6 + 2 + 4 * 10 + 2.5 = 50.5'$$

$$\therefore \text{Weight of wall on footing} = \left(50.5 \times \frac{10}{12}\right) \times 120 = 5050 \text{ lb/ft}$$

$$\text{Weight from slab} = \left(\frac{15 \times 15}{4}\right) \times \frac{6}{12} \times 150 = 4218.75 \text{ lb}$$

$$\therefore \text{Weight from slab on footing} = \frac{4 \times 4218.75}{15.83} = 1066 \text{ lb/ft}$$

$$\text{Weight from floor finish} = \left(\frac{15 \times 15}{4}\right) \times \frac{2.5}{12} \times 30 = 351.56 \text{ lb}$$

$$\therefore \text{Weight from slab on footing} = \frac{4 \times 351.56}{15.83} = 89 \text{ lb/ft}$$

$$\therefore \text{Total dead load on footing} = 5050 + 1066 + 89 = 6205 \text{ lb/ft}$$

(ii) Live load:

$$\text{Live load from slab} = \left(\frac{15 \times 15}{4}\right) \times 40 = 2250\ lb$$

$$\therefore \text{Live load from slab on footing} = \frac{4 \times 2250}{15.83} = 569 \text{ lb/ft}$$

(iii) Self-weight:

$$\text{Foundation self weight} = 0.10 \times (6205 + 569) = 678 \text{ lb/ft}$$

$$\therefore \text{Total load on footing, } P = 6205 + 569 + 678 = 7452 \approx 7500 \text{ lb/ft} = 7.5 \text{ k/ft}$$

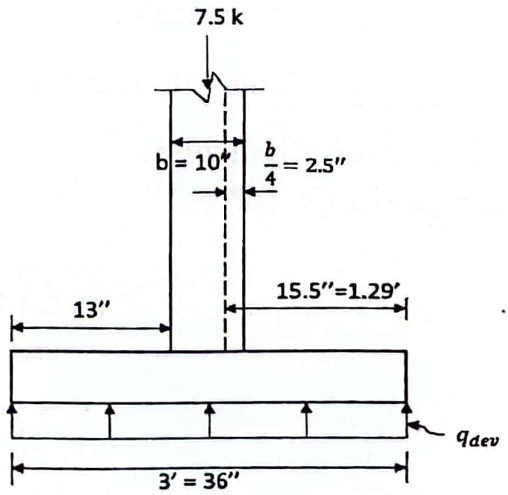
$$\text{Now, Foundation area} = \frac{P}{q_{all}} = \frac{7.5}{3} = 2.5 \text{ ft}^2$$

$$\therefore \text{Width of foundation} = \frac{2.5 \text{ ft}^2}{1 \text{ ft}} = 2.5 \text{ ft} \approx 3 \text{ ft}$$

$$\text{Ultimate soil pressure, } q_u = \frac{1.2DL + 1.6LL}{A} = \frac{1.2 \times 6205 + 1.6 \times 569 + 1.2 \times 678}{3 \times 1}$$

$$\therefore q_u = 3.06\ ksf$$

$$\therefore M_u = \frac{wl^2}{2} = \frac{3.06 \times 1.29^2}{2} = 2.55\ k - ft$$



Depth check:

(i) Flexure check:

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{3}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0135$$

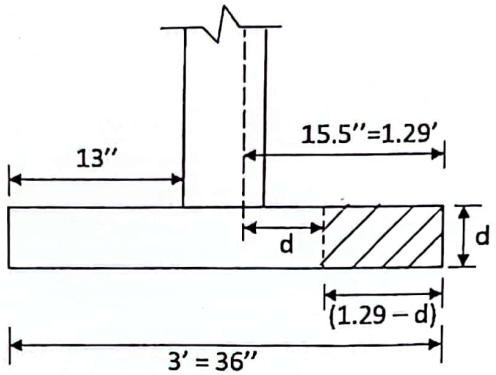
$$\text{Now, } M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

$$\Rightarrow 2.55 \times 12 = 0.90 \times 0.0135 \times 60 \times 12 \times d^2 \times \left(1 - 0.59 \times 0.0135 \times \frac{60}{3}\right)$$

$$\Rightarrow 30.6 = 8.748 \times d^2 \times 0.8407$$

$$\therefore d = 2.09\ in.$$

(ii) Wide beam shear check:



Ultimate shear, $V_u = q_u \times bd' = 3.06 \times 1' \times (1.29 - d)$

Concrete shear stress, $v = 2\varphi\sqrt{f_c'} = 2 \times 0.75 \times \sqrt{3000} = 82.16\ psi = 11.83\ ksf$

Allowable shear, $V_{all} = vbd = 11.83 \times 1' \times d$

Now, $V_u = V_{all}$

$\Rightarrow 3.06 \times 1' \times (1.29 - d) = 8.68 \times 1' \times d$

$\therefore d = 0.336\ ft = 4.03\ in.$

Taking maximum $d = 4.03\ in. \approx 5\ in.$

$\therefore$ Minimum foundation thickness, $t = d + cc = 5 + 3 = 8\ in.$

Reinforcement calculation:

(i) Longitudinal steel

$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{2.55 \times 12}{0.90 \times 60 \times \left(5 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 12}$$

$$\therefore A_s = 0.121\ in^2/ft$$

Provide #3 bar @ 10 in. c/c

Development length check (for #3 bar):

$$u = \frac{9.5\sqrt{f_c'}}{d_b} \leq 800; \quad \therefore u = \frac{9.5\sqrt{3000}}{0.375} = 1387.6 \approx 800$$

$$L_d = \frac{A_s f_y}{u\varepsilon_o} = \frac{0.11 \times 60000}{800 \times \pi * 0.375} = 7\ in.$$

$$\therefore L_d(provided) = \frac{1}{2}(b_f - b_w - 2cc) = \frac{1}{2} \times (36 - 10 - 2 * 3) = 10\ in. > L_d\ (7\ in.)\ (ok)$$

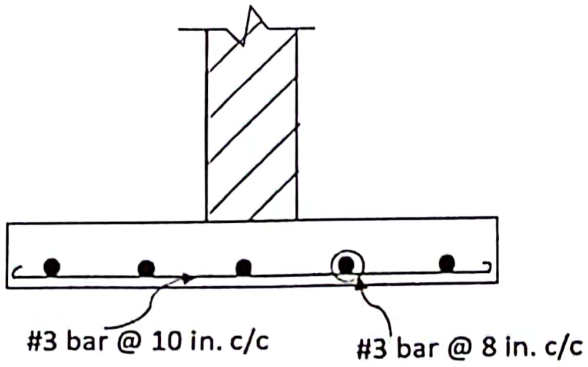
(ii) Distribution steel

For shallow foundation,

$$A_s(\min) = 0.0018\ bt = 0.0018 \times 12 \times 7 = 0.1512\ in^2/ft$$

Provide #3 bar @ 8 in. c/c

Reinforcement detailing:



3.4.3 Single Column Footing – WSD

3.4.3.1 Square Single Column Footing – WSD

Pb-5. Design a square single column footing to support a 15 in. square tied column that supports a dead load of 100 kips and a live load of 65 kips. The allowable soil pressure is 1.8 ksf. Material strengths are $f_c' = 3$ ksi and $f_y = 60$ ksi. [*CE-2013 (similar)*]

Sol^n:

Self weight of foundation $= 0.15 \times (100 + 65) = 24.75$ kips

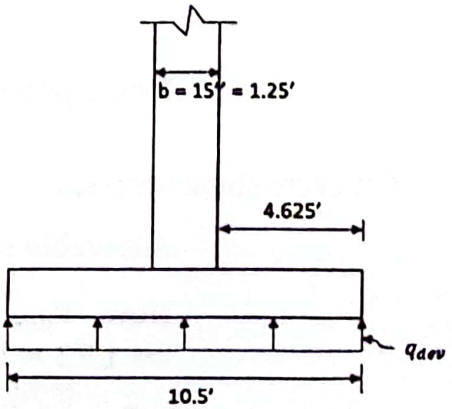
$\therefore$ Total load on ground $= 100 + 65 + 24.75 = 189.75$ kips

Now, Foundation area $= \frac{P}{q_{all}} = \frac{189.75}{1.8} = 105.42\ ft^2$

$\therefore$ Size of foundation $= \sqrt{105.42} = 10.27\ ft$

Let, Foundation size is $10.5' \times 10.5'$

$\therefore$ Developed soil pressure, $q_{dev} = \frac{189.75}{10.5 \times 10.5} = 1.72$ ksf



$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{1.72 \times 4.625^2}{2} = 18.4\ k-ft$$

Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \approx 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 3} = \frac{24}{1.35} = 17.8$$

$$k = \frac{n}{n+r} = 0.336$$

$$j = 1 - \frac{k}{3} = 0.888$$

Depth check:

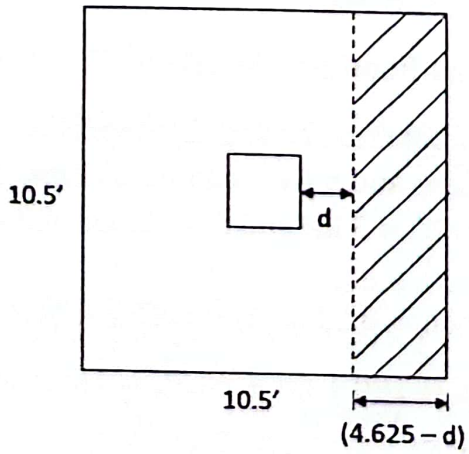
(i) Flexure check:

$$M = \frac{1}{2} f_c kjbd^2$$

$$\Rightarrow 18.4 \times 12 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 12 \times d^2$$

$$\therefore d = 9.56\ in.$$

(ii) Wide beam shear check:



Developed shear, $V_{dev} = q_{dev} \times bd' = 1.72 \times 1' \times (4.625 - d)$

Concrete shear stress, $v = 1.1\sqrt{f_c'} = 1.1\sqrt{3000} = 60.25\ psi = 60.25 \times \frac{144}{1000} = 8.68\ ksf$

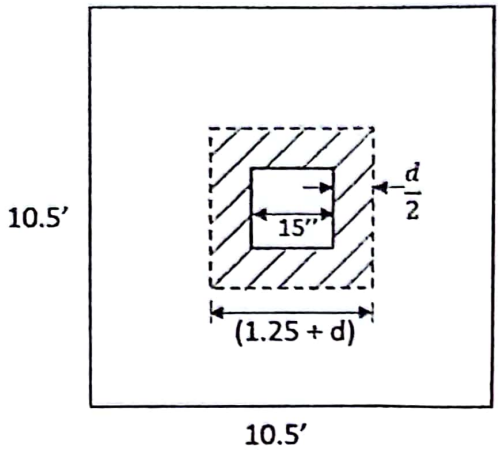
Allowable shear, $V_{all} = vbd = 8.68 \times 1' \times d$

Now, $V_{dev} = V_{all}$

$\Rightarrow 1.72 \times 1' \times (4.625 - d) = 8.68 \times 1' \times d$

$\therefore d = 0.765\ ft = 9.2\ in.$

(iii) Punching shear check:



Developed punching force, $V_{dev} = q_{dev} \times [A_f - A_p] = 1.72 \times [10.5^2 - (1.25 + d)^2]$

Punching shear stress, $v = 2\sqrt{f_c'} = 2\sqrt{3000} = 109.54\ psi = 109.54 \times \frac{144}{1000} = 15.77\ ksf$

Allowable punching shear force, $V_{all} = vb_o d = 15.77 \times \{4 * (1.25 + d)\} \times d$

Now, $V_{dev} = V_{all}$

$\Rightarrow 1.72 \times [10.5^2 - (1.25 + d)^2] = 15.77 \times \{4 * (1.25 + d)\} \times d$

$\therefore d = 1.174\ ft = 14.1\ in.$

Taking maximum $d = 14.1\ in. \approx 14.5\ in.$

$\therefore$ Minimum foundation thickness, $t = d + cc = 14.5 + 3 = 17.5\ in.$

Checking,

Foundation self weight $= \left(10.5^2 \times \frac{17.5}{12}\right) \times 150 = 24.12\ kips < 24.75$ kips (ok)

Reinforcement calculation:

(i) Longitudinal steel

$$\therefore A_s = \frac{M}{f_s j d} = \frac{18.4 \times 12}{24 \times 0.888 \times 14.5} = 0.715\ in^2/ft$$

Provide #6 bar @ 7 in. c/c

Development length check (for #6 bar):

$$u = \frac{4.8\sqrt{f_c'}}{d_b} \leq 500; \quad \therefore u = \frac{4.8\sqrt{3000}}{0.75} = 350.5$$

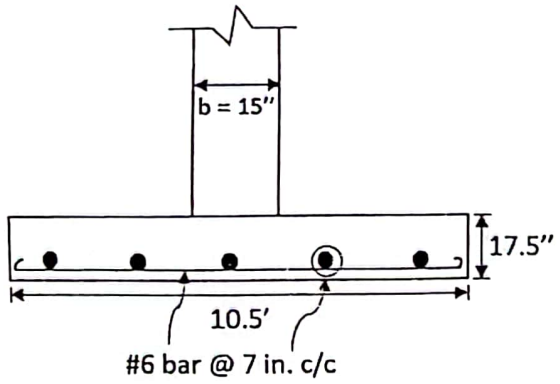
$$L_d = \frac{A_s f_s}{u \varepsilon_o} = \frac{0.44 \times 24000}{350.5 \times \pi * 0.75} = 12.79\ in.$$

$\therefore L_d(provided) = (4.625 \times 12) - 3 = 52.5\ in. > L_d\ (12.79\ in.)\ \ (ok)$

(ii) Distribution steel

Providing steel as same as longitudinal steel in square single column footing.

Reinforcement detailing:



3.4.3.2 Rectangular Single Column Footing – WSD

Pb-6. Design a rectangular footing to support a 16 in. square tied column that supports a dead load of 100 kips and a live load of 65 kips. The one side of the footing is limited to 8 ft. The allowable soil pressure is 1.7 ksf. Material strengths are $f_c' = 3$ ksi and $f_y = 60$ ksi.

Sol^n:

Self weight of foundation $= 0.15 \times (100 + 65) = 24.75$ kips

$\therefore$ Total load on ground $= 100 + 65 + 24.75 = 189.75 \approx 190$ kips

Now, Foundation area $= \dfrac{P}{q_{all}} = \dfrac{190}{1.7} = 111.76\ ft^2$

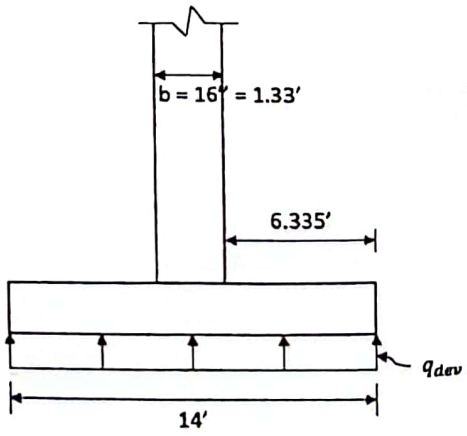
One side of foundation $= 8\ ft$

$\therefore$ Other side of foundation $= \dfrac{111.76}{8} = 13.97 \approx 14\ ft$

So, Foundation size is $14' \times 8'$

$\therefore$ Developed soil pressure, $q_{dev} = \dfrac{190}{14 \times 8} = 1.69$ ksf

$\therefore M_{dev} = \dfrac{wl^2}{2} = \dfrac{1.69 \times 6.335^2}{2} = 33.91\ k-ft$



Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \approx 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 3} = \frac{24}{1.35} = 17.8$$

$$k = \frac{n}{n+r} = 0.336$$

$$j = 1 - \frac{k}{3} = 0.888$$

Depth check:

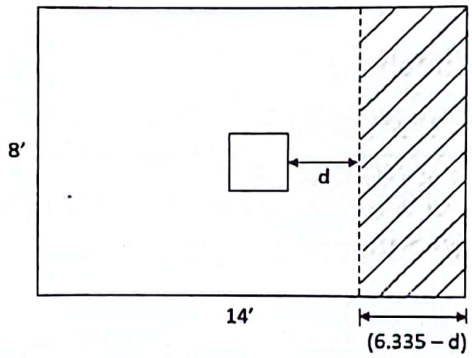
(i) Flexure check:

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Rightarrow 33.91 \times 12 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 12 \times d^2$$

$$\therefore d = 12.98\ in.$$

(ii) Wide beam shear check:



Developed shear, $V_{dev} = q_{dev} \times bd' = 1.69 \times 1' \times (6.335 - d)$

Concrete shear stress, $v = 1.1\sqrt{f_c'} = 1.1\sqrt{3000} = 60.25\ psi = 60.25 \times \frac{144}{1000} = 8.68\ ksf$

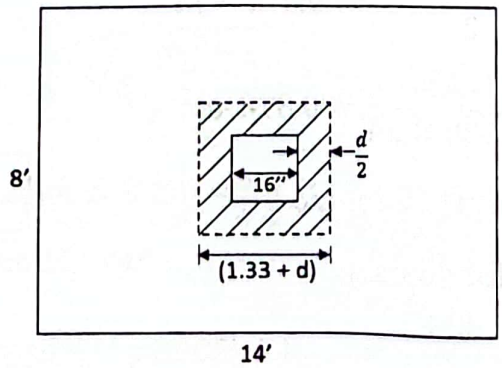
Allowable shear, $V_{all} = vbd = 8.68 \times 1' \times d$

Now, $V_{dev} = V_{all}$

$\Rightarrow 1.69 \times 1' \times (6.335 - d) = 8.68 \times 1' \times d$

$\therefore d = 1.032\ ft = 12.39\ in.$

(iii) Punching shear check:



Developed punching force, $V_{dev} = q_{dev} \times [A_f - A_p] = 1.69 \times [(14 \times 8) - (1.33 + d)^2]$

Punching shear stress, $v = 2\sqrt{f_c'} = 2\sqrt{3000} = 109.54\ psi = 109.54 \times \frac{144}{1000} = 15.77\ ksf$

Allowable punching shear force, $V_{all} = vb_o d = 15.77 \times \{4 * (1.33 + d)\} \times d$

Now, $V_{dev} = V_{all}$

$\Rightarrow 1.69 \times [(14 \times 8) - (1.33 + d)^2] = 15.77 \times \{4 * (1.33 + d)\} \times d$

$\therefore d = 1.146\ ft = 13.75\ in.$

Taking maximum $d = 13.75\ in. \approx 14\ in.$

$\therefore$ Minimum foundation thickness, $t = d + cc = 14 + 3 = 17\ in.$

Checking,

Foundation self weight $= \left(14 \times 8 \times \frac{17}{12}\right) \times 150 = 23.8\ kips < 24.75\ kips\ \ (ok)$

Reinforcement calculation:

(i) Longitudinal steel (in long direction)

$$\therefore A_s = \frac{M}{f_s jd} = \frac{33.91 \times 12}{24 \times 0.888 \times 14} = 1.36\ in^2/ft$$

Provide #7 bar @ 5 in. c/c

Development length check (for #7 bar):

$$u = \frac{4.8\sqrt{f_c'}}{d_b} \leq 500; \quad \therefore u = \frac{4.8\sqrt{3000}}{0.875} = 300.5$$

$$L_d = \frac{A_s f_s}{u\varepsilon_o} = \frac{0.6 \times 24000}{300.5 \times \pi * 0.875} = 17.43\ in.$$

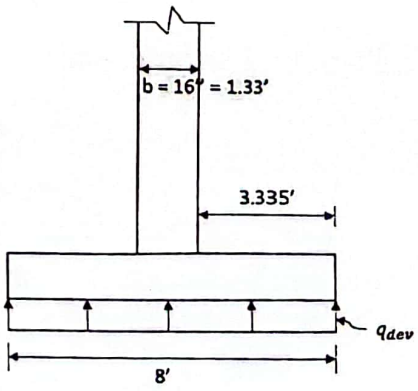
$\therefore L_d(provided) = (6.335 \times 12) - 3 = 73.02\ in. > L_d\ (17.43\ in.)\ \ (ok)$

(ii) Distribution steel (in short direction)

$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{1.69 \times 3.335^2}{2} = 9.4\ k - ft$$

$$\therefore A_s = \frac{M}{f_s jd} = \frac{9.4 \times 12}{24 \times 0.888 \times 14} = 0.378\ in^2/ft$$

Provide #6 bar @ 10 in. c/c



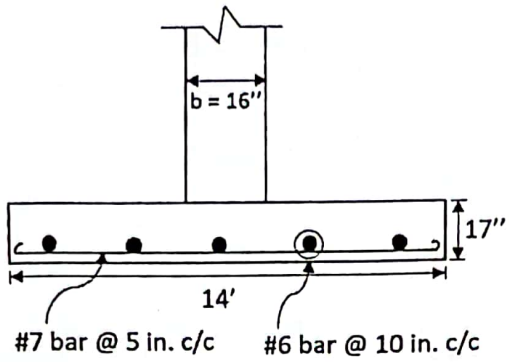
Development length check (for #6 bar):

$$u = \frac{4.8\sqrt{f_c'}}{d_b} \leq 500; \quad \therefore u = \frac{4.8\sqrt{3000}}{0.75} = 420.65$$

$$L_d = \frac{A_s f_s}{u\varepsilon_o} = \frac{0.44 \times 24000}{420.65 \times \pi * 0.75} = 10.65\ in.$$

$\therefore L_d(provided) = (3.335 \times 12) - 3 = 37.02\ in. > L_d\ (10.65\ in.)\ \ (ok)$

Reinforcement detailing:



Pb-7. Design a rectangular footing for an 18 in. square interior column with a dead load of 180 kips and a live load of 150 kips. Make the length of the long side equal to twice the width of the short side. The allowable soil pressure is 4.0 ksf. Assume the material strengths are $f_c' = 4$ ksi and $f_y = 60$ ksi. The base of the footing is 5 ft. below the grade.

Sol^n:

Here, Allowable soil pressure, $q_{all} = 4.0\ ksf = 4000\ psf$

Average unit weight of concrete and soil below the grade $= \frac{150 + 100}{2} = 125\ pcf$

$\therefore$ Effective soil pressure, $q_e = 4000 - 5 \times 125 = 3375\ psf = 3.375\ ksf$

Self weight of foundation $= 0.15 \times (180 + 150) = 49.5$ kips

$\therefore$ Total load on ground $= 180 + 150 + 49.5 = 379.5 \approx 380$ kips

Now, Foundation area $= \frac{P}{q_e} = \frac{380}{3.375} = 112.6\ ft^2 = b \times 2b$

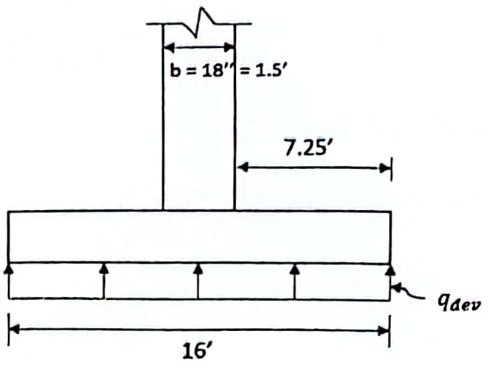
Short side of foundation, $b = \sqrt{\frac{112.6}{2}} = 7.503 \approx 8\ ft$

$\therefore$ Long side of foundation $= 2 \times 8 = 16\ ft$

So, Foundation size is $16' \times 8'$

$\therefore$ Developed soil pressure, $q_{dev} = \frac{380}{16 \times 8} = 2.97$ ksf

$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{2.97 \times 7.25^2}{2} = 78.06\ k-ft$$



Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8.04 \approx 8$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 4} = \frac{24}{1.8} = 13.33$$

$$k = \frac{n}{n+r} = 0.375$$

$$j = 1 - \frac{k}{3} = 0.875$$

Depth check:

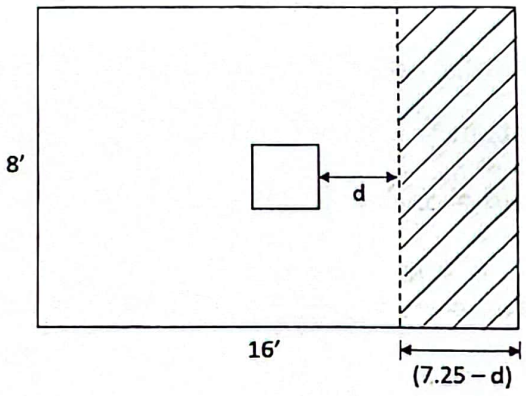
(i) Flexure check:

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Longrightarrow 78.06 \times 12 = \frac{1}{2} \times 1.8 \times 0.375 \times 0.875 \times 12 \times d^2$$

$$\therefore d = 16.26\ in.$$

(ii) Wide beam shear check:



Developed shear, $V_{dev} = q_{dev} \times bd' = 2.97 \times 1' \times (7.25 - d)$

Concrete shear stress, $v = 1.1\sqrt{f_c'} = 1.1\sqrt{4000} = 69.57\ psi = 69.57 \times \frac{144}{1000} = 10.02\ ksf$

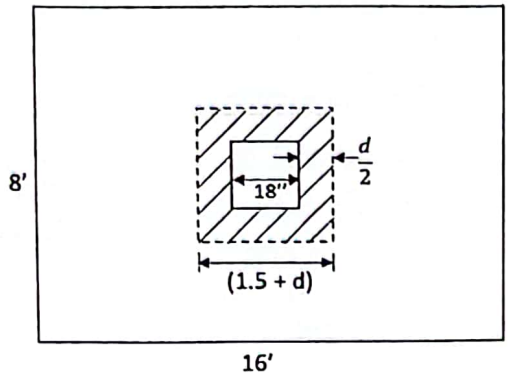
Allowable shear, $V_{all} = vbd = 10.02 \times 1' \times d$

Now, $V_{dev} = V_{all}$

$\Longrightarrow 2.97 \times 1' \times (7.25 - d) = 10.02 \times 1' \times d$

$\therefore d = 1.658\ ft = 19.9\ in.$

(iii) Punching shear check:



Developed punching force, $V_{dev} = q_{dev} \times [A_f - A_p] = 2.97 \times [(16 \times 8) - (1.5 + d)^2]$

Punching shear stress, $v = 2\sqrt{f_c'} = 2\sqrt{4000} = 126.49\ psi = 126.49 \times \frac{144}{1000} = 18.21\ ksf$

Allowable punching shear force, $V_{all} = vb_o d = 18.21 \times \{4 * (1.5 + d)\} \times d$

Now, $V_{dev} = V_{all}$

$\Longrightarrow 2.97 \times [(16 \times 8) - (1.5 + d)^2] = 18.21 \times \{4 * (1.5 + d)\} \times d$

$\therefore d = 1.573\ ft = 18.88\ in.$

Taking maximum $d = 19.9\ in. \approx 20\ in.$

$\therefore$ Minimum foundation thickness, $t = d + cc = 20 + 3 = 23\ in.$

Checking,

Foundation self weight $= \left(16 \times 8 \times \frac{23}{12}\right) \times 150 = 36.8\ kips < 49.5\ kips\ \ (ok)$

Reinforcement calculation:

(i) Longitudinal steel (in long direction)

$$\therefore A_s = \frac{M}{f_s jd} = \frac{78.06 \times 12}{24 \times 0.875 \times 20} = 2.23\ in^2/ft$$

Provide #9 bar @ 5 in. c/c

Development length check (for #9 bar):

$$u = \frac{4.8\sqrt{f_c'}}{d_b} \leq 500; \quad \therefore u = \frac{4.8\sqrt{4000}}{1.125} = 269.85$$

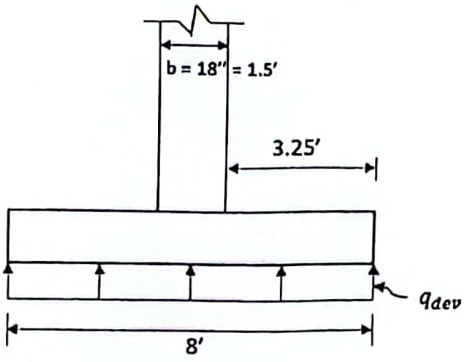
$$L_d = \frac{A_s f_s}{u\varepsilon_o} = \frac{1.0 \times 24000}{269.85 \times \pi * 1.125} = 25.16\ in.$$

$\therefore L_d(provided) = (7.25 \times 12) - 3 = 84\ in. > L_d\ (25.16\ in.)\ \ (ok)$

(ii) Distribution steel (in short direction)

$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{2.97 \times 3.25^2}{2}$$
$$= 15.69\ k-ft$$

$$\therefore A_s = \frac{M}{f_s jd} = \frac{15.69 \times 12}{24 \times 0.875 \times 20}$$
$$= 0.448\ in^2/ft$$



Provide #6 bar @ 11 in. c/c

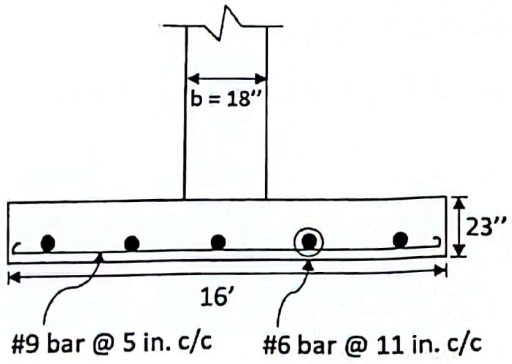
Development length check (for #6 bar):

$$u = \frac{4.8\sqrt{f_c'}}{d_b} \leq 500; \quad \therefore u = \frac{4.8\sqrt{4000}}{0.75} = 404.77$$

$$L_d = \frac{A_s f_s}{u\varepsilon_o} = \frac{0.44 \times 24000}{404.77 \times \pi * 0.75} = 11.07\ in.$$

$$\therefore L_d(provided) = (3.25 \times 12) - 3 = 36\ in. > L_d\ (11.07\ in.) \quad (ok)$$

Reinforcement detailing:



3.4.4 Single Column Footing – USD

3.4.4.1 Square Single Column Footing – USD

Pb-8. Design a square column footing under a 8'' diameter circular column that supports a dead load of 160 kips and a live load of 160 kips. The foundation is placed 5′ from ground level and allowable soil bearing capacity at this level is 2.5 ksf. Use $f_c' = 3$ ksi and $f_y = 60$ ksi. Follow USD method. [*CE-2018, 2013 (similar)*]

Sol^n:

Here, Allowable soil pressure, $q_{all} = 2.5\ ksf = 2500\ psf$

Average unit weight of concrete and soil below the grade $= \dfrac{150 + 100}{2} = 125\ pcf$

$\therefore$ Effective soil pressure, $q_e = 2500 - 5 \times 125 = 1875\ psf = 1.875\ ksf$

Self weight of foundation $= 0.15 \times (160 + 160) = 48$ kips

$\therefore$ Total load on ground $= 160 + 160 + 48 = 368$ kips

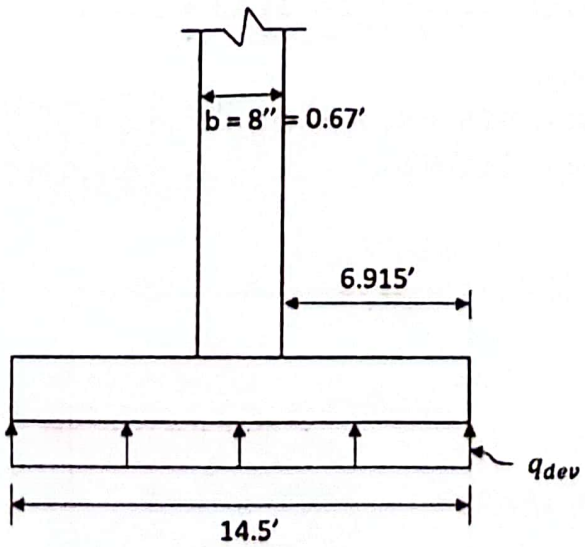
Now, Foundation area $= \dfrac{P}{q_e} = \dfrac{368}{1.875} = 196.3\ ft^2$

$\therefore$ Size of foundation $= \sqrt{196.3} = 14.01\ ft$

Let, Foundation size is $14.5' \times 14.5'$

$\therefore$ Ultimate soil pressure, $q_u = \dfrac{1.2\ DL + 1.6\ LL}{A} = \dfrac{1.2 \times 160 + 1.6 \times 160 + 1.2 \times 48}{14.5^2}$

$\therefore q_u = 2.4\ ksf$



$$\therefore M_u = \frac{wl^2}{2} = \frac{2.4 \times 6.915^2}{2} = 57.38\ k - ft$$

Depth check:

(i) Flexure check:

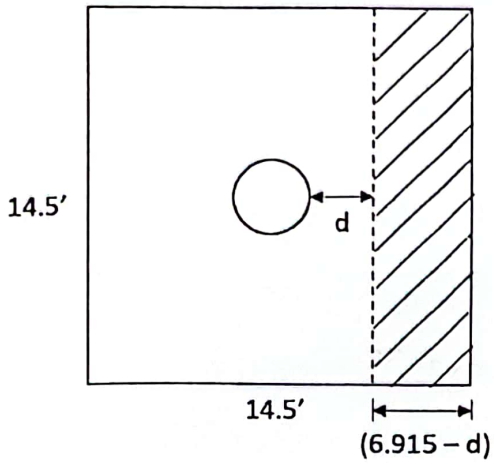
$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{3}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0135$$

$$\text{Now, } M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

$$\Rightarrow 57.38 \times 12 = 0.90 \times 0.0135 \times 60 \times 12 \times d^2 \times \left(1 - 0.59 \times 0.0135 \times \frac{60}{3}\right)$$

$$\therefore d = 9.68\ in.$$

(ii) Wide beam shear check:



$$\text{Ultimate shear, } V_u = q_u \times bd' = 2.4 \times 1' \times (6.915 - d)$$

$$\text{Concrete shear stress, } v = 2\varphi\sqrt{f_c'} = 2 \times 0.75 \times \sqrt{3000} = 82.16\ psi = 11.83\ ksf$$

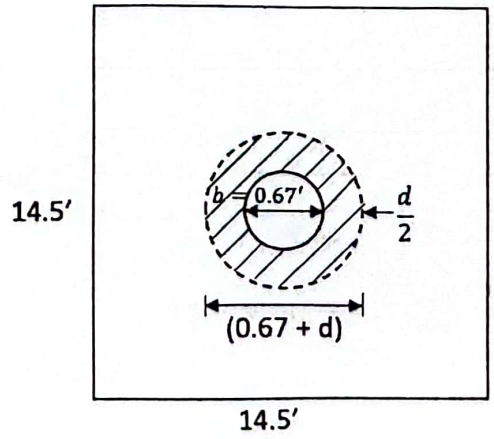
$$\text{Allowable shear, } V_{all} = vbd = 11.83 \times 1' \times d$$

$$\text{Now, } V_u = V_{all}$$

$$\Rightarrow 2.4 \times 1' \times (6.915 - d) = 11.83 \times 1' \times d$$

$$\therefore d = 1.17\ ft = 14.04\ in.$$

(iii) Punching shear check:



Ultimate punching force, $V_u = q_u \times [A_f - A_p] = 2.4 \times [(14.5)^2 - \left\{\frac{\pi \times (0.67+d)^2}{4}\right\}]$

Punching shear stress, $v = 4\varphi\sqrt{f_c'} = 4 \times 0.75 \times \sqrt{3000} = 164.32\ psi = 23.66\ ksf$

Allowable punching shear force, $V_{all} = vb_o d = 23.66 \times \{\pi \times (0.67+d)\} \times d$

Now, $V_u = V_{all}$

$\Rightarrow 2.4 \times [(14.5)^2 - \left\{\frac{\pi \times (0.67+d)^2}{4}\right\}] = 23.66 \times \{\pi \times (0.67+d)\} \times d$

$\therefore d = 2.25\ ft = 27\ in.$

Taking maximum $d = 27\ in.$

$\therefore$ Minimum foundation thickness, $t = d + cc = 27 + 3 = 30\ in.$

Checking,

Foundation self weight $= \left(14.5 \times 14.5 \times \frac{30}{12}\right) \times 150 = 78.84\ kips < 48\ kips$ (Not ok)

Here, Revised foundation self weight and checked.

Reinforcement calculation:

(i) Longitudinal steel (in long direction)

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{57.38 \times 12}{0.90 \times 60 \times \left(27 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 12}$$

$$\therefore A_s = 0.514\ in^2/ft$$

Provide #5 bar @ 7 in. c/c

Development length check (for #5 bar):

$$u = \frac{9.5\sqrt{f_c'}}{d_b} \leq 800; \quad \therefore u = \frac{9.5\sqrt{3000}}{0.625} = 832.54 \approx 800$$

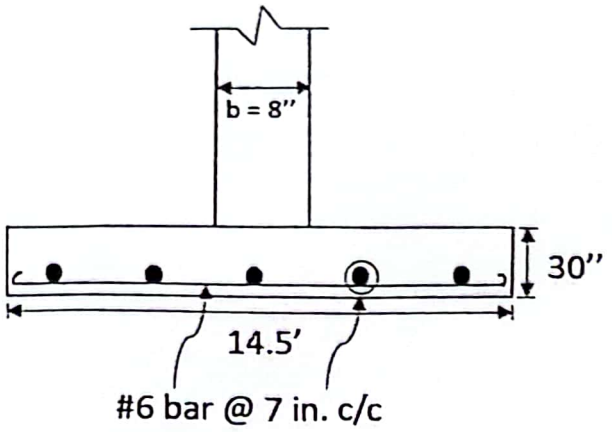
$$L_d = \frac{A_s f_y}{u\varepsilon_o} = \frac{0.31 \times 60000}{800 \times \pi * 0.625} = 11.84\ in.$$

$\therefore L_d(provided) = (6.915 \times 12) - 3 = 80\ in. > L_d\ (11.84\ in.)$ (ok)

(ii) Distribution steel

Providing steel as same as longitudinal steel in square single column footing.

Reinforcement detailing:



3.4.4.2 Rectangular Single Column Footing – USD

Pb-9. Design a rectangular footing to support a 16 in. square tied column that supports a dead load of 100 kips and a live load of 65 kips. The one side of the footing is limited to 8 ft. The allowable soil pressure is 1.7 ksf. Material strengths are $f_c' = 3$ ksi and $f_y = 60$ ksi.

Sol^n:

Self weight of foundation $= 0.15 \times (100 + 65) = 24.75$ kips

$\therefore$ Total load on ground $= 100 + 65 + 24.75 = 189.75 \approx 190$ kips

Now, Foundation area $= \dfrac{P}{q_{all}} = \dfrac{190}{1.7} = 111.76\ ft^2$

One side of foundation $= 8\ ft$

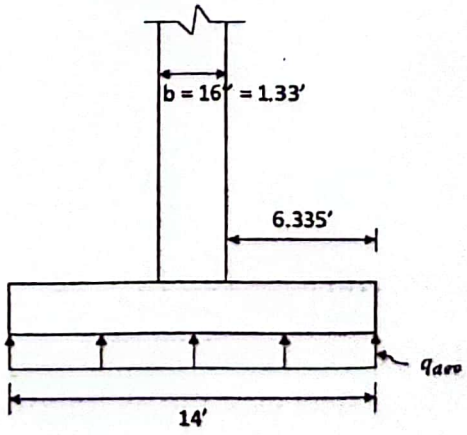
$\therefore$ Other side of foundation $= \dfrac{111.76}{8} = 13.97 \approx 14\ ft$

So, Foundation size is $14' \times 8'$

$\therefore$ Ultimate soil pressure, $q_u = \dfrac{1.2DL + 1.6LL}{A} = \dfrac{1.2 \times 100 + 1.6 \times 65 + 1.2 \times 24.75}{14 \times 8}$

$\therefore q_u = 2.3\ ksf$

$\therefore M_u = \dfrac{wl^2}{2} = \dfrac{2.3 \times 6.335^2}{2} = 46.15\ k - ft$



Depth check:

(i) Flexure check:

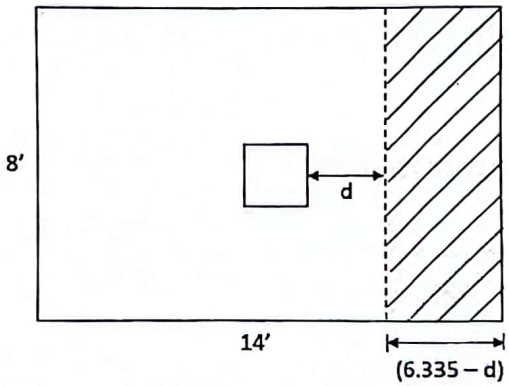
$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{3}{60} \times \frac{0.003}{0.003 + 0.005} = \mathbf{0.0135}$$

$$\text{Now, } M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

$$\Rightarrow 46.15 \times 12 = 0.90 \times 0.0135 \times 60 \times 12 \times d^2 \times \left(1 - 0.59 \times 0.0135 \times \frac{60}{3}\right)$$

$$\therefore d = 8.1\ in.$$

(ii) Wide beam shear check:



$$\text{Ultimate shear, } V_u = q_u \times bd' = 2.3 \times 1' \times (6.335 - d)$$

$$\text{Concrete shear stress, } v = 2\varphi\sqrt{f_c'} = 2 \times 0.75 \times \sqrt{3000} = 82.16\ psi = 11.83\ ksf$$

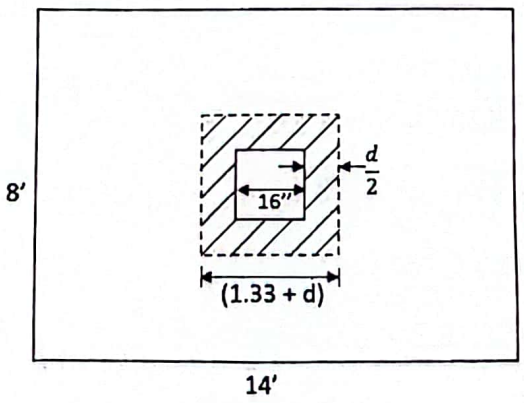
$$\text{Allowable shear, } V_{all} = vbd = 11.83 \times 1' \times d$$

$$\text{Now, } V_u = V_{all}$$

$$\Rightarrow 2.3 \times 1' \times (6.335 - d) = 11.83 \times 1' \times d$$

$$\therefore d = 1.031\ ft = 12.37\ in.$$

(iii) Punching shear check:



$$\text{Ultimate punching force, } V_u = q_u \times [A_f - A_p] = 2.3 \times [(14 \times 8) - (1.33 + d)^2]$$

Punching shear stress, $v = 4\varphi\sqrt{f_c'} = 4 \times 0.75 \times \sqrt{3000} = 164.32\ psi = 23.66\ ksf$

Allowable punching shear force, $V_{all} = vb_o d = 23.66 \times \{4 * (1.33 + d)\} \times d$

Now, $V_u = V_{all}$

$\Rightarrow 2.3 \times [(14 \times 8) - (1.33 + d)^2] = 23.66 \times \{4 * (1.33 + d)\} \times d$

$\therefore d = 1.074\ ft = 12.89\ in.$

Taking maximum $d = 12.89\ in. \approx 13\ in.$

$\therefore$ Minimum foundation thickness, $t = d + cc = 13 + 3 = 16\ in.$

Checking,

Foundation self weight $= \left(14 \times 8 \times \frac{16}{12}\right) \times 150 = 22.4\ kips < 24.75\ kips\ \ (ok)$

Reinforcement calculation:

(i) Longitudinal steel (in long direction)

$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{46.15 \times 12}{0.90 \times 60 \times \left(13 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 12}$$

$$\therefore A_s = 0.858\ in^2/ft$$

Provide #8 bar @ 11 in. c/c

Development length check (for #8 bar):

$$u = \frac{9.5\sqrt{f_c'}}{d_b} \leq 800;$$

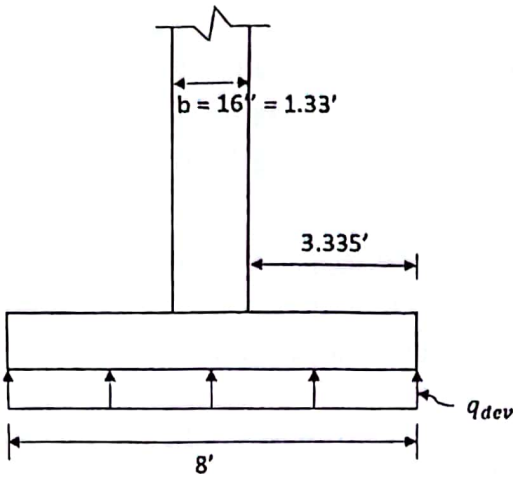
$$\therefore u = \frac{9.5\sqrt{3000}}{1.0} = 520.34$$

$$L_d = \frac{A_s f_y}{u\varepsilon_o} = \frac{0.79 \times 60000}{520.34 \times \pi * 1.0} = 29\ in.$$

$\therefore L_d(provided) = (6.335 \times 12) - 3 = 73.02\ in. > L_d\ (29\ in.)\ \ (ok)$

(ii) Distribution steel (in short direction)

$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{2.3 \times 3.335^2}{2} = 12.79\ k - ft$$



$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{12.79 \times 12}{0.90 \times 60 \times \left(13 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c{}' b} = \frac{A_s \times 60}{0.85 \times 3 \times 12}$$

$$\therefore A_s = 0.234\ in^2/ft$$

Provide #5 bar @ 15 in. c/c

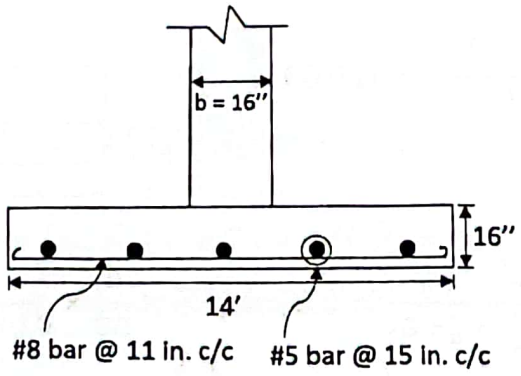
Development length check (for #5 bar):

$$u = \frac{9.5\sqrt{f_c{}'}}{d_b} \leq 800; \quad \therefore u = \frac{9.5\sqrt{3000}}{0.625} = 832.54 \approx 800$$

$$L_d = \frac{A_s f_y}{u\varepsilon_o} = \frac{0.31 \times 60000}{800 \times \pi * 0.625} = 11.84\ in.$$

$$\therefore L_d(provided) = (3.335 \times 12) - 3 = 37.02\ in. > L_d\ (11.84\ in.) \quad (ok)$$

Reinforcement detailing:



Pb-10. Design a rectangular footing for an 18 in. square interior column with a dead load of 180 kips and a live load of 150 kips. Make the length of the long side equal to twice the width of the short side. The allowable soil pressure is 4.0 ksf. Assume the material strengths are $f_c' = 4$ ksi and $f_y = 60$ ksi. The base of the footing is 5 ft. below the grade.

Sol^n:

Here, Allowable soil pressure, $q_{all} = 4.0\ ksf = 4000\ psf$

Average unit weight of concrete and soil below the grade $= \frac{150 + 100}{2} = 125\ pcf$

$\therefore$ Effective soil pressure, $q_e = 4000 - 5 \times 125 = 3375\ psf = 3.375\ ksf$

Self weight of foundation $= 0.15 \times (180 + 150) = 49.5$ kips

$\therefore$ Total load on ground $= 180 + 150 + 49.5 = 379.5 \approx 380$ kips

Now, Foundation area $= \frac{P}{q_e} = \frac{380}{3.375} = 112.6\ ft^2 = b \times 2b$

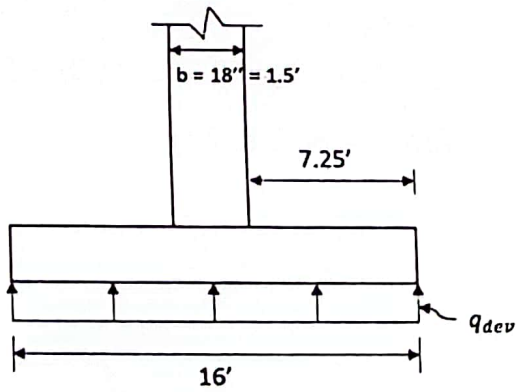
Short side of foundation, $b = \sqrt{\frac{112.6}{2}} = 7.503 \approx 8\ ft$

$\therefore$ Long side of foundation $= 2 \times 8 = 16\ ft$

So, Foundation size is $16' \times 8'$

$\therefore$ Ultimate soil pressure, $q_u = \frac{1.2DL + 1.6LL}{A} = \frac{1.2 \times 180 + 1.6 \times 150 + 1.2 \times 49.5}{14 \times 8}$

$\therefore q_u = 4.6\ ksf$



$\therefore M_u = \frac{wl^2}{2} = \frac{4.6 \times 7.25^2}{2} = 120.89\ k-ft$

Depth check:

(i) Flexure check:

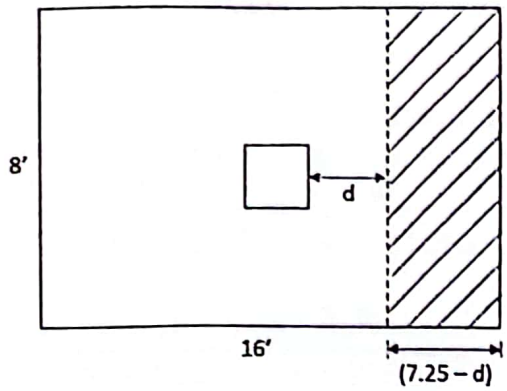
$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{4}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0181$$

Now, $M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$

$\Rightarrow 120.89 \times 12 = 0.90 \times 0.0181 \times 60 \times 12 \times d^2 \times \left(1 - 0.59 \times 0.0181 \times \frac{60}{4}\right)$

$\therefore d = 12.14\ in.$

(ii) Wide beam shear check:



Ultimate shear, $V_u = q_u \times bd' = 4.6 \times 1' \times (7.25 - d)$

Concrete shear stress, $v = 2\varphi\sqrt{f_c'} = 2 \times 0.75 \times \sqrt{4000} = 94.87\ psi = 13.66\ ksf$

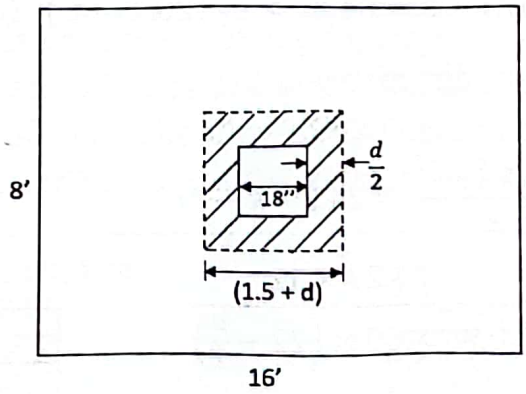
Allowable shear, $V_{all} = vbd = 13.66 \times 1' \times d$

Now, $V_u = V_{all}$

$\Longrightarrow 4.6 \times 1' \times (7.25 - d) = 13.66 \times 1' \times d$

$\therefore d = 1.826\ ft = 21.92\ in.$

(iii) Punching shear check:



Ultimate punching force, $V_u = q_u \times [A_f - A_p] = 4.6 \times [(16 \times 8) - (1.5 + d)^2]$

Punching shear stress, $v = 4\varphi\sqrt{f_c'} = 4 \times 0.75 \times \sqrt{4000} = 189.74\ psi = 27.32\ ksf$

Allowable punching shear force, $V_{all} = vb_o d = 27.325 \times \{4 * (1.5 + d)\} \times d$

Now, $V_u = V_{all}$

$\Longrightarrow 4.6 \times [(16 \times 8) - (1.5 + d)^2] = 27.32 \times \{4 * (1.5 + d)\} \times d$

$\therefore d = 1.605\ ft = 19.26\ in.$

Taking maximum $d = 21.92\ in. \approx 22\ in.$

$\therefore$ Minimum foundation thickness, $t = d + cc = 22 + 3 = 25\ in.$

Checking,

$$\text{Foundation self weight} = \left(16 \times 8 \times \frac{25}{12}\right) \times 150 = 40\ kips < 49.5\ kips\ \ (ok)$$

Reinforcement calculation:

(i) Longitudinal steel (in long direction)

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{120.89 \times 12}{0.90 \times 60 \times \left(22 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 4 \times 12}$$

$$\therefore A_s = 1.27\ in^2/ft$$

Provide #8 bar @ 7 in. c/c

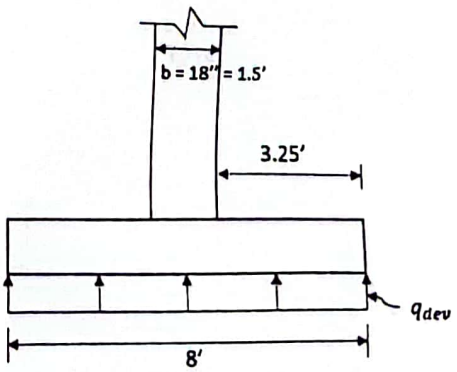
Development length check (for #8 bar):

$$u = \frac{9.5\sqrt{f_c'}}{d_b} \leq 800; \quad \therefore u = \frac{9.5\sqrt{4000}}{1.0} = 600.83$$

$$L_d = \frac{A_s f_y}{u \varepsilon_o} = \frac{0.79 \times 60000}{600.83 \times \pi * 1.0} = 25.11\ in.$$

$$\therefore L_d(provided) = (7.25 \times 12) - 3 = 84\ in. > L_d\ (25.11\ in.)\ \ (ok)$$

(ii) Distribution steel (in short direction)



$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{4.6 \times 3.25^2}{2} = 24.29\ k - ft$$

$$\therefore A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{24.29 \times 12}{0.90 \times 60 \times \left(22 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 4 \times 12}$$

$$\therefore A_s = 0.256\ in^2/ft$$

$$\therefore A_s(min) = \frac{200}{f_y} bd = \frac{200}{60000} \times 12 \times 22 = 0.88\ in^2/ft$$

Provide #7 bar @ 8 in. c/c

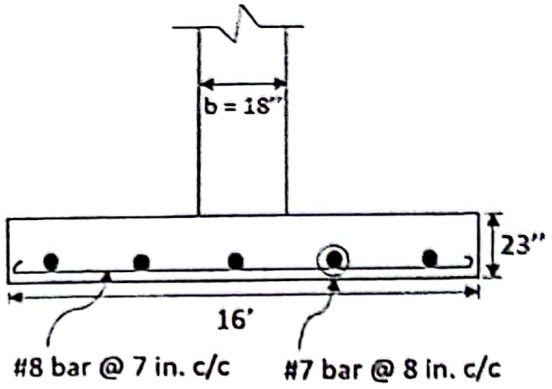
Development length check (for #7 bar):

$$u = \frac{9.5\sqrt{f_c'}}{d_b} \leq 800; \quad \therefore u = \frac{9.5\sqrt{4000}}{0.875} = 686.67$$

$$L_d = \frac{A_s f_y}{u\varepsilon_o} = \frac{0.60 \times 60000}{686.67 \times \pi * 0.875} = 19.07\ in.$$

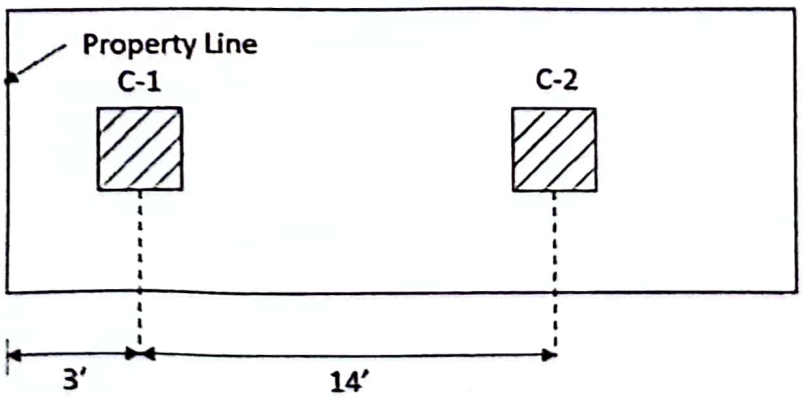
$$\therefore L_d(provided) = (3.25 \times 12) - 3 = 36\ in. > L_d\ (19.07\ in.)\ \ (ok)$$

Reinforcement detailing:



3.4.5 Combined Footing – WSD

Pb-11. Design a rectangular combined footing to support two 18 in. square columns as shown in figure. The C-1 carries DL of 250 kips and LL of 140 kips and the C-2 carries DL of 330 kips and LL of 180 kips. The allowable soil pressure is 3 ksf. Material strengths are $f_c' = 3$ ksi and $f_y = 60$ ksi. [*CE-2019 (similar), 2018, 2017(similar)*]



Sol^n:

Total load of column C1 & C2 = 390 + 510 = 900 kips

Self weight of foundation = 0.15 × 900 = 135 kips

∴ Total load on ground = 900 + 135 = 1035 kips

Now, Foundation area $= \frac{P}{q_{all}} = \frac{1035}{3} = 345\ ft^2$

$\therefore$ Load c. g $= \frac{390 \times 0 + 510 \times 14}{390 + 510} = 7.93\ ft$ (from C1)

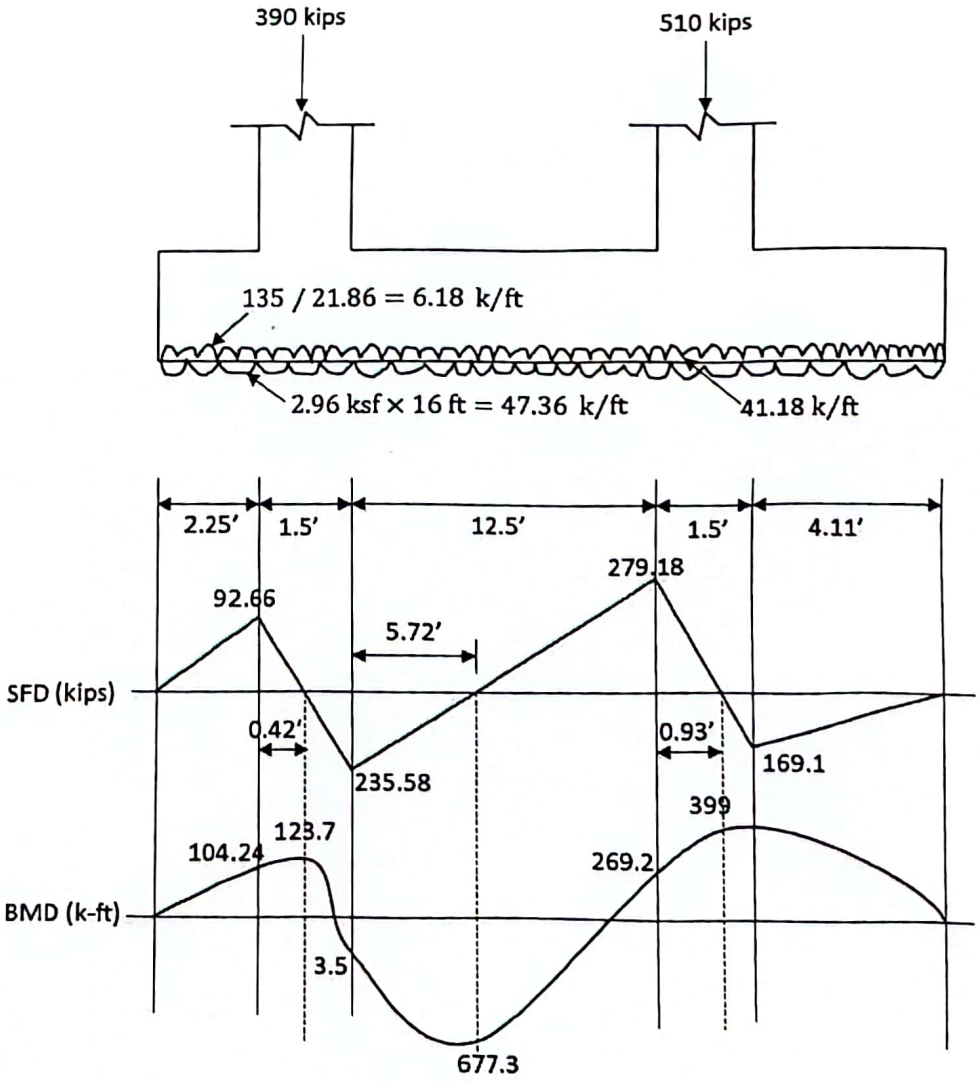
Foundation length $= (3 + 7.93) \times 2 = 21.86\ ft$

$\therefore$ Foundation width $= \frac{345}{21.86} = 15.78 \approx 16\ ft$

So, Foundation size is $21.86' \times 16'$

$\therefore$ Developed soil pressure, $q_{dev} = \frac{1035}{21.86 \times 16} = 2.96$ ksf

Diagram of SFD and BMD:



Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \approx 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 3} = \frac{24}{1.35} = 17.77$$

$$k = \frac{n}{n+r} = 0.336$$

$$j = 1 - \frac{k}{3} = 0.888$$

Depth check:

(i) Flexure check:

Maximum moment = 677.3 k-ft.

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Rightarrow 677.3 \times 12 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 16 * 12 \times d^2$$

$$\therefore d = 14.5\ in.$$

(ii) Wide beam shear check:

Maximum shear occurs at interior face of C-2,

Developed shear, $V_{dev} = 279.18 - 41.18 \times d$

Concrete shear stress, $v = 1.1\sqrt{f_c'} = 1.1\sqrt{3000} = 60.25\ psi = 8.68\ ksf$

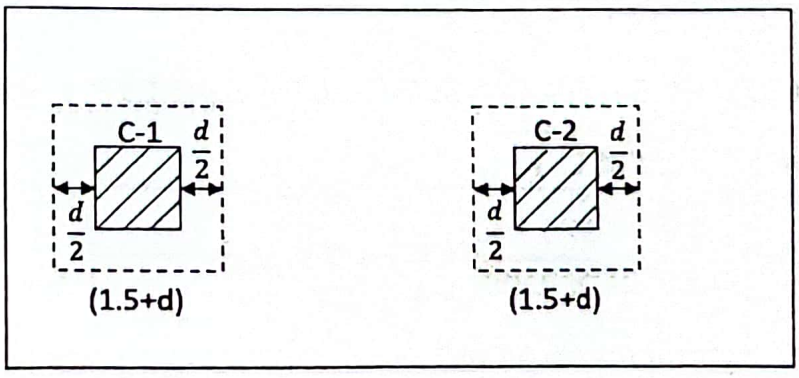
Allowable shear, $V_{all} = vbd = 8.68 \times 16' \times d$

Now, $V_{dev} = V_{all}$

$$\Rightarrow 279.18 - 41.18 \times d = 8.68 \times 16' \times d$$

$$\therefore d = 18.6\ in.$$

(iii) Punching shear check:



The punching shear section is more critical for C-2,

Developed punching force, $V_{dev} = 510 - (1.5 + d)^2 \times 2.96$

Punching shear stress, $v = 2\sqrt{f_c'} = 2\sqrt{3000} = 109.54\ psi = 15.77\ ksf$

Allowable punching shear force, $V_{all} = vb_o d = 15.77 \times \{4 * (1.5 + d)\} \times d$

Now, $V_{dev} = V_{all}$

$\Rightarrow 510 - (1.5 + d)^2 \times 2.96 = 15.77 \times \{4 * (1.5 + d)\} \times d$

$\therefore d = 2.09\ ft = 25.08\ in.$

Taking maximum $d = 25.08\ in. \approx 26\ in.$

$\therefore$ Minimum foundation thickness, $t = d + cc = 26 + 3 = 29\ in.$

Checking,

Foundation self weight $= \left(21.86 \times 16 \times \frac{29}{12}\right) \times 150 = 126.8\ kips < 135\ kips\ (ok)$

Reinforcement calculation:

(a) Longitudinal steel (in long direction)

(i) For (-)ve moment (top bar)

$$\therefore A_s = \frac{M}{f_s jd} = \frac{677.3 \times 12}{24 \times 0.888 \times 26} = 14.67\ in^2$$

Here, $A_s(min) = \frac{200}{f_y} bd = \frac{200}{60000} \times (16 * 12) \times 26 = 16.64\ in^2$

Provide 14 - #10 bars

Development length check (for #10 bar):

$$u = \frac{3.4\sqrt{f_c'}}{d_b} \leq 350; \quad \therefore u = \frac{3.4\sqrt{3000}}{1.27} = 147\ psi$$

$$L_d = \frac{A_s f_s}{u\varepsilon_o} = \frac{1.27 \times 24000}{147 \times \pi * 1.27} = 51.97''$$

$\therefore L_d(provided) = (5.72 + 1.5 - 0.42) \times 12 = 81.6'' > L_d\ (51.97'')\ (ok)$

(ii) For (+)ve moment at C-1

$$\therefore A_s = \frac{M}{f_s jd} = \frac{123.7 \times 12}{24 \times 0.888 \times 26} = 2.68\ in^2$$

Here, $A_s(min) = 16.64\ in^2$

Provide 17 - #9 bars

Development length check (for #9 bar):

$$u = \frac{4.8\sqrt{f_c'}}{d_b} \leq 500; \quad \therefore u = \frac{4.8\sqrt{3000}}{1.125} = 234\ psi$$

$$L_d = \frac{A_s f_s}{u\varepsilon_o} = \frac{1.0 \times 24000}{234 \times \pi * 1.125} = 29''$$

$\therefore L_d(provided) = (2.25 + 0.42) \times 12 - 3 = 29.04'' > L_d\ (29'')\ \ (ok)$

(iii) For (+)ve moment at C-2

$$\therefore A_s = \frac{M}{f_s jd} = \frac{399 \times 12}{24 \times 0.888 \times 26} = 8.64\ in^2$$

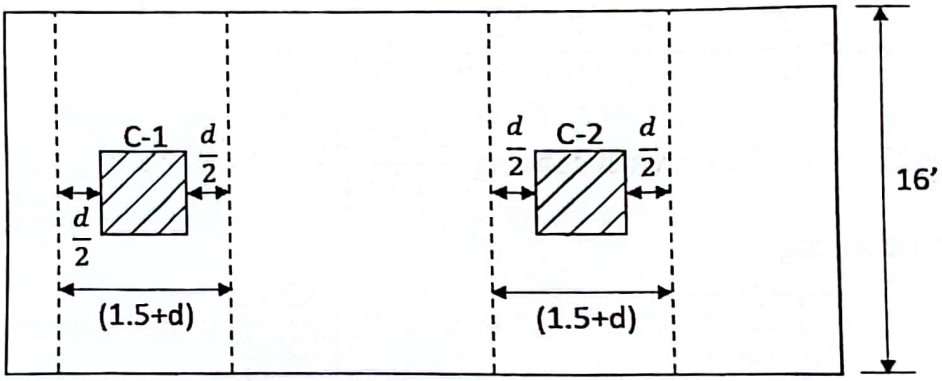
Here, $A_s(min) = 16.64\ in^2$

Provide 17 - #9 bars

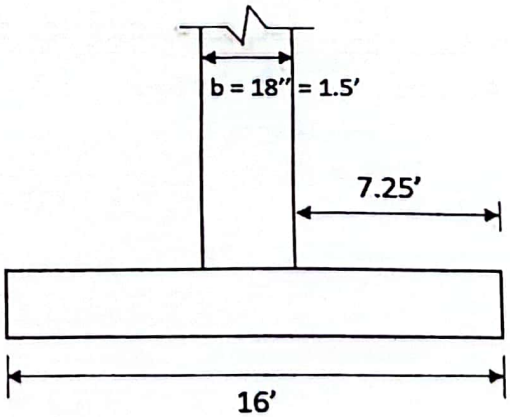
$\therefore L_d(provided) = (4.11 + 1.5 - 0.93) \times 12 - 3 = 53'' > L_d\ (29'')\ \ (ok)$

(b) Distribution steel

Design of transverse beam under interior column:



For transverse beam under C-1,



Load on C-1 = 390 kips

$$\text{UDL on beam} = \frac{390}{16} = 24.375\ k/ft$$

Effective width of transverse beam $= 1.5 + \text{d} = 1.5 + \frac{26}{12} = 3.67' = 44''$

$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{24.375 \times 7.25^2}{2} = 640.6\ k-ft$$

$$\therefore A_s = \frac{M}{f_s jd} = \frac{640.6 \times 12}{24 \times 0.888 \times 26} = 13.87\ in^2$$

Provide $11 - \#10$ bars with spacing $\frac{44 \times 1.27}{13.87} = 4\ in.\ c/c$

For transverse beam under C-2,

Load on C-2 = 510 kips

$$\text{UDL on beam} = \frac{510}{16} = 31.875\ k/ft$$

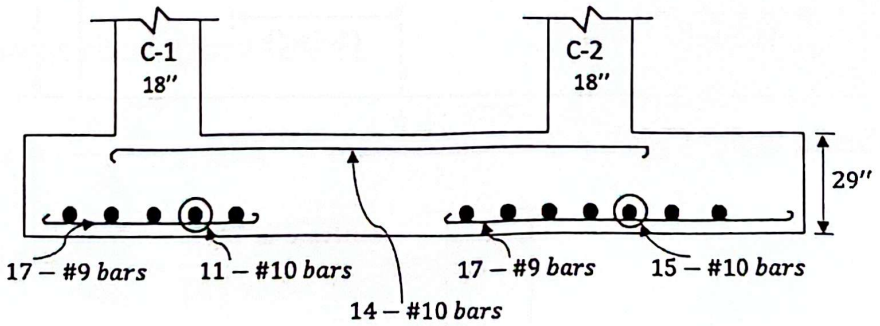
Effective width of transverse beam $= 1.5 + \text{d} = 44''$

$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{31.875 \times 7.25^2}{2} = 837.7\ k-ft$$

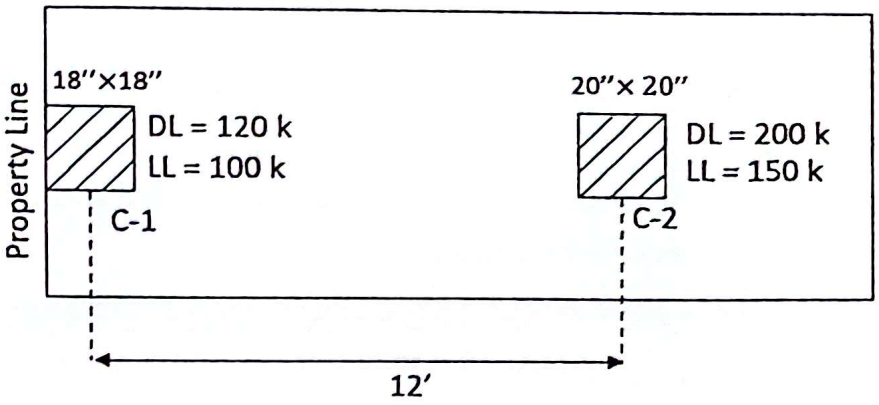
$$\therefore A_s = \frac{M}{f_s jd} = \frac{837.7 \times 12}{24 \times 0.888 \times 26} = 18.14\ in^2$$

Provide $15 - \#10$ bars with spacing $\frac{44 \times 1.27}{18.14} = 3\ in.\ c/c$

Reinforcement detailing:



Pb-12. Design a rectangular combined footing for the two columns as shown in figure. The allowable soil pressure is 5 ksf. Material strengths are $f_c' = 3000$ psi and $f_y = 60000$ psi.



Sol^n:

Total load of column C1 & C2 $= 220 + 350 = 570$ kips

Self weight of foundation $= 0.15 \times 570 = 85.5$ kips

$\therefore$ Total load on ground $= 570 + 85.5 = 655.5$ kips

Now, Foundation area $= \dfrac{P}{q_{all}} = \dfrac{655.5}{5} = 131.1\ ft^2$

$\therefore$ Load c.g $= \dfrac{220 \times 0 + 350 \times 12}{220 + 350} = 7.37\ ft$ (from C1)

Foundation length $= \left(\dfrac{9}{12} + 7.37\right) \times 2 = 16.24\ ft$

$\therefore$ Foundation width $= \dfrac{131.1}{16.24} = 8.07 \approx 9\ ft$

So, Foundation size is $16.24' \times 9'$

$\therefore$ Developed soil pressure, $q_{dev} = \dfrac{655.5}{16.24 \times 9} = 4.48$ ksf

Design constants:

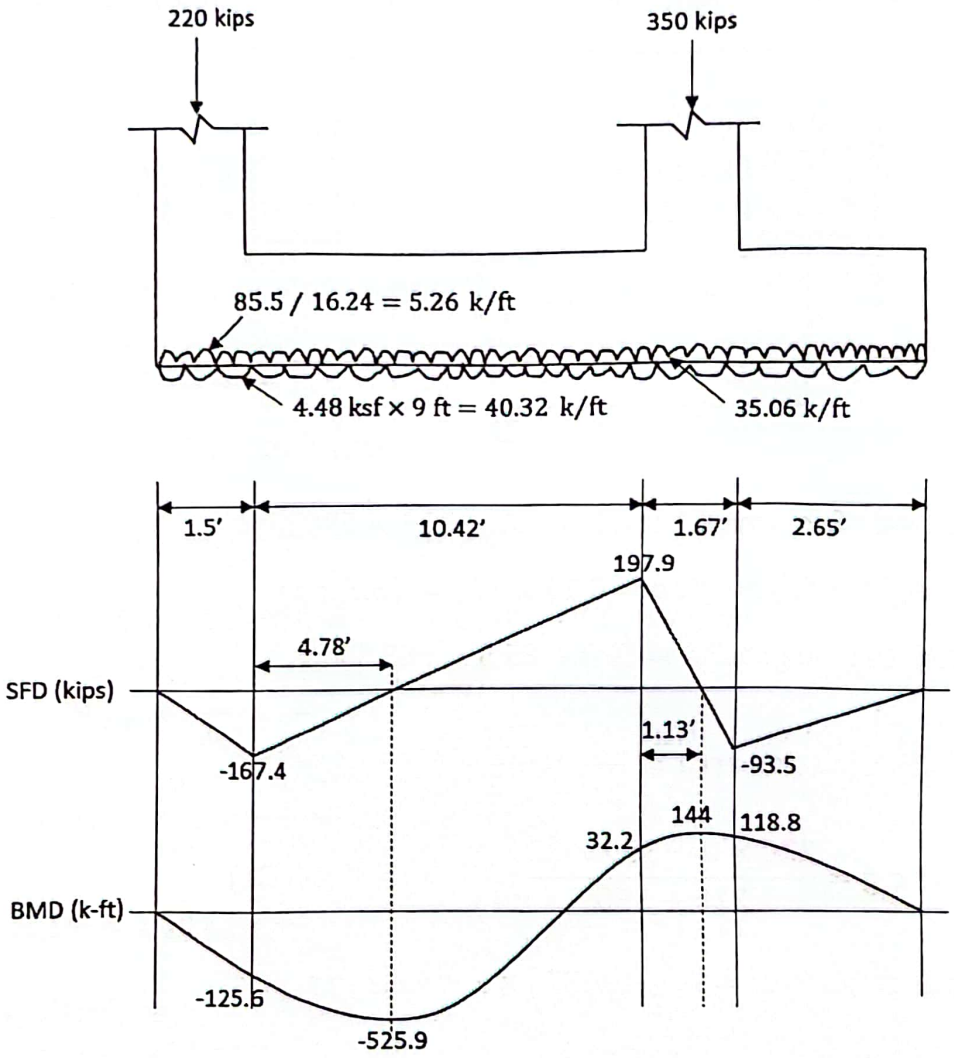
$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \approx 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 3} = \frac{24}{1.35} = 17.77$$

$$k = \frac{n}{n + r} = 0.336$$

$$j = 1 - \frac{k}{3} = 0.888$$

Diagram of SFD and BMD:



Depth check:

(i) Flexure check:

Maximum moment = 525.9 k-ft.

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Rightarrow 525.9 \times 12 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 9 * 12 \times d^2$$

$$\therefore d = 17.03\ in.$$

(ii) Wide beam shear check:

Maximum shear occurs at interior face of C-2,

Developed shear, $V_{dev} = 197.9 - 35.06 \times d$

Concrete shear stress, $v = 1.1\sqrt{f_c'} = 1.1\sqrt{3000} = 60.25\ psi = 8.68\ ksf$

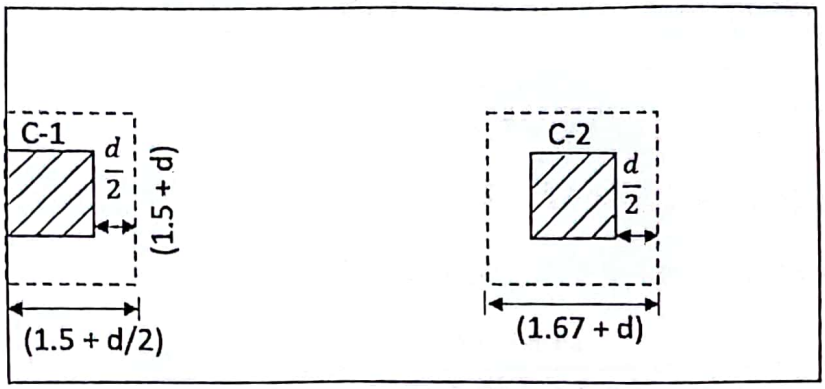
Allowable shear, $V_{all} = vbd = 8.68 \times 9' \times d$

Now, $V_{dev} = V_{all}$

$\Rightarrow 197.9 - 35.06 \times d = 8.68 \times 9' \times d$

$\therefore d = 1.749\ ft = 20.98\ in.$

(iii) Punching shear check:



Punching shear stress, $v = 2\sqrt{f_c'} = 2\sqrt{3000} = 109.54\ psi = 15.77\ ksf$

For, C-1

Developed punching force, $V_{dev} = 220 - \left\{\left(1.5 + {}^{d}/_{2}\right) \times (1.5 + d)\right\} \times 4.48$

Allowable punching shear force, $V_{all} = vb_o d = 15.77 \times \left\{2\left(1.5 + {}^{d}/_{2}\right) + (1.5 + d)\right\} \times d$

Now, $V_{dev} = V_{all}$

$\Rightarrow 220 - \left\{\left(1.5 + {}^{d}/_{2}\right) \times (1.5 + d)\right\} \times 4.48 = 15.77 \times \left\{2\left(1.5 + {}^{d}/_{2}\right) + (1.5 + d)\right\} \times d$

$\therefore d = 1.93\ ft = 23.16\ in.$

For, C-2

Developed punching force, $V_{dev} = 350 - (1.67 + d)^2 \times 4.48$

Allowable punching shear force, $V_{all} = vb_o d = 15.77 \times \{4 * (1.67 + d)\} \times d$

Now, $V_{dev} = V_{all}$

$\Rightarrow 350 - (1.67 + d)^2 \times 4.48 = 15.77 \times \{4 * (1.67 + d)\} \times d$

$\therefore d = 1.52\ ft = 18.19\ in.$

Taking maximum $d = 23.16\ in. \approx 24\ in.$

$\therefore$ Minimum foundation thickness, $t = d + cc = 24 + 3 = 27\ in.$

Checking,

Foundation self weight $= \left(16.24 \times 9 \times \frac{27}{12}\right) \times 150 = 49.33\ kips < 85.5\ kips\ \ (ok)$

Reinforcement calculation:

(a) Longitudinal steel (in long direction)

(i) For (-)ve moment (top bar)

$$\therefore A_s = \frac{M}{f_s jd} = \frac{525.9 \times 12}{24 \times 0.888 \times 24} = 12.34\ in^2$$

Provide 10 - #10 bars

Development length check (for #10 bar):

$$u = \frac{3.4\sqrt{f_c'}}{d_b} \leq 350; \quad \therefore u = \frac{3.4\sqrt{3000}}{1.27} = 147\ psi$$

$$L_d = \frac{A_s f_s}{u\varepsilon_o} = \frac{1.27 \times 24000}{147 \times \pi * 1.27} = 51.97''$$

$$\therefore L_d(provided) = (4.78 + 1.5) \times 12 - 3 = 72.36'' > L_d\ (51.97'')\ \ (ok)$$

(ii) For (+)ve moment at C-2 (bottom bar)

$$\therefore A_s = \frac{M}{f_s jd} = \frac{144 \times 12}{24 \times 0.888 \times 24} = 3.38\ in^2$$

Here, $A_s(min) = \frac{200}{f_y} bd = \frac{200}{60000} \times (9 * 12) \times 24 = 8.64\ in^2$

Provide 9 - #9 bars

Development length check (for #9 bar):

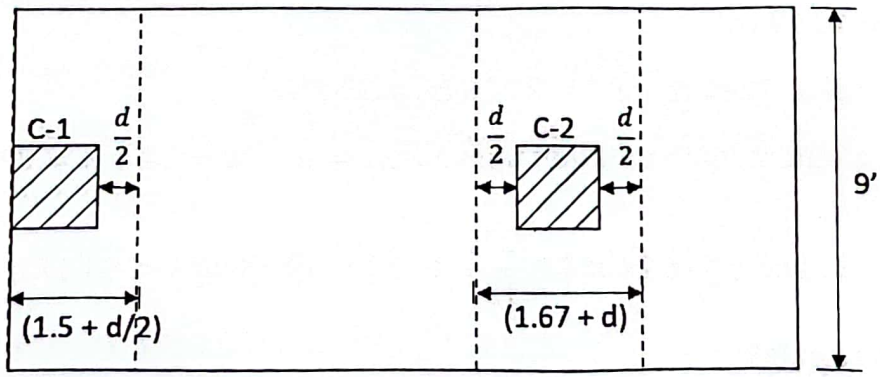
$$u = \frac{4.8\sqrt{f_c'}}{d_b} \leq 500; \quad \therefore u = \frac{4.8\sqrt{3000}}{1.125} = 234\ psi$$

$$L_d = \frac{A_s f_s}{u\varepsilon_o} = \frac{1.0 \times 24000}{234 \times \pi * 1.125} = 29''$$

$$\therefore L_d(provided) = (2.65 + 1.67 - 1.13) \times 12 - 3 = 35.28'' > L_d\ (29'')\ \ (ok)$$

(b) Distribution steel

Design of transverse beam under interior column:



For transverse beam under C-1,

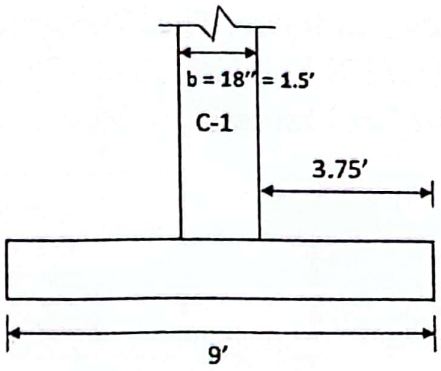
Load on C-1 = 220 kips

UDL on beam $= \frac{220}{9} = 24.44\ k/ft$

Effective width of transverse beam $= 1.5 + \frac{d}{2} = 1.5 + \frac{24}{2 \times 12} = 2.5' = 30''$

$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{24.44 \times 3.75^2}{2} = 171.8\ k-ft$$

$$\therefore A_s = \frac{M}{f_s jd} = \frac{171.8 \times 12}{24 \times 0.888 \times 24} = 4.03\ in^2$$



Provide 7 – #7 bars with spacing $\frac{30 \times 0.60}{4.03} = 4\ in.\ c/c$

For transverse beam under C-2,

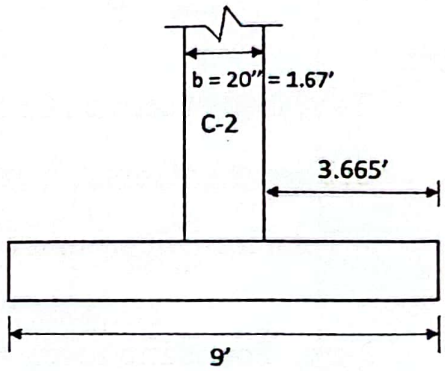
Load on C-2 = 350 kips

UDL on beam $= \frac{350}{9} = 38.89\ k/ft$

Effective width of transverse beam = 1.5 + d = 44″

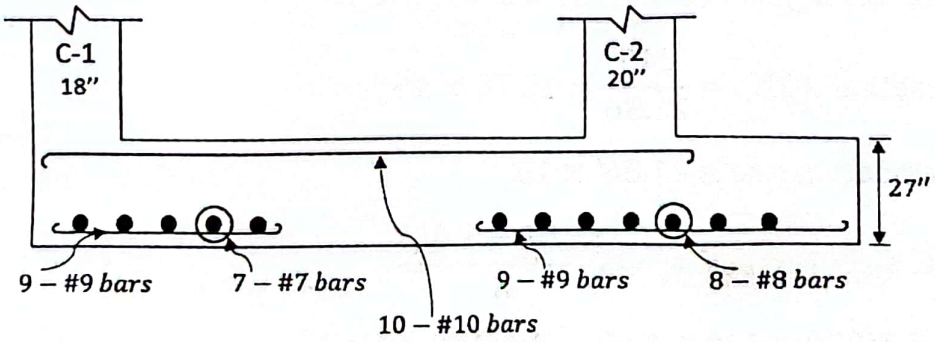
$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{38.89 \times 3.665^2}{2} = 261.49\ k-ft$$

$$\therefore A_s = \frac{M}{f_s jd} = \frac{261.19 \times 12}{24 \times 0.888 \times 24} = 6.13\ in^2$$



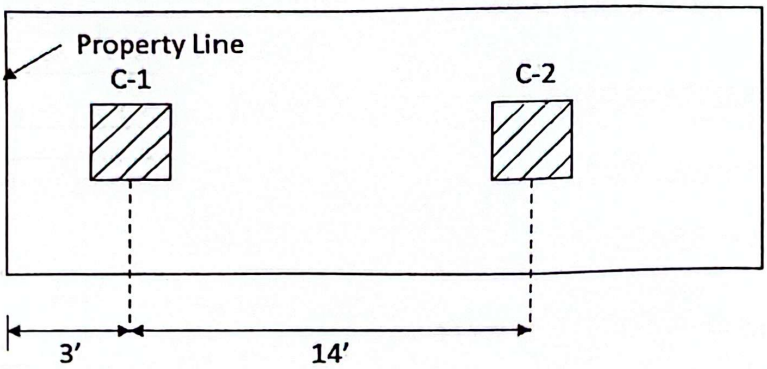
Provide 8 – #8 bars with spacing $\frac{44 \times 0.79}{6.13} = 5\ in.\ c/c$

Reinforcement detailing:



3.4.6 Combined Footing – USD

Pb-13. Design a rectangular combined footing to support two 18 in. square columns as shown in figure. The C-1 carries DL of 250 kips and LL of 140 kips and the C-2 carries DL of 330 kips and LL of 180 kips. The allowable soil pressure is 3 ksf. Material strengths are $f_c' = 3$ ksi and $f_y = 60$ ksi. [*CE-2016*]



Sol^n:

Total load of column C1 & C2 $= 390 + 510 = 900$ kips

Self weight of foundation $= 0.15 \times 900 = 135$ kips

$\therefore$ Total load on ground $= 900 + 135 = 1035$ kips

Now, Foundation area $= \dfrac{P}{q_{all}} = \dfrac{1035}{3} = 345\ ft^2$

$\therefore$ Load c.g $= \dfrac{390 \times 0 + 510 \times 14}{390 + 510} = 7.93\ ft$ (from C1)

Foundation length $= (3 + 7.93) \times 2 = 21.86\ ft$

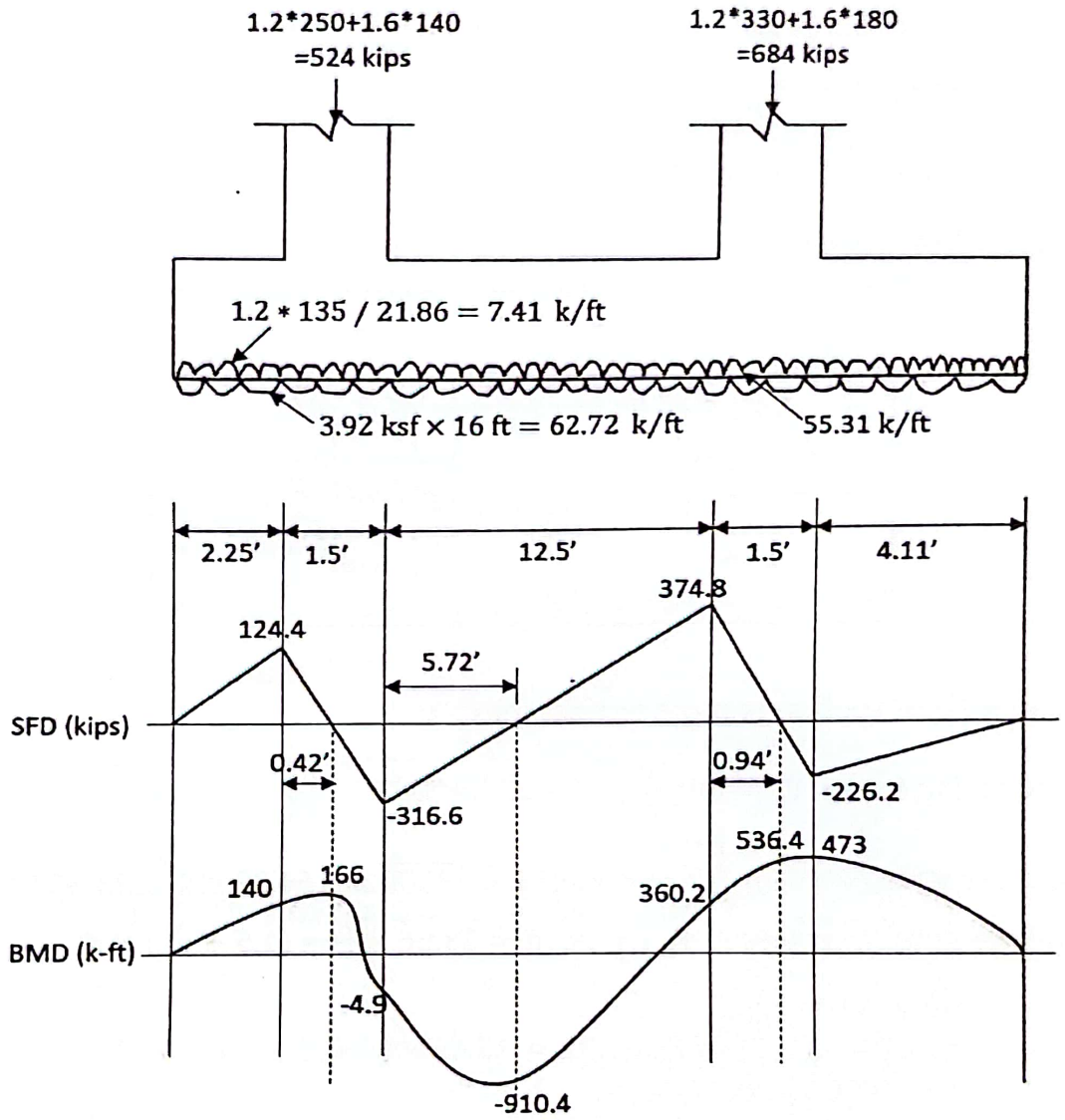
$\therefore$ Foundation width $= \dfrac{345}{21.86} = 15.78 \approx 16\ ft$

So, Foundation size is $21.86' \times 16'$

Ultimate soil pressure, $q_u = \dfrac{1.2DL + 1.6LL}{A}$

$\therefore q_u = \dfrac{1.2(250 + 330 + 135) + 1.6(140 + 180)}{21.86 \times 16} = 3.92$ ksf

Diagram of SFD and BMD:



Depth check:

(i) Flexure check:

Maximum moment = 910.4 k-ft.

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{4}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0181$$

Now, $M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$

$$\Rightarrow 910.4 \times 12 = 0.90 \times 0.0181 \times 60 \times (16 * 12) \times d^2 \times \left(1 - 0.59 \times 0.0181 \times \frac{60}{3}\right)$$

$$\therefore d = 8.6\ in.$$

(ii) Wide beam shear check:

Maximum shear occurs at interior face of C-2,

Ultimate shear, $V_u = 374.8 - 55.31 \times d$

Concrete shear stress, $v = 2\varphi\sqrt{f_c'} = 2 \times 0.75 \times \sqrt{3000} = 82.16\ psi = 11.83\ ksf$

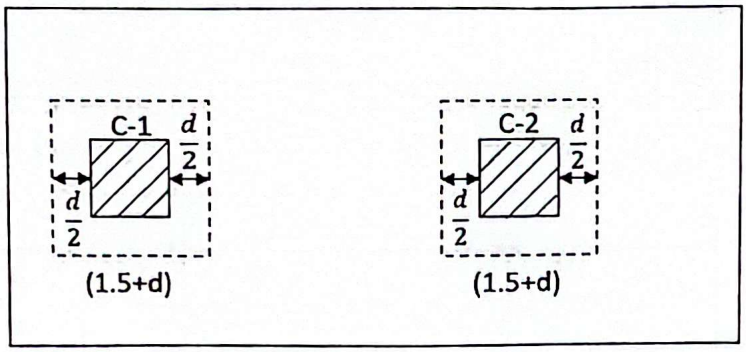
Allowable shear, $V_{all} = vbd = 11.83 \times 16' \times d$

Now, $V_u = V_{all}$

$\Rightarrow 374.8 - 55.31 \times d = 11.83 \times 16' \times d$

$\therefore d = 1.53' = 18.4\ in.$

(iii) Punching shear check:



The punching shear section is more critical for C-2,

Ultimate punching force, $V_u = 684 - (1.5 + d)^2 \times 3.92$

Punching shear stress, $v = 4\varphi\sqrt{f_c'} = 4 \times 0.75 \times \sqrt{3000} = 164.32\ psi = 23.66\ ksf$

Allowable punching shear force, $V_{all} = vb_o d = 23.66 \times \{4 * (1.5 + d)\} \times d$

Now, $V_u = V_{all}$

$\Rightarrow 684 - (1.5 + d)^2 \times 3.92 = 23.66 \times \{4 * (1.5 + d)\} \times d$

$\therefore d = 1.95\ ft = 23.41\ in.$

Taking maximum $d = 23.41\ in. \approx 24\ in.$

$\therefore$ Minimum foundation thickness, $t = d + cc = 24 + 3 = 27\ in.$

Checking,

Foundation self weight $= \left(21.86 \times 16 \times \frac{27}{12}\right) \times 150 = 118\ kips < 135\ kips\ \ (ok)$

Reinforcement calculation:

(a) Longitudinal steel (in long direction)

(i) For (-)ve moment (top bar)

$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{910.4 \times 12}{0.90 \times 60 \times \left(24 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times (16 * 12)}$$

$$\therefore A_s = 8.62\ in^2$$

Here, $A_s(min) = \frac{200}{f_y} bd = \frac{200}{60000} \times (16 * 12) \times 24 = 15.36\ in^2$

Provide 13 - #10 bars

Development length check (for #10 bar):

$$u = \frac{6.7\sqrt{f_c'}}{d_b} \leq 560; \quad \therefore u = \frac{6.7\sqrt{3000}}{1.27} = 289\ psi$$

$$L_d = \frac{A_s f_s}{u \varepsilon_o} = \frac{1.27 \times 24000}{289 \times \pi * 1.27} = 26.43''$$

$\therefore L_d(provided) = (5.72 + 1.5 - 0.42) \times 12 = 81.6'' > L_d\ (26.43'')\ \ (ok)$

(ii) For (+)ve moment at C-1

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{166 \times 12}{0.90 \times 60 \times \left(24 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times (16 * 12)}$$

$$\therefore A_s = 1.54\ in^2$$

Here, $A_s(min) = 15.36\ in^2$

Provide 13 - #10 bars

Development length check (for #10 bar):

$$u = \frac{9.5\sqrt{f_c'}}{d_b} \leq 800; \quad \therefore u = \frac{9.5\sqrt{3000}}{1.27} = 410\ psi$$

$$L_d = \frac{A_s f_s}{u \varepsilon_o} = \frac{1.27 \times 24000}{410 \times \pi * 1.27} = 18.6''$$

$\therefore L_d(provided) = (2.25 + 0.42) \times 12 - 3 = 29.04'' > L_d\ (18'')\ \ (ok)$

(iii) For (+)ve moment at C-2

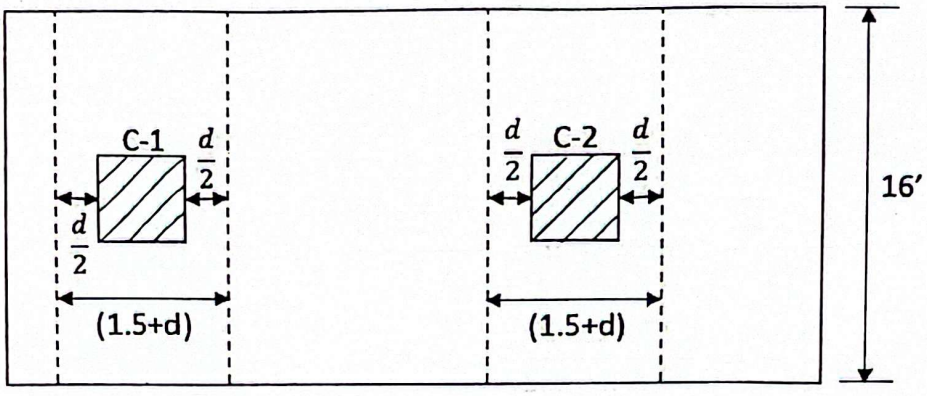
Here, $A_s(min) = 15.36\ in^2$

Provide 13 - #10 bars

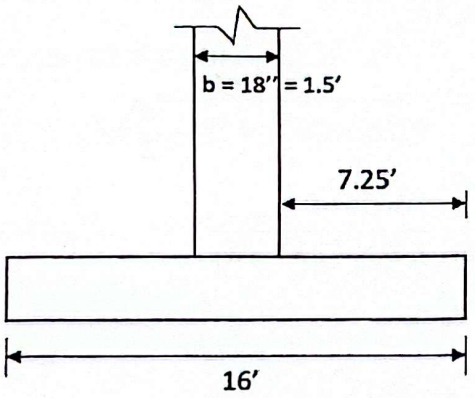
$\therefore L_d(provided) = (4.11 + 1.5 - 0.94) \times 12 - 3 = 53'' > L_d\ (18'')\ \ (ok)$

(b) Distribution steel

Design of transverse beam under interior column:



For transverse beam under C-1,



Load on C-1 = 524 kips

$$\text{UDL on beam} = \frac{524}{16} = 32.75\ k/ft$$

$$\text{Effective width of transverse beam} = 1.5 + \text{d} = 1.5 + \frac{24}{12} = 3.5' = 42''$$

$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{32.75 \times 7.25^2}{2} = 860.7\ k - ft$$

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{860.7 \times 12}{0.90 \times 60 \times \left(24 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 42}$$

$$\therefore A_s = 9.11\ in^2$$

Provide 12 – #8 bars with spacing $\frac{42 \times 0.79}{9.11} = 3\ in.\ c/c$

For transverse beam under C-2,

Load on C-2 = 684 kips

UDL on beam $= \frac{510}{16} = 42.75\ k/ft$

Effective width of transverse beam = 1.5 + d = 42″

$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{42.75 \times 7.25^2}{2} = 1123.5\ k-ft$$

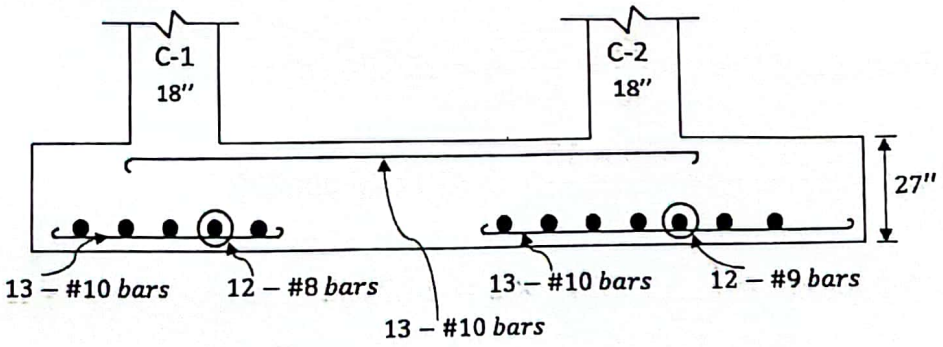
$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{1123.5 \times 12}{0.90 \times 60 \times \left(24 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 42}$$

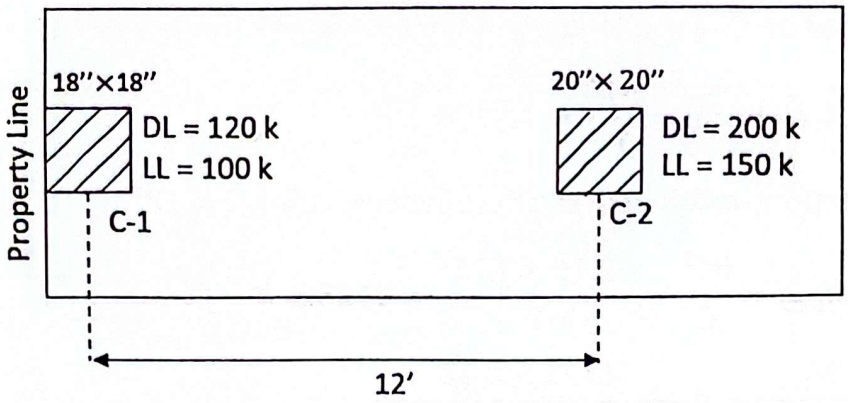
$$\therefore A_s = 11.64\ in^2$$

Provide 12 – #9 bars with spacing $\frac{42 \times 1.0}{11.64} = 3\ in.\ c/c$

Reinforcement detailing:



Pb-14. Design a rectangular combined footing for the two columns as shown in figure. The allowable soil pressure is 5 ksf. Material strengths are $f_c' = 3000$ psi and $f_y = 60000$ psi. The bottom of the footing is to be 6 ft. below grade.



Sol^n:

Here, Allowable soil pressure, $q_{all} = 5\ ksf = 5000\ psf$

Avg. unit weight of concrete and soil below the grade $= (150 + 100)/2 = 125\ pcf$

$\therefore$ Effective soil pressure, $q_e = 5000 - 6 \times 125 = 4250\ psf = 4.25\ ksf$

Total load of column C1 & C2 $= 220 + 350 = 570$ kips

Self weight of foundation $= 0.15 \times 570 = 85.5$ kips

$\therefore$ Total load on ground $= 570 + 85.5 = 655.5$ kips

Now, Foundation area $= \dfrac{P}{q_e} = \dfrac{655.5}{4.25} = 154.24\ ft^2$

$\therefore$ Load c.g $= \dfrac{220 \times 0 + 350 \times 12}{220 + 350} = 7.37\ ft$ (from C1)

Foundation length $= \left(\dfrac{9}{12} + 7.37\right) \times 2 = 16.24\ ft$

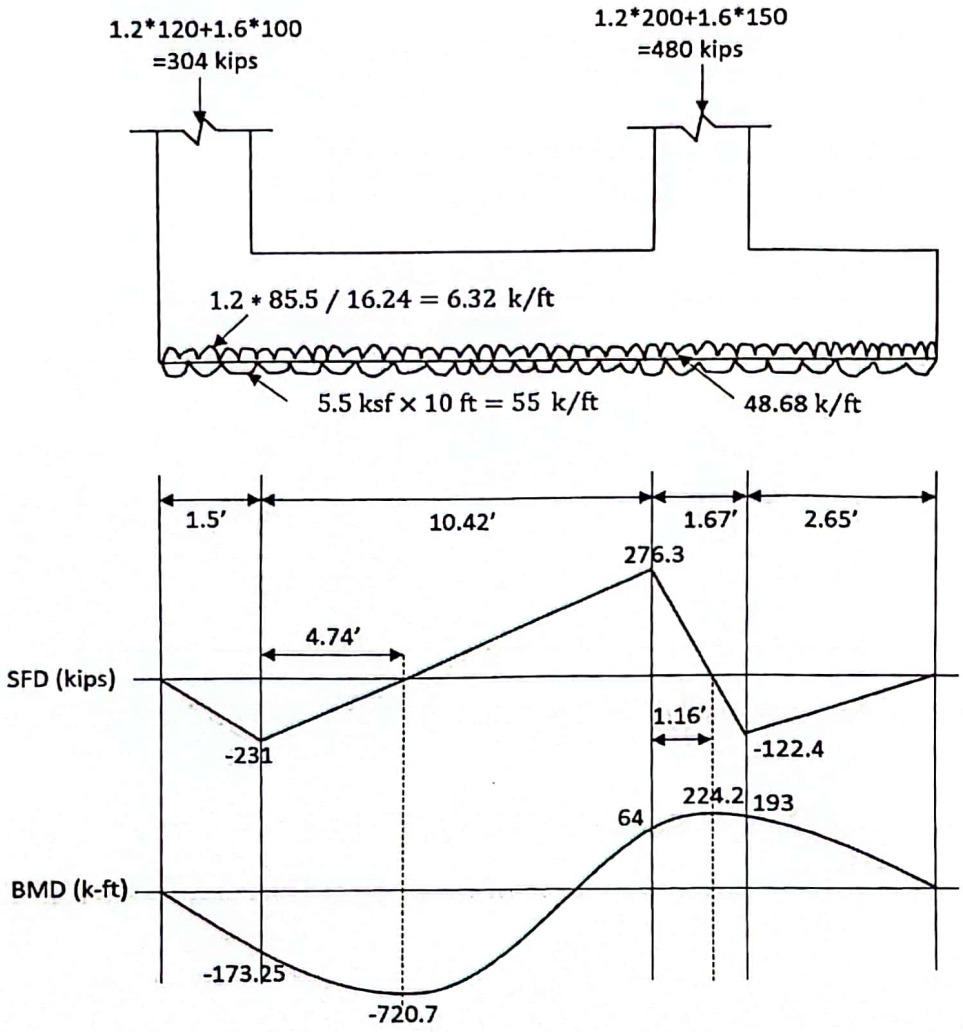
$\therefore$ Foundation width $= \dfrac{154.24}{16.24} = 9.50 \approx 10\ ft$

So, Foundation size is $16.24' \times 10'$

Ultimate soil pressure, $q_u = \dfrac{1.2DL + 1.6LL}{A}$

$\therefore q_u = \dfrac{1.2(120 + 200 + 85.5) + 1.6(100 + 150)}{16.24 \times 10} = 5.5$ ksf

Diagram of SFD and BMD:



Depth check:

(i) Flexure check:

Maximum moment = 720.7 k-ft.

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{4}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0181$$

$$\text{Now, } M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

$$\Rightarrow 720.7 \times 12 = 0.90 \times 0.0181 \times 60 \times (10 * 12) \times d^2 \times \left(1 - 0.59 \times 0.0181 \times \frac{60}{3}\right)$$

$$\therefore d = 9.68\ in.$$

(ii) Wide beam shear check:

Maximum shear occurs at interior face of C-2,

Ultimate shear, $V_u = 276.3 - 48.68 \times d$

Concrete shear stress, $v = 2\varphi\sqrt{f_c'} = 2 \times 0.75 \times \sqrt{3000} = 82.16\ psi = 11.83\ ksf$

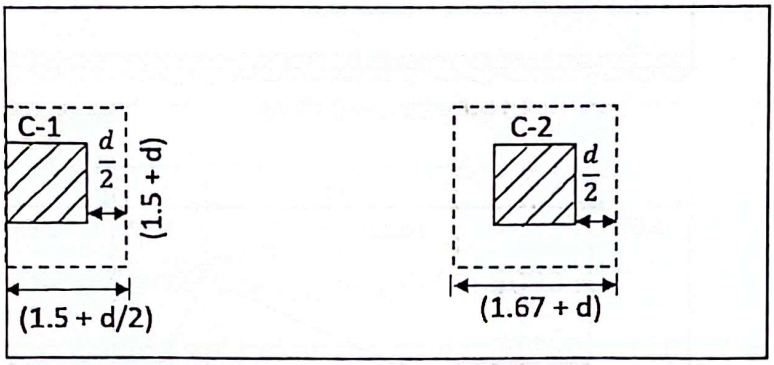
Allowable shear, $V_{all} = vbd = 11.83 \times 10' \times d$

Now, $V_{dev} = V_{all}$

$\Rightarrow 276.3 - 48.68 \times d = 11.83 \times 10' \times d$

$\therefore d = 1.65\ ft = 19.8\ in.$

(iii) Punching shear check:



Punching shear stress, $v = 4\varphi\sqrt{f_c'} = 4 \times 0.75 \times \sqrt{3000} = 164.32\ psi = 23.66\ ksf$

For, C-1

Ultimate punching force, $V_u = 304 - \left\{\left(1.5 + {}^{d}/_{2}\right) \times (1.5 + d)\right\} \times 5.5$

Allowable punching shear force, $V_{all} = vb_o d = 23.66 \times \left\{2\left(1.5 + {}^{d}/_{2}\right) + (1.5 + d)\right\} \times d$

Now, $V_u = V_{all}$

$\Rightarrow 304 - \left\{\left(1.5 + {}^{d}/_{2}\right) \times (1.5 + d)\right\} \times 5.5 = 23.66 \times \left\{2\left(1.5 + {}^{d}/_{2}\right) + (1.5 + d)\right\} \times d$

$\therefore d = 1.5\ ft = 18\ in.$

For, C-2

Developed punching force, $V_u = 480 - (1.67 + d)^2 \times 5.5$

Allowable punching shear force, $V_{all} = vb_o d = 23.66 \times \{4 * (1.67 + d)\} \times d$

Now, $V_u = V_{all}$

$\Rightarrow 480 - (1.67 + d)^2 \times 5.5 = 23.66 \times \{4 * (1.67 + d)\} \times d$

$\therefore d = 1.45\ ft = 17.4\ in.$

Taking maximum $d = 19.8\ in. \approx 20\ in.$

$\therefore$ Minimum foundation thickness, $t = d + cc = 20 + 3 = 23\ in.$

Checking,

Foundation self weight $= \left(16.24 \times 10 \times \frac{23}{12}\right) \times 150 = 46.7\ kips < 85.5\ kips\ (ok)$

Reinforcement calculation:

(a) Longitudinal steel (in long direction)

(i) For (-)ve moment (top bar)

$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{720.7 \times 12}{0.90 \times 60 \times \left(20 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times (10 * 12)}$$

$$\therefore A_s = 8.06\ in^2$$

Here, $A_s(min) = \frac{200}{f_y} bd = \frac{200}{60000} \times (10 * 12) \times 20 = 8.0\ in^2$

Provide 11 - #8 bars

Development length check (for #8 bar):

$$u = \frac{6.7\sqrt{f_c'}}{d_b} \leq 560; \quad \therefore u = \frac{6.7\sqrt{3000}}{1.0} = 367\ psi$$

$$L_d = \frac{A_s f_s}{u \varepsilon_o} = \frac{0.79 \times 24000}{367 \times \pi * 1.0} = 16.44''$$

$\therefore L_d(provided) = (4.74 + 1.5) \times 12 - 3 = 71.88'' > L_d\ (16.44'')$ (ok)

(ii) For (+)ve moment at C-2 (bottom bar)

$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{224.2 \times 12}{0.90 \times 60 \times \left(20 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times (10 * 12)}$$

$$\therefore A_s = 2.59\ in^2$$

Here, $A_s(min) = 8.0\ in^2$

Provide 8 - #9 bars

Development length check (for #9 bar):

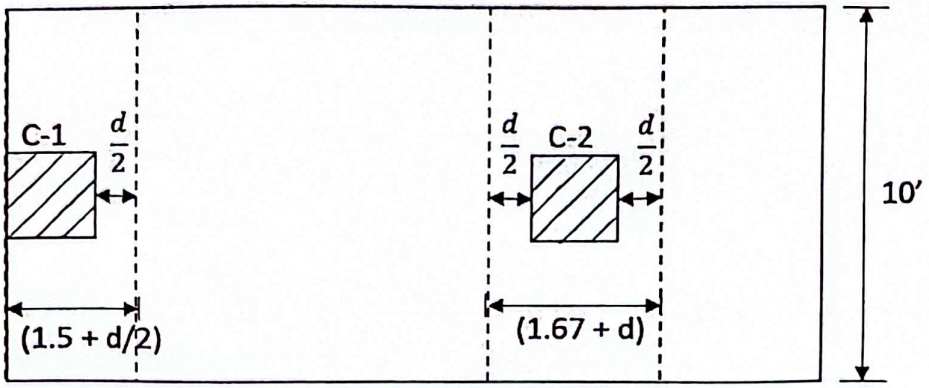
$$u = \frac{9.5\sqrt{f_c'}}{d_b} \leq 800; \quad \therefore u = \frac{9.5\sqrt{3000}}{1.125} = 462.5\ psi$$

$$L_d = \frac{A_s f_s}{u \varepsilon_o} = \frac{1.0 \times 24000}{462.5 \times \pi * 1.125} = 14.7''$$

$\therefore L_d(provided) = (2.65 + 1.67 - 1.13) \times 12 - 3 = 35.28'' > L_d\ (14.7'')$ (ok)

(b) Distribution steel

Design of transverse beam under interior column:



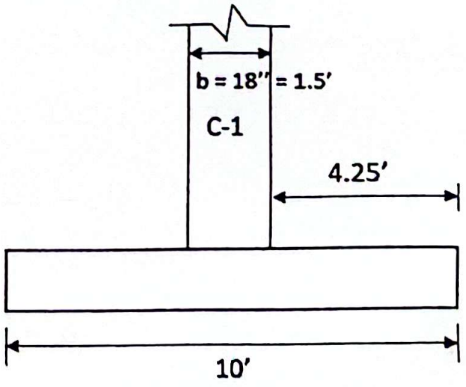
For transverse beam under C-1,

Load on C-1 = 304 kips

$$\text{UDL on beam} = \frac{304}{10} = 30.4\ k/ft$$

$$\text{Effective width of transverse beam} = 1.5 + \frac{d}{2} = 1.5 + \frac{20}{2 \times 12} = 2.33' = 28''$$

$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{30.4 \times 4.25^2}{2} = 274.55\ k-ft$$



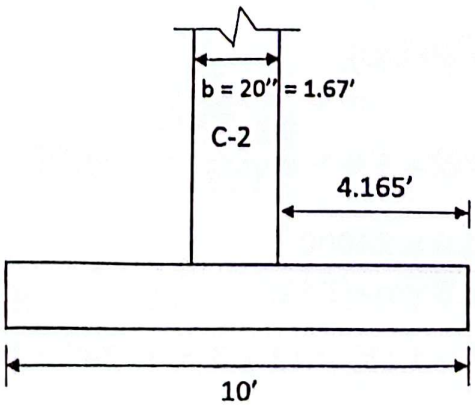
$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{274.55 \times 12}{0.90 \times 60 \times \left(20 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 28}$$

$$\therefore A_s = 4.04\ in^2$$

$$\text{Provide } 7 - \#7 \text{ bars with spacing } \frac{28 \times 0.60}{4.04} = 4\ in.\ c/c$$

For transverse beam under C-2,



Load on C-2 = 480 kips

$$\text{UDL on beam} = \frac{480}{10} = 48\ k/ft$$

Effective width of transverse beam = 1.67 + d = 1.67 * 12 + 20 = 40″

$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{48 \times 4.165^2}{2} = 416.33\ k-ft$$

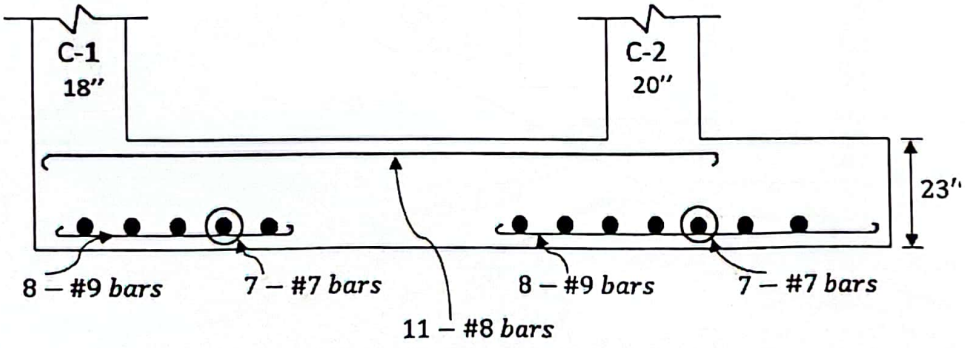
$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{416.33 \times 12}{0.90 \times 60 \times \left(20 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 40}$$

$$\therefore A_s = 4.92\ in^2$$

Provide 7 – #7 bars with spacing $\frac{40 \times 1.0}{8.0} = 5\ in.\ c/c$

Reinforcement detailing:



Chapter - 4
Design of Stairs

4.1 Theory Analysis

Q-1. Introduce a staircase as a reinforced concrete structure. Describe the structural component of the staircase briefly.

Ans: Introduction: Stair consists of rises, treads (runs) and landings. The total steps and landings are called a *staircase*. The *rise* is defined as the vertical distance between two steps, and the *tread* is the depth (width) of the step. The *landing* is the horizontal part of the staircase without rises.

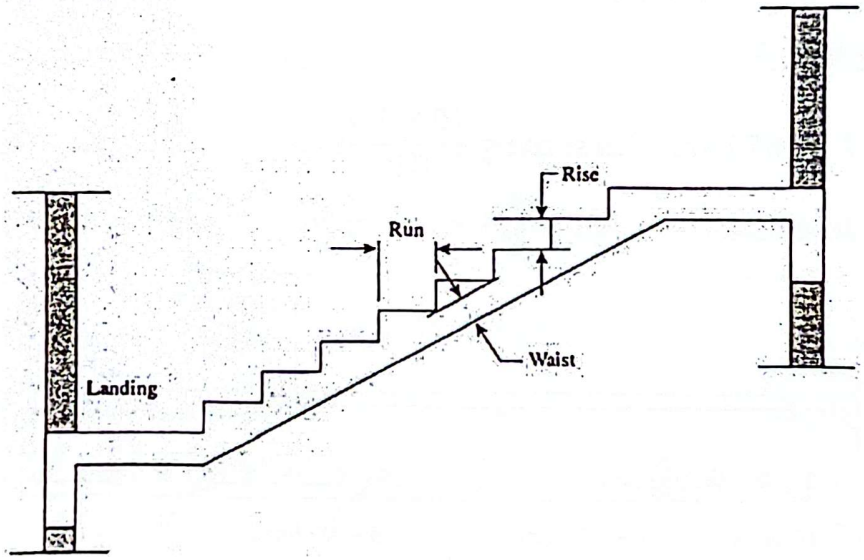


Figure 4.1: Section of a staircase

Description of the structural component:

i) Tread: The horizontal portion of a step where the foot rests is referred to as tread or run. 10 in. (250 mm) to 12 in. (300 mm) is the typical dimensions of a tread.

ii) Rise: Rise is the vertical distance between the adjacent treads or the vertical projection of the step. 6 in. (150 mm) to 7.5 in. (190 mm) is the typical dimensions of a rise.

iii) Going and Landing: Going is the horizontal projection of an inclined flight of steps between the first and last rise. A typical flight comprises two landings and one going as shown in figure 4.1. To break the monotony of climbing, the number of steps in a flight should not exceed 10 to 12.

The tread-rise combination in a flight can be provided in conjunction with –

(a) Waist slab or stair slab
(b) Tread-Rise type (continuous folded plate)
(c) Isolated cantilever tread slab

(d) Double cantilever precast tread slab with a central inclined beam.

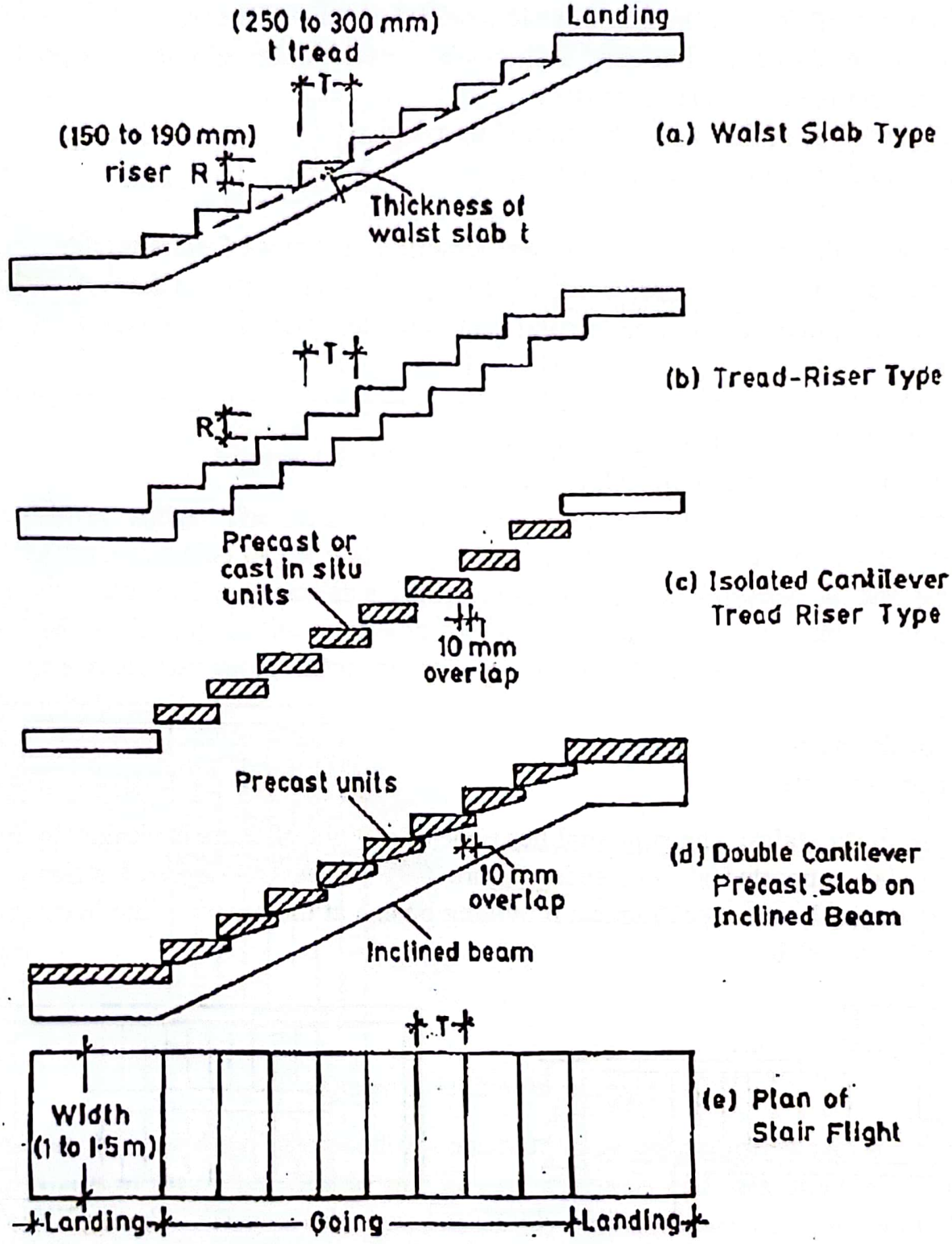


Figure 4.2: Typical flight in a staircase

Q-2. What are the requirements of a good stair?

Ans: The requirements of a good stair are described below:

(i) It should be well ventilated.

(ii) It should be easily approachable.

(iii) It should have sufficiently wide landing at the end of each flight.
(iv) The width of landing should not be less than the width of the stair.
(v) Rise of the steps should be easy. It should be neither too small nor too large.
(vi) Steps should be sufficiently wide.
(vii) Winder steps as far as possible should be avoided.
(viii) It should be strong durable and stiff.

Q-3. What are the different types of stairs? Describe the types of stairs briefly.
Ans: There are different types of stairs, which depend upon mainly on the type and function of the building and on the architectural requirements. The most common types are as follows:

(a) Single flight stairs
(b) Double flight stairs / Dog-legged stairs
(c) Three or more flights stairs / Open well stairs
(d) Cantilever stairs
(e) Precast flight stairs
(f) Free-standing stairs
(g) Run-rise stairs
(h) Spiral stairs
(i) Helical stairs
(j) Helicoidal stairs
(k) Triangular stairs

(i) Single flight stairs: The structural behavior of a flight of stairs is similar to that of a one-way slab supported at both ends (Figure 4.3). When the flight of stairs contains landings, it may be more economical providing beams at the junction line between waist slab and landing slab.

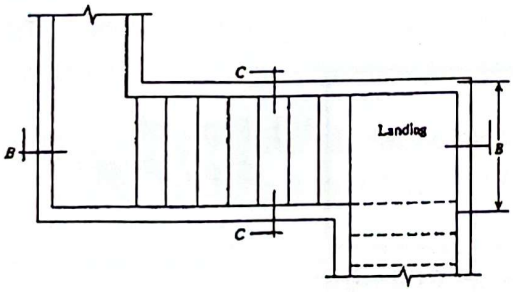


Figure 4.3: Single flight stair

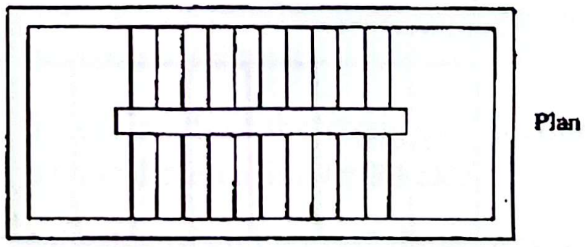


Figure 4.4: Dog-legged stair

(b) Double flight stairs / Dog-legged stairs: The most common type of stairs arranged with two adjacent flights running parallel with a mid-landing (Figure 4.4). For the structural analysis of the stairs, each flight is treated as a single flight and is considered supported on two or more beams.

(c) Three or more flights stairs / Open well stairs: In some cases, where the overall dimensions of the staircase are limited, three or four flights may be adopted (Figure 4.5). Each flight will be treated separately, as in the case of double flight staircase.

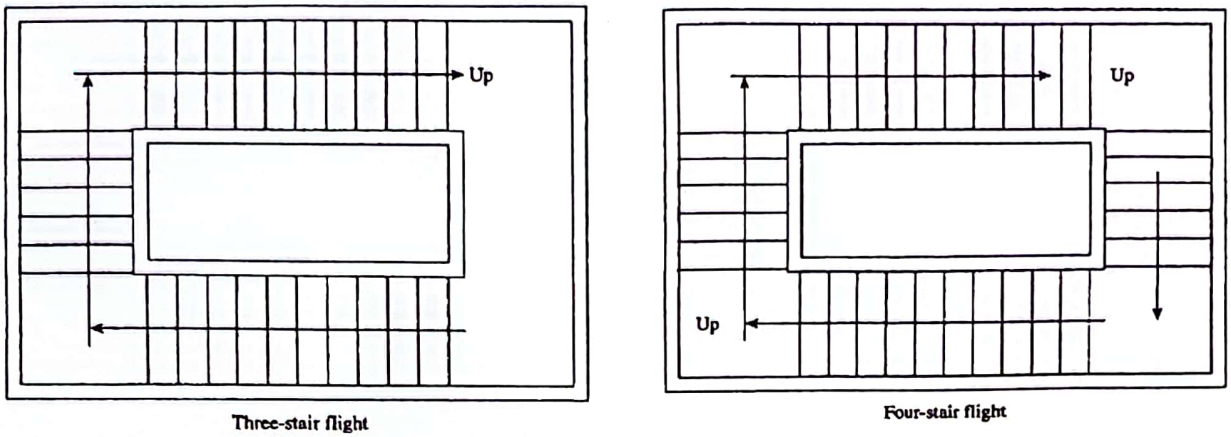


Figure 4.5: More flight or Open well stairs

(d) Cantilever stairs: The stair which is supported by walls or beams and stair steps projecting from one side of the wall or beam is said to be cantilever stair (Figure 4.6). In this type of stairs, each step acts as a cantilever and the main reinforcement is placed in the tension side at the top of the tread and the bars are anchored within the wall or beam. Shrinkage and temperature reinforcement is provided in the transverse direction.

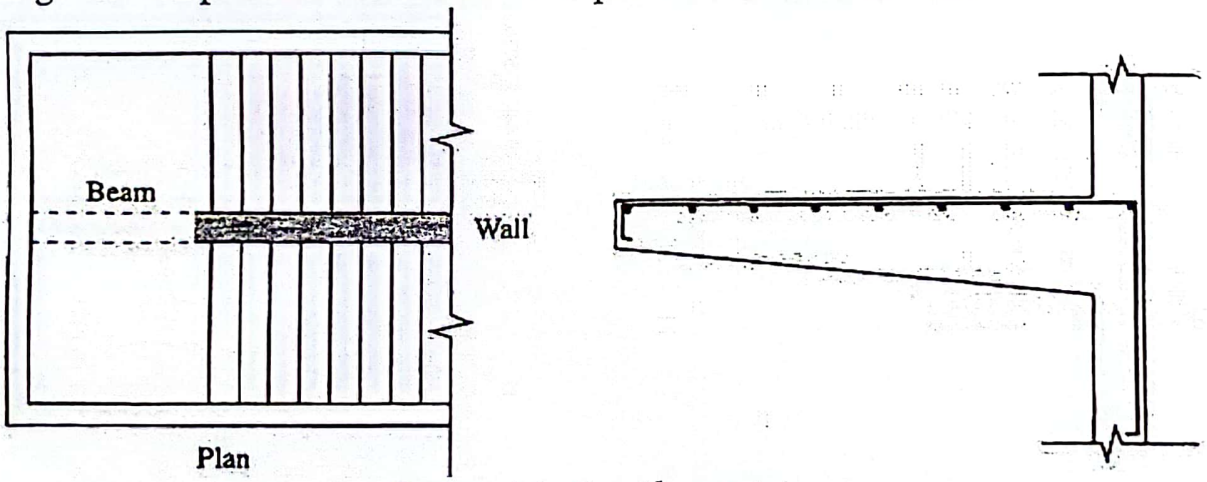


Figure 4.6: Cantilever stairs

(e) Precast flight stairs: The speed of construction in some projects requires the use of precast flights of stairs. The flights may be cast separately and then fixed to cast-in-place landings (Figure 4.7). They are designed as simply supported one-way slabs with the main reinforcement at the bottom of the stair waist.

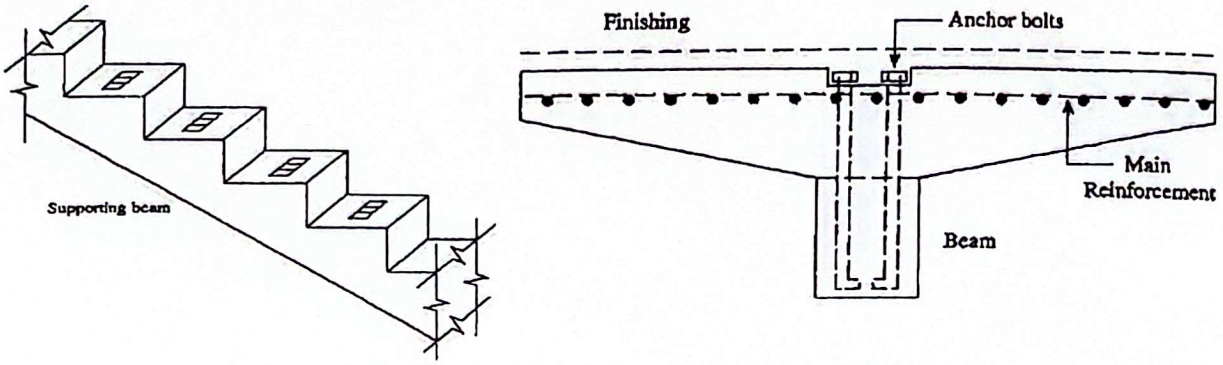


Figure 4.7: Precast flight stair and its reinforcement

(f) Free-standing stairs: The stair in which the landing projects into the air without any support at its end is said to be free-standing stair (Figure 4.8). The stairs behave in a springboard manner, causing torsional stresses in the slab.

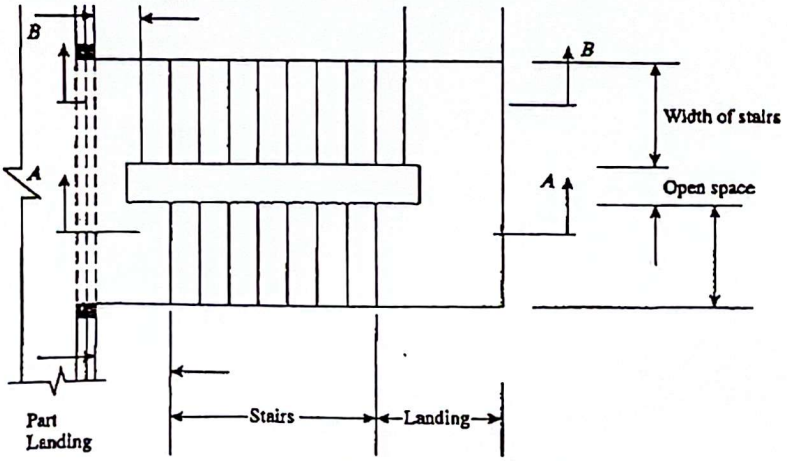


Figure 4.8: Free-standing stair

(g) Run-rise stairs: Run-rise stairs are stepped underside stairs that consist of a number of runs (treads) and rises rigidly connected without the normal waist slab (Figure 4.9). This type of stairs has an elegant appearance and is sometimes favored by architects.

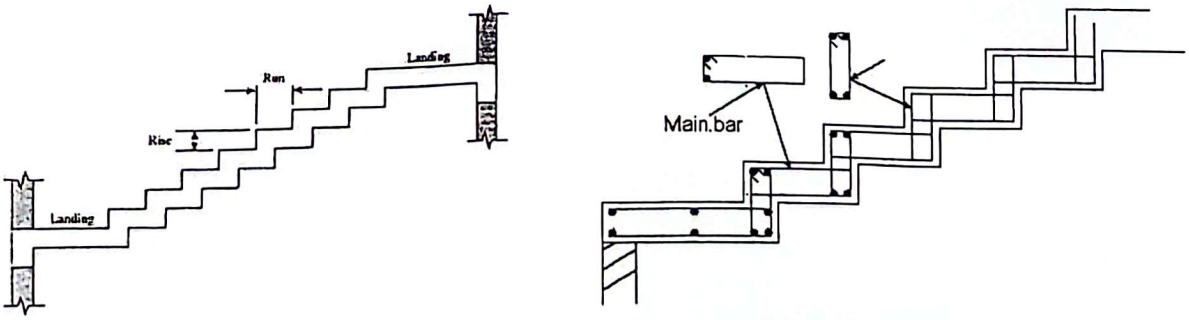


Figure 4.9: Run-rise stair or its reinforcement

(h) Spiral stairs: In congested locations, where space available is small, spiral stairs are ideally suited. A typical spiral staircase comprises a central post with precast slab treads anchored to the central column (Figure 4.10). It is not user friendly due to the reduced tread width near the port and is suitable only for single person to use the staircase at a time.

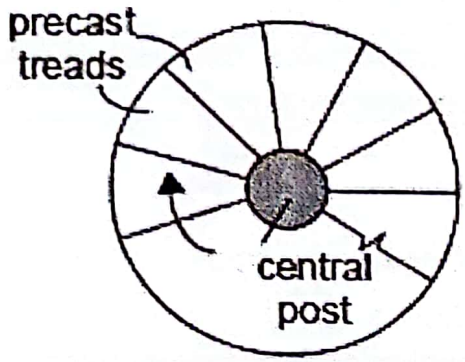


Figure 4.10: Spiral stair

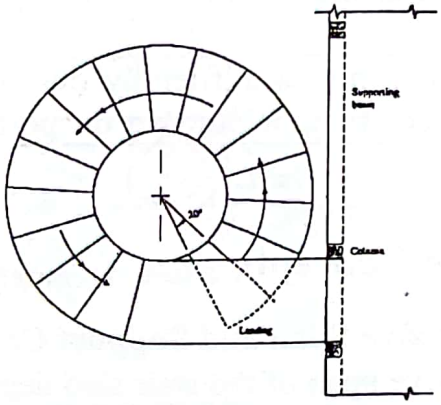


Figure 4.11: Helical Stair

(i) Helical stairs: The stair which is three-dimensional structure, has a circular shape in plan and may be supported at some edges within adjacent walls is said to be helical stairs (Figure 4.11). The main longitudinal reinforcement consists of helical bars placed in the concrete waist slab of the stairs and runs from the top landing to the bottom support. The transverse reinforcement must be in a closed stirrup form to resist torsional stresses or in a U-shaped lapped at about the mid width of the stairs.

(j) Helicoidal stairs: The stair which is built as a ramp following the helicoidal curve with supports at ground and first floor or with intermediate supports involves rigorous structural computations for the determination of design moments and shear forces is said to be helicoidal stair (Figure 4.12).

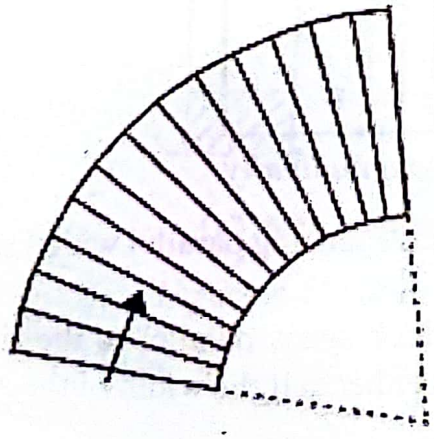


Figure 4.12: Helicoidal Stair

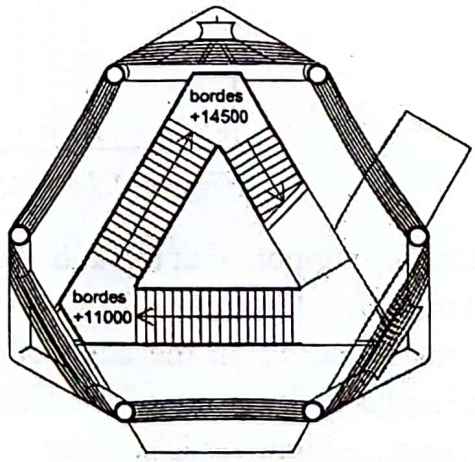


Figure 4.13: Triangular Stair

(k) Triangular stairs: The stair in which three flights are connected in triangular shape and makes a triangular open space is said to be triangular stair (Figure 4.13).

Q-4. What are the differences between spiral and helical stair?

Ans: The differences between spiral and helical stair are described below:

Spiral stair	Helical stair
i) Spiral stairs have a central column.	i) Helical stairs do not need a central column.
ii) It is not user friendly due to the reduced tread width near the port.	ii) It has more spaces than a spiral stair and more user friendly.

4.2 Basic Concepts

4.2.1 Effective Span and Support Condition of Stair:

The effective span of the stair slab depends on the position of the supports over on which the slab rests. Depending upon the method of support provided, the stair slab may be divided into two types:

(i) Stair slab spanning longitudinally (along the slope line)

(ii) Stair slab spanning transversely (slab width wise with central or side supports)

(i) Stair slab spanning longitudinally:

The effective span in longitudinal direction is considered in between the center to center of supports as the following distance:

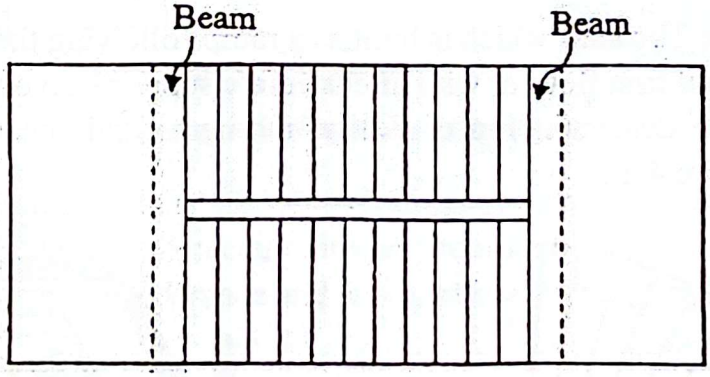


Figure 4.14: Stair slab spanning longitudinally

(a) Where the supported at top and bottom risers by beams spanning parallel with the risers, the distance c/c of beams.

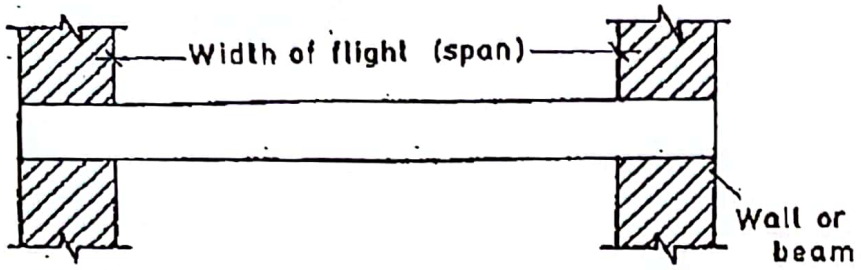
(b) Where spanning on the edge of a landing slab, which spans parallel to the risers, a distance equal to the going of the stairs plus at each end either half the width of the landing or one meter, whichever is smaller.

(c) Where the landing slab spans in the same direction as the stairs, they should be considered as acting together to form a single slab, the distance c/c of beams.

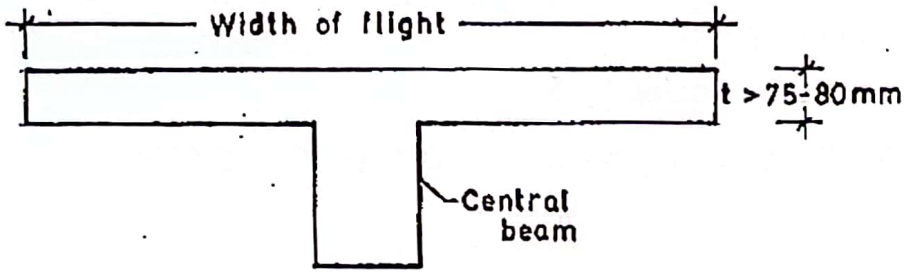
(ii) Stair slab spanning transversely:

The effective span in transverse or horizontal direction is considered as the following distance:

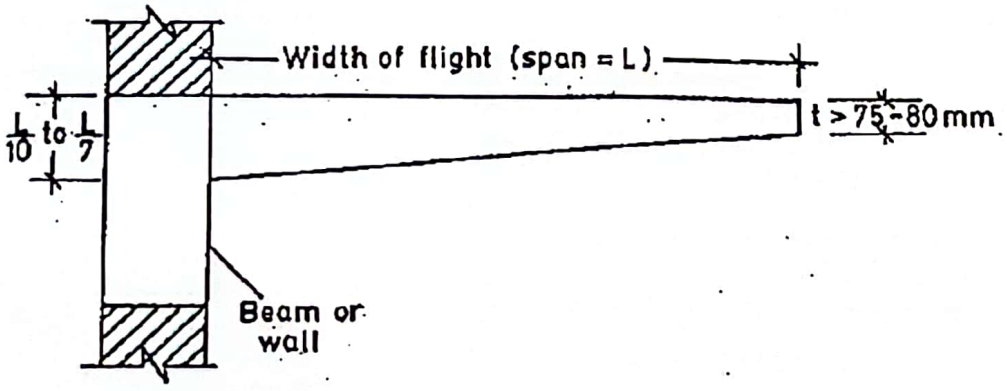
(a) Slab supported between two edges beams are walls. [Figure 4.15 (a)]
(b) Slab cantilevering on either side of a central beam. [Figure 4.15 (b)]
(c) A cantilevered slab from a wall or a spandrel beam. [Figure 4.15 (c)]



(a) Slab Supported on Edge Beam or Walls



(b) Slab Cantilevered From a Central Beam



(c) Slab Cantilevered From Wall or Beam

Figure 4.15: Stair slabs spanning in transverse direction

Bangla Terminology:
Stair বা Beam design করার সময়ে তাদের যে span দেওয়া থাকে, তার থেকে দুই পাশে support এর জন্য 6″ করে ভিতরে বাড়িয়ে ধরা হয়। অর্থাৎ effective span, given span থেকে always 1′ বেশি হবে।

4.2.2 Minimum Thickness of Stair Slab or Waist Slab:

Support condition	Minimum thickness, $t_{min} \geq$	
	WSD	*USD*
Simply supported	$\frac{L}{25}$	$\frac{L}{20}$
One end continuous	$\frac{L}{30}$	$\frac{L}{24}$
Both ends continuous	$\frac{L}{35}$	$\frac{L}{28}$
Cantilever	$\frac{L}{12}$	$\frac{L}{10}$

*L = span length of a panel

*In WSD 25, 30, 35 increasing by **5**.

*In USD 20, 24, 28 increasing by **4**. *(Design is very easy)*

4.3 Solution Techniques

4.3.1 Stair Slab Spanning Longitudinally

4.3.1.1 Double Flight Stairs / Dog-legged Stairs

WSD

1. Thickness of stair slab
Stair effective span length, L
Stair length, S
According to support condition of the stair,
$\therefore$ Thickness of stair slab, $t = \frac{L}{*}$

2. Stair height, h
$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2}$

3. Load calculation
i)Dead load
Self weight of stair slab
Weight due to stair step
ii)Live load

4. Diagram of SFD and BMD
Considering unit (1 ft or 1 m) strip

5. Design constants: $n,\ r,\ k,\ j$

6. Depth check
$M = \frac{1}{2} f_c k j b d^2$

7. Reinforcement calculation
i) Longitudinal bar
$\therefore A_s(+ve) = \frac{M}{f_s j d}$

$\therefore A_s(-ve) = \frac{M}{f_s j d}$
Here, A_s (min) $= 0.0018\ bt$
ii) Distribution bar, A_s (min)

8. Shear Check
Developed shear, $V_{dev(1)} = R_1 - w_1 d$
$V_{dev(2)R} = R_1 - w_1 d$
$V_{dev(2)L} = R_1 - w_2 d$

USD

1. Thickness of stair slab
Stair effective span length, L
Stair length, S
According to support condition of the stair,
$\therefore$ Thickness of stair slab, $t = \frac{L}{*}$

2. Stair height, h
$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2}$

3. Load calculation
i)Dead load
Self weight of stair slab
Weight due to stair step
ii)Live load

4. Diagram of SFD and BMD
Considering unit (1 ft or 1 m) strip

5. Depth check
$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t}$
$M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$

6. Reinforcement calculation
i) Longitudinal bar
$\therefore A_s(+ve) = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$

$a = \frac{A_s f_y}{0.85 f_c' b}$

$A_s(-ve) = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$

Here, A_s (min) $= \frac{200}{f_y} bd$

Allowable shear, $V_{all} = 1.1\sqrt{f_c'}\, bd$

9. Beam design (if need)

i) Load calculation

ii) Diagram of SFD and BMD

iii) Depth check

$$M = \frac{1}{2} f_c kjbd^2$$

iv) Reinforcement calculation

$$A_s = \frac{M}{f_s jd}$$

10. Reinforcement deatiling

ii) Distribution bar, A_s (min)

7. Shear Check

Developed shear, $V_{dev(1)} = R_1 - w_1 d$

$V_{dev(2)R} = R_2 - w_1 d$

$V_{dev(2)L} = R_2 - w_2 d$

Allowable shear, $V_{all} = 2\varphi\sqrt{f_c'}\, bd$

8. Beam design (if need)

i) Load calculation

ii) Diagram of SFD and BMD

iii) Depth check

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t}$$

$$M = \varphi\rho f_y bd^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

iv) Reinforcement calculation

$$\therefore A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

9. Reinforcement deatiling

4.3.1.2 Three Flight Stairs

WSD

1. Thickness of stair slab
Stair effective span length, L
Stair length, S
According to support condition of the stair,
$\therefore$ Thickness of stair slab, $t = \frac{L}{*}$

2. Stair height, h
$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2}$

3. Load calculation
i)Dead load
Self weight of stair slab
Weight due to stair step
ii)Live load

4. Diagram of SFD and BMD
Considering unit (1 ft or 1 m) strip

5. Design constants: $n,\ r,\ k,\ j$

6. Depth check
$M = \frac{1}{2} f_c k j b d^2$

7. Reinforcement calculation
i) Longitudinal bar
$\therefore A_s(+ve) = \frac{M}{f_s j d}$

$\therefore A_s(-ve) = \frac{M}{f_s j d}$ (if need)
Here, A_s (min) $= 0.0018\ bt$
ii) Distribution bar, A_s (min)

8. Shear Check
Developed shear, $V_{dev(1)} = R_1 - w_1 d$
$V_{dev(2)R} = R_2 - w_1 d$
$V_{dev(2)L} = R_2 - w_2 d$

Allowable shear, $V_{all} = 1.1\sqrt{f_c'}\ bd$

USD

1. Thickness of stair slab
Stair effective span length, L
Stair length, S
According to support condition of the stair,
$\therefore$ Thickness of stair slab, $t = \frac{L}{*}$

2. Stair height, h
$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2}$

3. Load calculation
i)Dead load
Self weight of stair slab
Weight due to stair step
ii)Live load

4. Diagram of SFD and BMD
Considering unit (1 ft or 1 m) strip

5. Depth check
$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t}$
$M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$

6. Reinforcement calculation
i) Longitudinal bar
$\therefore A_s(+ve) = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$

$a = \frac{A_s f_y}{0.85 f_c' b}$

$A_s(-ve) = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$

Here, A_s (min) $= \frac{200}{f_y} bd$
ii) Distribution bar, A_s (min)

7. Shear Check
Developed shear, $V_{dev(1)} = R_1 - w_1 d$
$V_{dev(2)R} = R_2 - w_1 d$

9. Beam design (if need)
i) Load calculation
ii) Diagram of SFD and BMD
iii) Depth check

$$M = \frac{1}{2} f_c k j b d^2$$

iv) Reinforcement calculation

$$A_s = \frac{M}{f_s j d}$$

10. Reinforcement deatiling

* = constant from code [see section 4.2.2]

$$V_{dev(2)L} = R_2 - w_2 d$$

Allowable shear, $V_{all} = 2\varphi\sqrt{f_c'}\, bd$

8. Beam design (if need)
i) Load calculation
ii) Diagram of SFD and BMD
iii) Depth check

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t}$$

$$M = \varphi\rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

iv) Reinforcement calculation

$$\therefore A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

9. Reinforcement deatiling

4.3.2 Stair Slab Spanning Transversely

4.3.2.1 Cantilever Slab Span from Edge Wall or Beam

WSD

1. Thickness of stair slab
Stair effective span length, L
Stair length, S
According to support condition of the stair,
$\therefore$ Thickness of stair slab, $t = \frac{L}{*}$

2. Stair height, h
$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2}$

3. Load calculation
i)Dead load
Self weight of stair slab
Weight due to stair step
ii)Live load

4. Diagram of SFD and BMD
Considering unit (1 ft or 1 m) strip
$\therefore$ Design Moment, $M = \frac{wl^2}{2}$

5. Design constants: $n,\ r,\ k,\ j$

6. Depth check
$M = \frac{1}{2} f_c k j b d^2$

7. Reinforcement calculation
i) Longitudinal bar (top bar)
$\therefore A_s(-ve) = \frac{M}{f_s j d}$

Here, A_s (min) $= 0.0018\ bt$
ii) Distribution bar, A_s (min)

8. Shear Check
Developed shear, $V_{dev} = R - wd$

Allowable shear, $V_{all} = 1.1\sqrt{f_c'}\ bd$

9. Beam design (if need)
i) Load calculation
ii) Diagram of SFD and BMD
iii) Depth check
$M = \frac{1}{2} f_c k j b d^2$
iv) Reinforcement calculation
$A_s = \frac{M}{f_s j d}$

10. Reinforcement deatiling

$*$ = constant from code [see section 4.2.2]

4.3.2.1 Cantilever Slab Span from Central Beam

WSD

1. Thickness of stair slab

Stair effective span length, $L = \frac{B}{2}$

Stair length, S

According to support condition of the stair,

$\therefore$ Thickness of stair slab, $t = \frac{L}{*}$

2. Stair height, h

$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2}$

3. Load calculation

i)Dead load

Self weight of stair slab

Weight due to stair step

ii)Live load

4. Diagram of SFD and BMD

Considering unit (1 ft or 1 m) strip

$\therefore$ Design Moment, $M = \frac{wl^2}{2}$

5. Design constants: $n,\ r,\ k,\ j$

6. Depth check

$M = \frac{1}{2} f_c k j b d^2$

7. Reinforcement calculation

i) Longitudinal bar (top bar)

$\therefore A_s(-ve) = \frac{M}{f_s j d}$

Here, A_s (min) $= 0.0018\, bt$

ii) Distribution bar, A_s (min)

8. Shear Check

Developed shear, $V_{dev} = R - wd$

Allowable shear, $V_{all} = 1.1\sqrt{f_c'}\, bd$

9. Beam design (if need)

i) Load calculation

ii) Diagram of SFD and BMD

iii) Depth check

$M = \frac{1}{2} f_c k j b d^2$

iv) Reinforcement calculation

$A_s = \frac{M}{f_s j d}$

10. Reinforcement deatiling

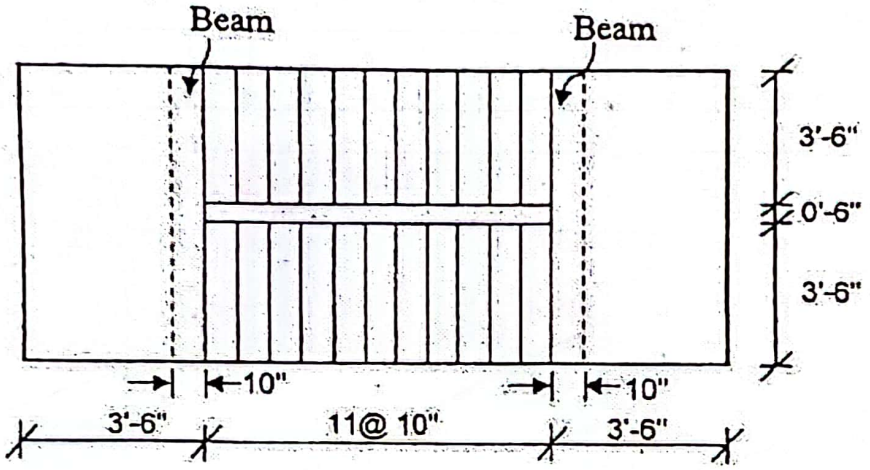
$*$ = constant from code [see section 4.2.2]

4.4 Problem Solution

4.4.1 Stair Slab Spanning Longitudinally – WSD

4.4.1.1 Double Flight Stairs / Dog-legged Stairs – WSD

Pb-1. Design any flight of the following stair and show reinforcement detailing. The dimension of the beams are $10'' \times 18''$ and height of each riser is $6''$. Live load on stair is 80 psf. Use materials of $f_c' = 4000$ psi and $f_y = 60{,}000$ psi. Follow WSD method. [*CE-2014*]



Sol^n:

Stair span length, $L = 6'' + (11 * 10'') + 6'' = 122'' = 10.2'$

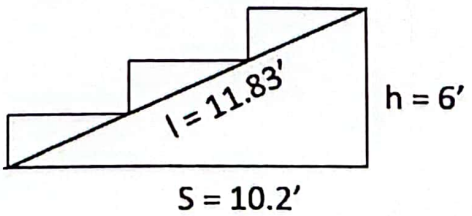
Stair length, $S = 10.2'$

Since the support condition of the stair is both end continuous,

$\therefore$ Thickness of stair slab, $t = \frac{L}{35} = \frac{122''}{35} = 3.49'' \approx 4''$

Stair height, $h =$ No. of rise $\times$ Riser $= (11 + 1) \times 6 = 72'' = 6'$

$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2} = \sqrt{10.2^2 + 6^2} = 11.83'$



Load Calculation:

Dead load:

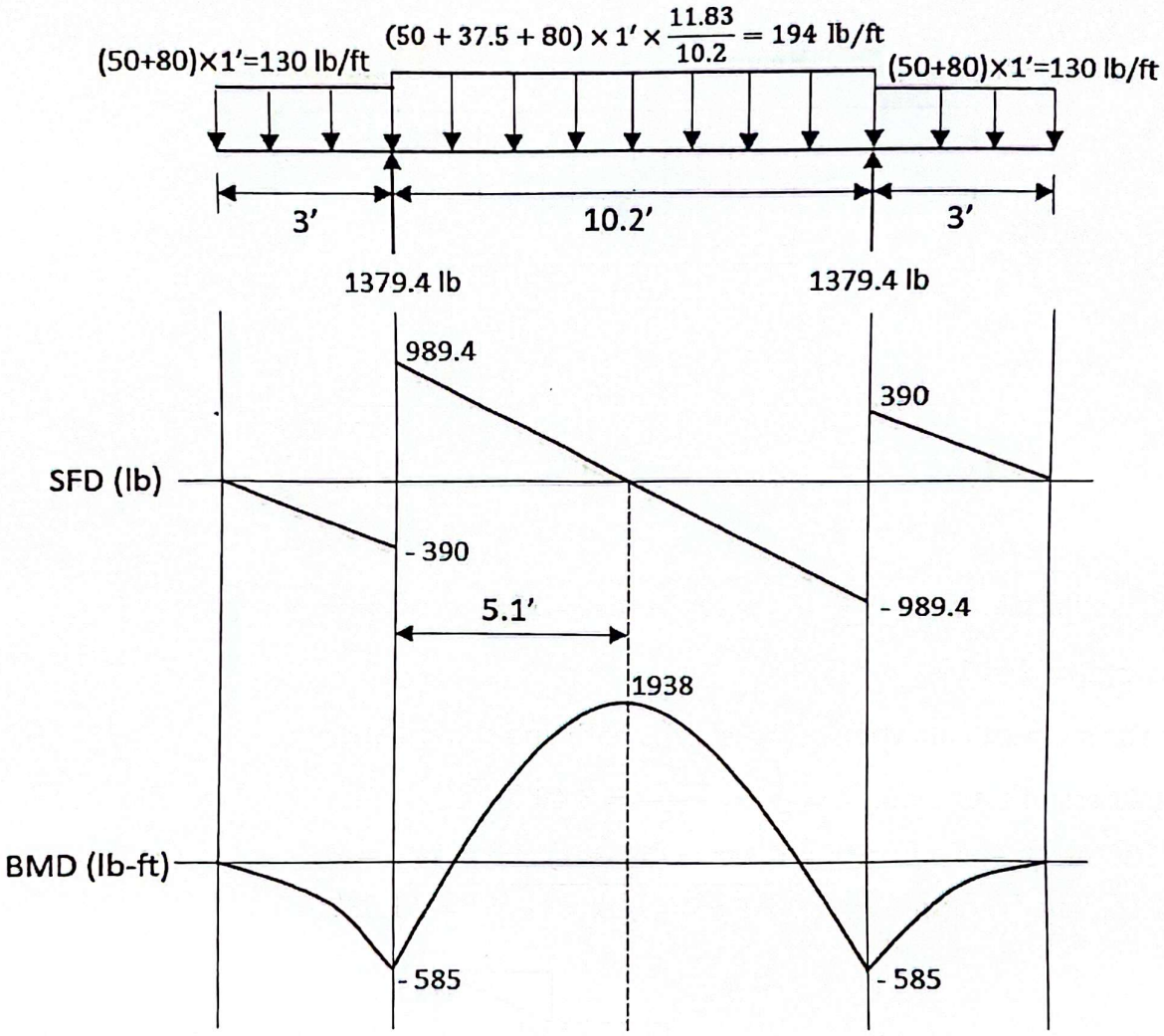
Self weight of stair slab $= \frac{4}{12} \times 150 = 50\ psf$

Weight due to stair step $= \frac{\left(\frac{6+0}{2}\right)}{12} \times 150 = 37.5\ psf$

Live load:

Live load = 80 psf

Considering 1 ft. strip along stair span length,



Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8.04 \approx 8$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 4} = \frac{24}{1.8} = 13.33$$

$$k = \frac{n}{n+r} = 0.375$$

$$j = 1 - \frac{k}{3} = 0.875$$

Depth check:

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Rightarrow 1.938 \times 12 = \frac{1}{2} \times 1.8 \times 0.375 \times 0.875 \times 12 \times d^2$$

$$\therefore d = 2.56\ in. \approx 3\ in.$$

$$d_{provided} = t - cc = 4 - 1 = 3\ in. \geq d \ \ (ok)$$

Reinforcement:

Longitudinal bar:

$$\therefore A_s(+ve) = \frac{M}{f_s j d} = \frac{1.938 \times 12}{24 \times 0.875 \times 3} = 0.337 \ \ in^2/ft$$

Provide #4 bar @ 7 in. c/c

$$\therefore A_s(-ve) = \frac{M}{f_s j d} = \frac{0.585 \times 12}{24 \times 0.875 \times 3} = 0.111 \ \ in^2/ft$$

Provide #3 bar @ 11 in. c/c

Here, A_s (min) $= 0.0018\ bt = 0.0018 \times 12 \times 4 = 0.0864 \ \ in^2/ft$

Distribution bar:

A_s (min) $= 0.0864 \ \ in^2/ft$

Provide #3 bar @ ~~15~~ 12 in. c/c

Shear check:

Developed shear, $V_{dev} = R - wd$

$$V_{dev(L)} = R_1 - w_1 d = 1379.4 - 130 \times \frac{3}{12} = 1347\ lb$$

$$V_{dev(R)} = R_1 - w_2 d = 1379.4 - 194 \times \frac{3}{12} = 1331\ lb$$

Allowable shear, $V_{all} = 1.1\sqrt{f_c'}\ bd = 1.1 \times \sqrt{4000} \times 12 \times 3 = 2505\ lb > V_{dev} \ \ (ok)$

Beam design:

Here,

Beam width, $b = 10''$

Beam height, $h = 18''$

$\therefore$ Beam span, $L = 6'' + (3' + 6'' + 6'' + 3 + 6'') + 6'' = 102'' = 8.5'$

Load Calculation:

Dead load:

$$\text{Self weight of beam} = \frac{10 \times 18}{144} \times 150 = 188\ lb/ft$$

Load from landing slab or stair slab $= 50\ psf$

Load from stair $= (50 + 37.5) = 87.5$

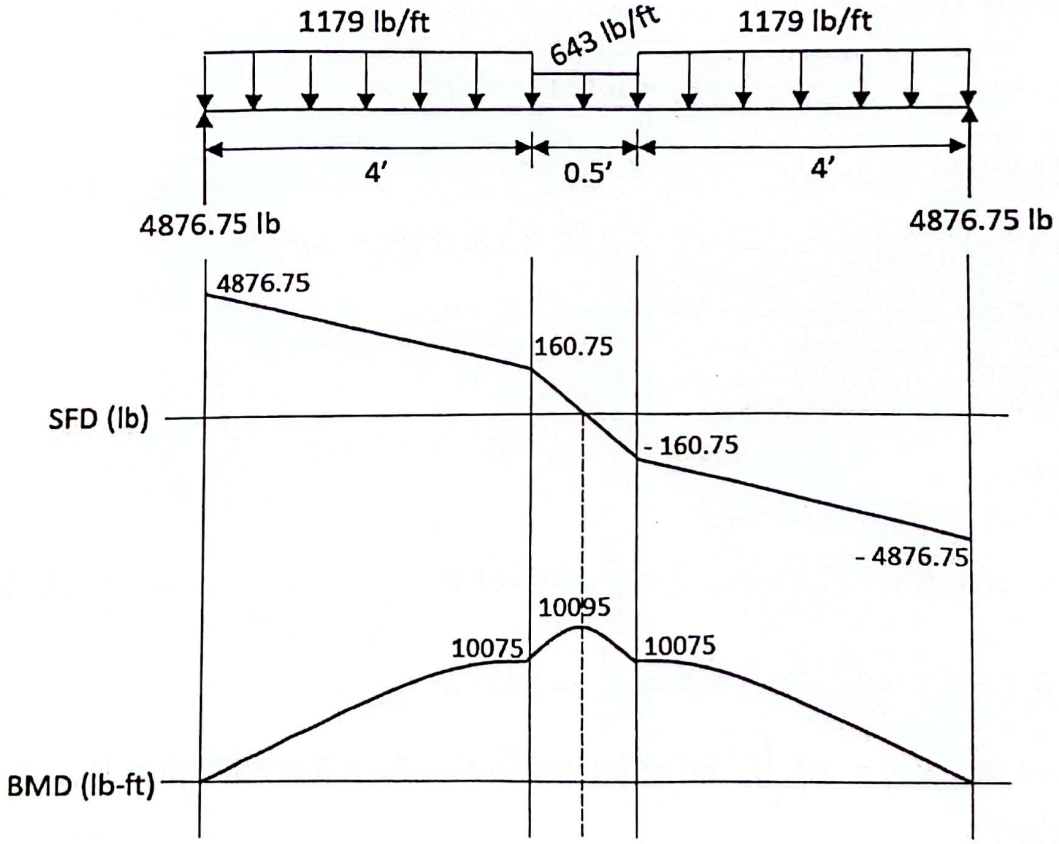
Live load:

Live load $= 80\ psf$

$\therefore$ Load from landing on beam $= 188 + (50 \times 3.5) + (80 \times 3.5) = 643\ lb/ft$

$\therefore$ Load from stair on beam $= 188 + (87.5 + 80) \times \frac{11.83}{2} = 1179\ lb/ft$

এখানে বলা রাখা ভালো, Beam এর load calculation করার সময় প্রতিটি psf load কে Beam এর transverse direction এর dimension দিয়ে গুণ করতে হবে। আর পুরো stair এর load half করে দুই পাশের beam তা carry করবে।



Depth check:

$M = \frac{1}{2} f_c k j b d^2$

$\Longrightarrow 10.095 \times 12 = \frac{1}{2} \times 1.8 \times 0.375 \times 0.875 \times 10 \times d^2$

$\therefore d = 6.4\ in. \approx 7\ in.$

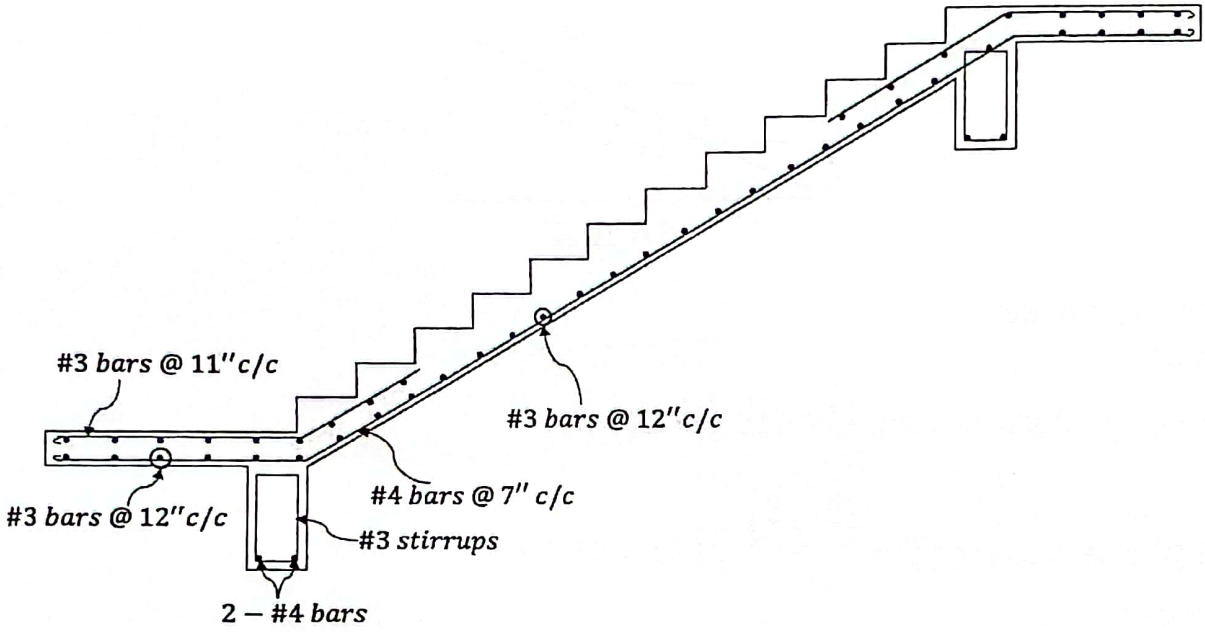
$d_{provided} = h - cc = 18 - 2 = 16\ in. \geq d\ \ (ok)$

Reinforcement:

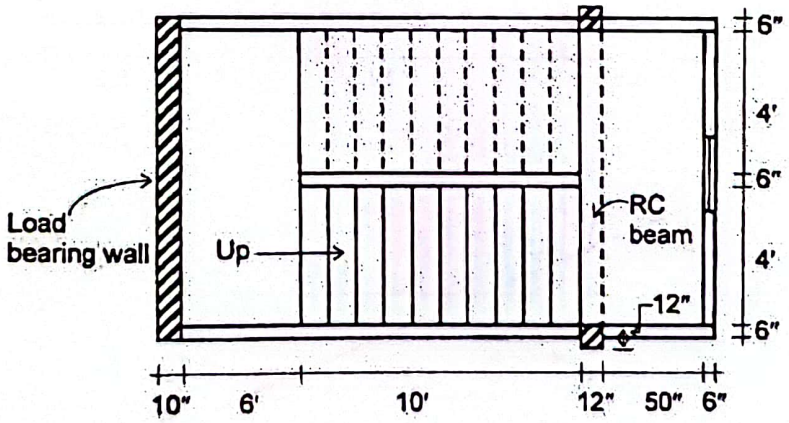
$$\therefore A_s = \frac{M}{f_s j d} = \frac{10.095 \times 12}{24 \times 0.875 \times 16} = 0.361\ in^2$$

Provide 2 - #4 bars

Reinforcement Detailing:



Pb-2. Design and draw reinforcement detailing of the following stair case. Live load on stair is 80 psf. Floor to floor height is 11'-0". Use $f_c' = 3.5$ ksi and $f_y = 60$ ksi. [*CE-2019, 2010*]



Sol^n:

Stair span length, $L = 6'' + 6' + 10' + 6'' = 204'' = 17'$

Stair length, $S = 10' + 6'' = 10.5'$

Since the support condition of the stair is one end continuous,

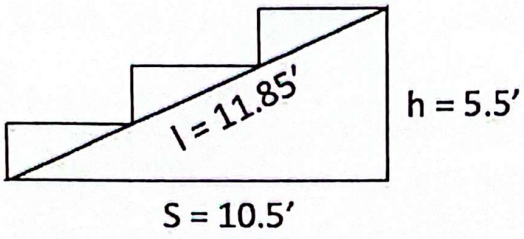
$$\therefore \text{Thickness of stair slab, } t = \frac{L}{30} = \frac{204''}{30} = 6.8'' \approx 7''$$

Height of each flight $= \frac{11'}{2} = 5.5' = 66''$

Here, No. of trade (run) $= 10$ & No. rise $= 10 + 1 = 11$

$\therefore$ Riser $= \frac{66''}{11} = 6''$

$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2} = \sqrt{10.5^2 + 5.5^2} = 11.85'$



Load Calculation:

Dead load:

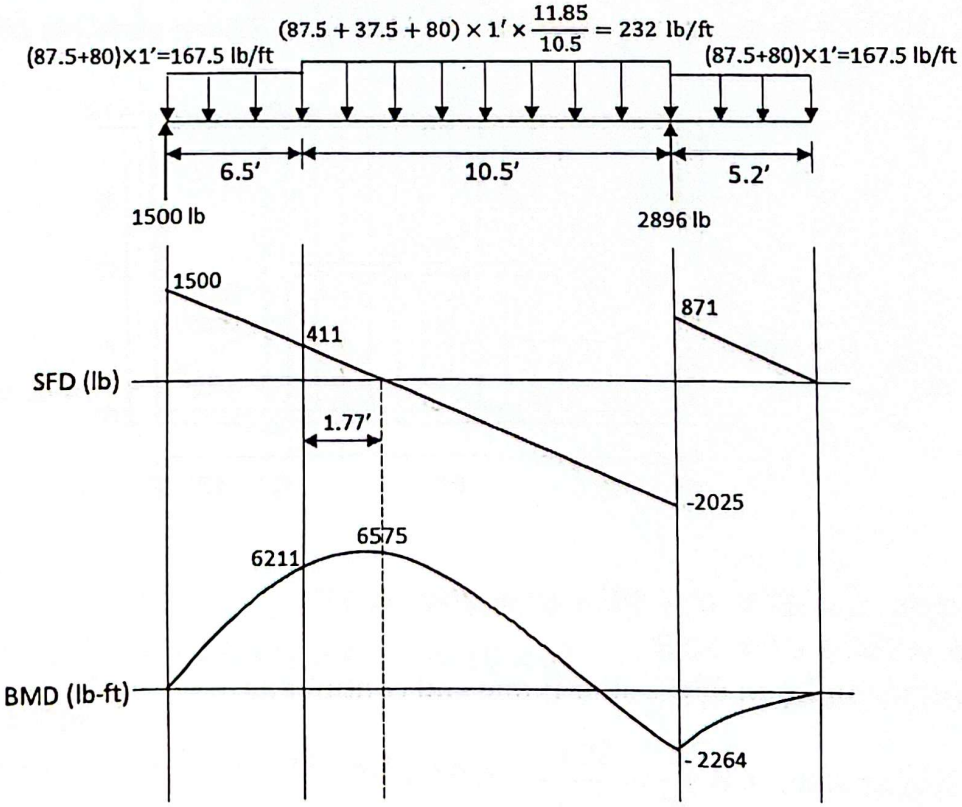
Self weight of stair slab $= \frac{7}{12} \times 150 = 87.5\ psf$

Weight due to stair step $= \frac{\left(\frac{6+0}{2}\right)}{12} \times 150 = 37.5\ psf$

Live load:

Live load $= 80\ psf$

Considering 1 ft. strip along stair span length,



Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3500}} = 8.6 \approx 8$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 3.5} = \frac{24}{1.575} = 15.24$$

$$k = \frac{n}{n+r} = 0.344$$

$$j = 1 - \frac{k}{3} = 0.885$$

Depth check:

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Longrightarrow 6.575 \times 12 = \frac{1}{2} \times 1.575 \times 0.344 \times 0.885 \times 12 \times d^2$$

$$\therefore d = 5.24\ in. \approx 5.5\ in.$$

$$d_{provided} = t - cc = 7 - 1 = 6\ in. \geq d\ \ (Ok)$$

Reinforcement:

Longitudinal bar:

$$\therefore A_s(+ve) = \frac{M}{f_s j d} = \frac{6.575 \times 12}{24 \times 0.885 \times 5.5} = 0.675\ \ in^2/ft$$

Provide #6 bar @ 7 in. c/c

$$\therefore A_s(-ve) = \frac{M}{f_s j d} = \frac{2.264 \times 12}{24 \times 0.885 \times 5.5} = 0.233\ \ in^2/ft$$

Here, A_s (min) $= 0.0018\, bt = 0.0018 \times 12 \times 7 = 0.1512\ \ in^2/ft$

Provide #4 bar @ 10 in. c/c

Distribution bar:

A_s (min) $= 0.1512\ \ in^2/ft$

Provide #3 bar @ 8 in. c/c

Shear check:

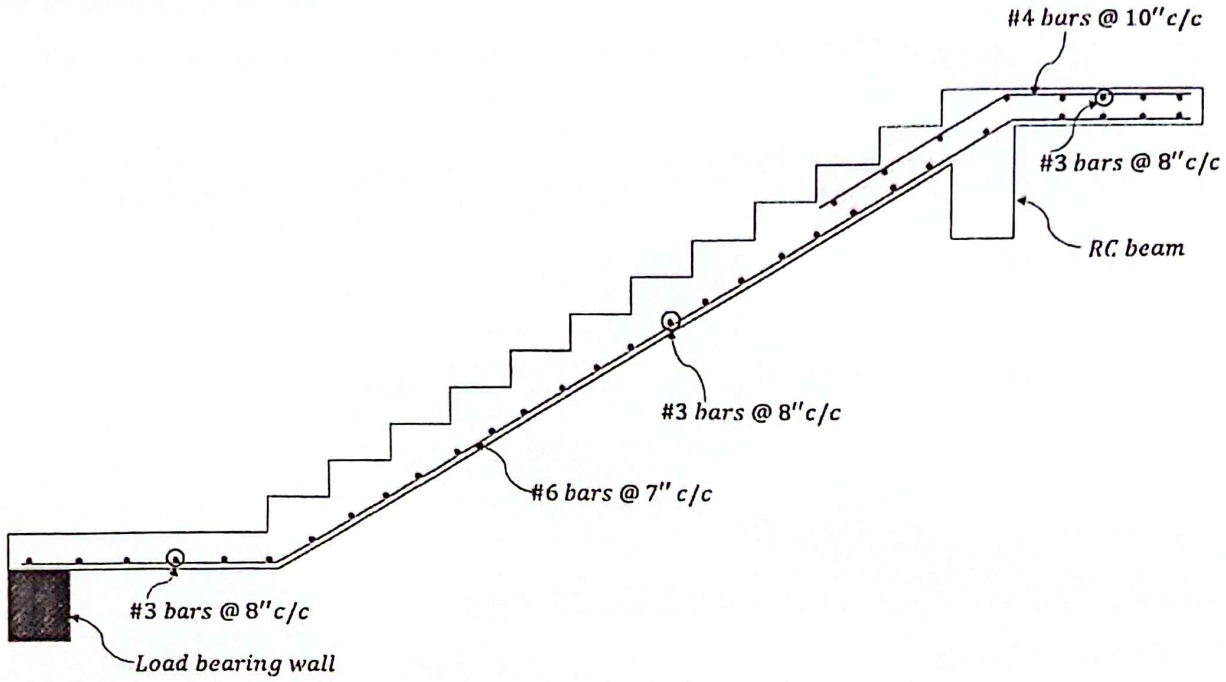
Developed shear, $V_{dev(1)} = R_1 - w_1 d = 1500 - 167.5 \times \frac{5.5}{12} = 1423\ lb$

$$V_{dev(2)R} = R_2 - w_1 d = 2896 - 167.5 \times \frac{5.5}{12} = 2819\ lb$$

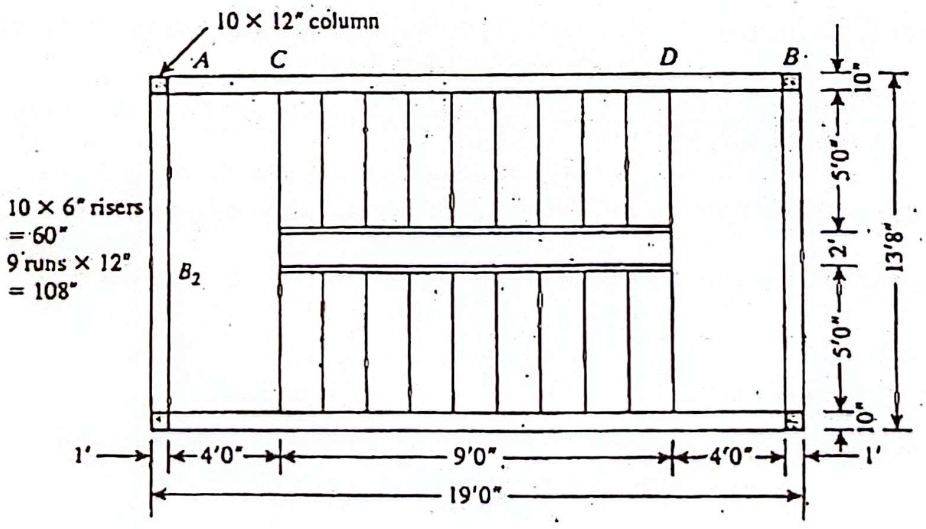
$$V_{dev(2)L} = R_2 - w_2 d = 2896 - 232 \times \frac{5.5}{12} = 2790\ lb$$

Allowable shear, $V_{all} = 1.1\sqrt{f_c'}\, bd = 1.1 \times \sqrt{3500} \times 12 \times 5.5 = 4295\ lb > V_{dev}\ \ (ok)$

Reinforcement Detailing:



Pb-3. Design a typical flight of the staircase shown in figure below, which is a part of a multistory building. The height between the concrete floors is 10 ft. The stair are supported at the ends of the landings and carry a live load equal to 120 psf. Use $f_c' = 3$ ksi and $f_y = 60$ ksi.



Sol^n:

Stair span length, $L = 6'' + 4' + 9' + 4' + 6'' = 216'' = 18'$

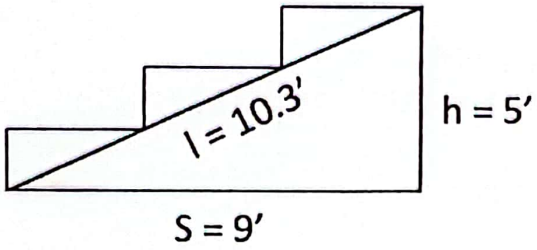
Stair length, $S = 9'$

Since the support condition of the stair is simply supported,

$\therefore$ Thickness of stair slab, $t = \frac{L}{25} = \frac{216''}{25} = 8.64'' \approx 9''$

Stair height, h = No. of rise × Riser = $10 \times 6'' = 60'' = 5'$

$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2} = \sqrt{9^2 + 5^2} = 10.3'$



Load Calculation:

Dead load:

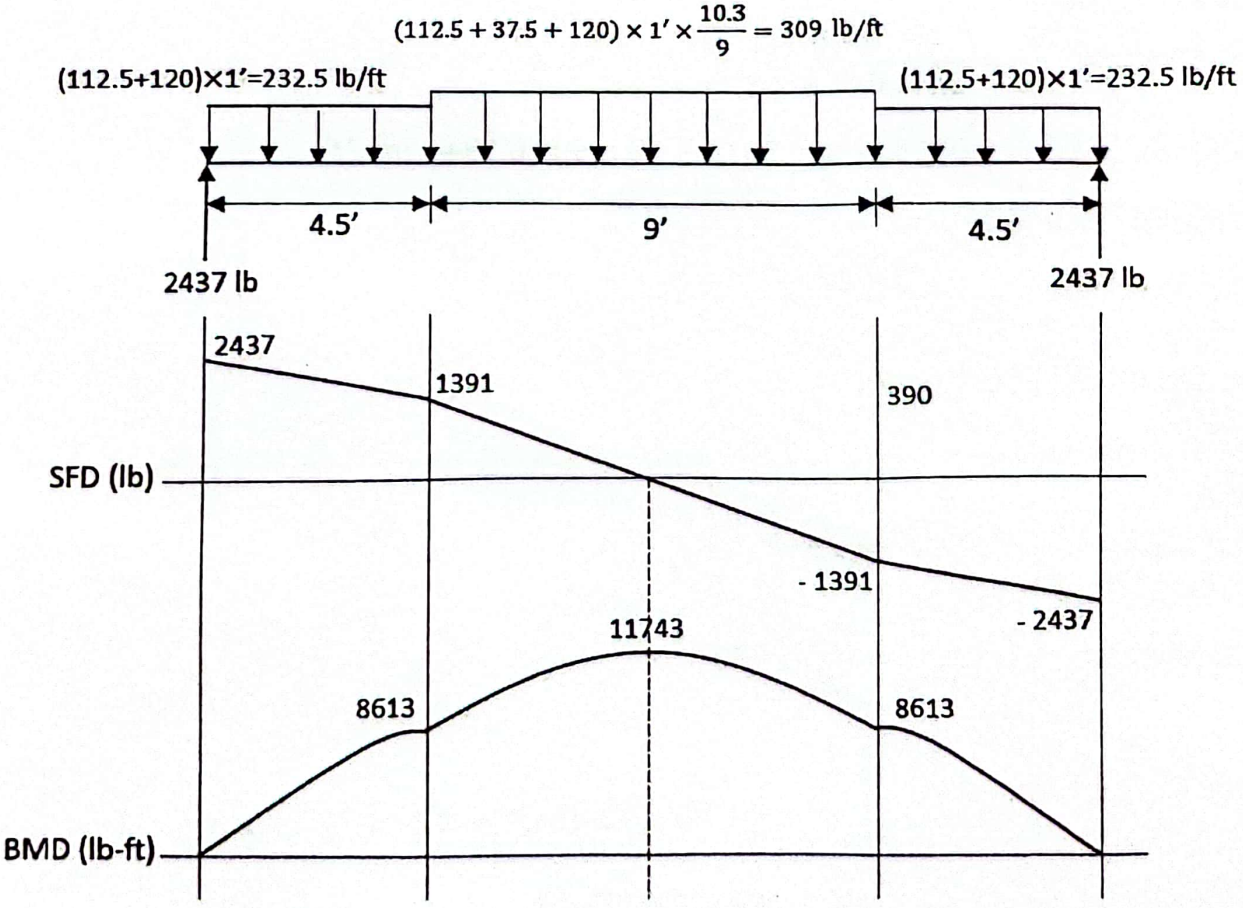
Self weight of stair slab $= \frac{9}{12} \times 150 = 112.5\ psf$

Weight due to stair step $= \frac{\left(\frac{6+0}{2}\right)}{12} \times 150 = 37.5\ psf$

Live load:

Live load = 120 psf

Considering 1 ft. strip along stair span length,



Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \approx 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 3} = \frac{24}{1.35} = 17.78$$

$$k = \frac{n}{n+r} = 0.336$$

$$j = 1 - \frac{k}{3} = 0.888$$

Depth check:

Maximum moment = 11.743 k – ft.

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Rightarrow 11.743 \times 12 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 12 \times d^2$$

$$\therefore d = 7.64\ in. \approx 8\ in.$$

$$d_{provided} = t - cc = 9 - 1 = 8\ in. \geq d\ \ (ok)$$

Reinforcement:

Longitudinal bar:

$$\therefore A_s(+ve) = \frac{M}{f_s j d} = \frac{11.743 \times 12}{24 \times 0.888 \times 8} = 0.827\ \ in^2/ft$$

Here, A_s (min) $= 0.0018\, bt = 0.0018 \times 12 \times 9 = 0.1944\ in^2/ft$

Provide #7 bar @ 8 in. c/c

Distribution bar:

A_s (min) $= 0.1944\ in^2/ft$

Provide #4 bar @ 12 in. c/c

Beam design:

Here,

Beam span, $L = 13' - 8'' = 13.67' \approx 14'$

Beam width, $b = 12''$

$$\therefore \text{Beam height, } h = \frac{b}{60\%} = \frac{12''}{0.6} = 20''$$

Load Calculation:

Dead load:

$$\text{Self weight of beam} = \frac{12 \times 20}{144} \times 150 = 250\ lb/ft$$

Load from landing slab or stair slab $= 112.5\ psf$

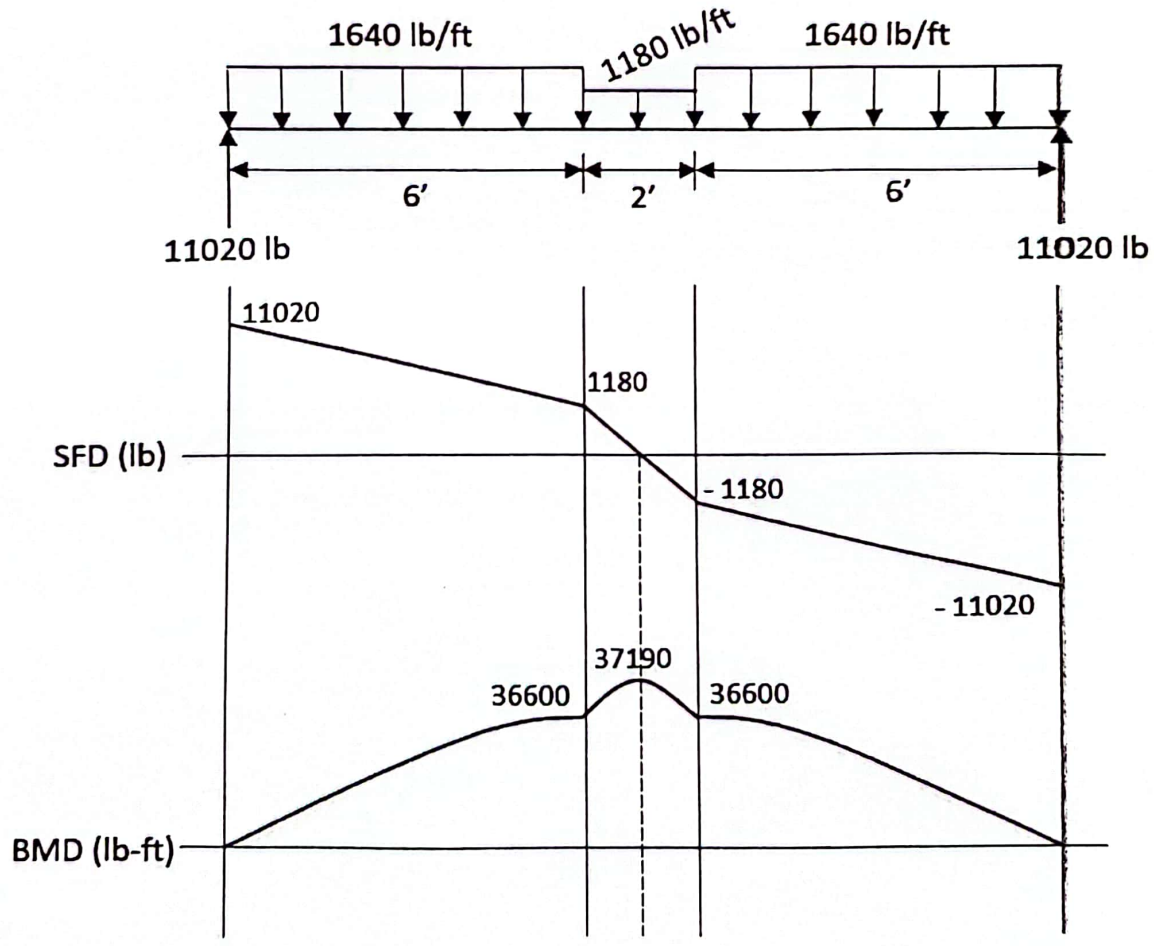
Load from stair $= (112.5 + 37.5) = 150\ psf$

Live load:

Live load $= 120\ psf$

$\therefore$ Load from landing on beam $= 250 + (112.5 \times 4) + (120 \times 4) = 1180\ lb/ft$

$\therefore$ Load from stair on beam $= 250 + (150 + 120) \times \frac{10.3}{2} = 1640\ lb/ft$



Depth check:

Maximum moment = 37.19 k – ft.

$M = \frac{1}{2} f_c k j b d^2$

$\Rightarrow 37.19 \times 12 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 12 \times d^2$

$\therefore d = 13.6\ in. \approx 14\ in.$

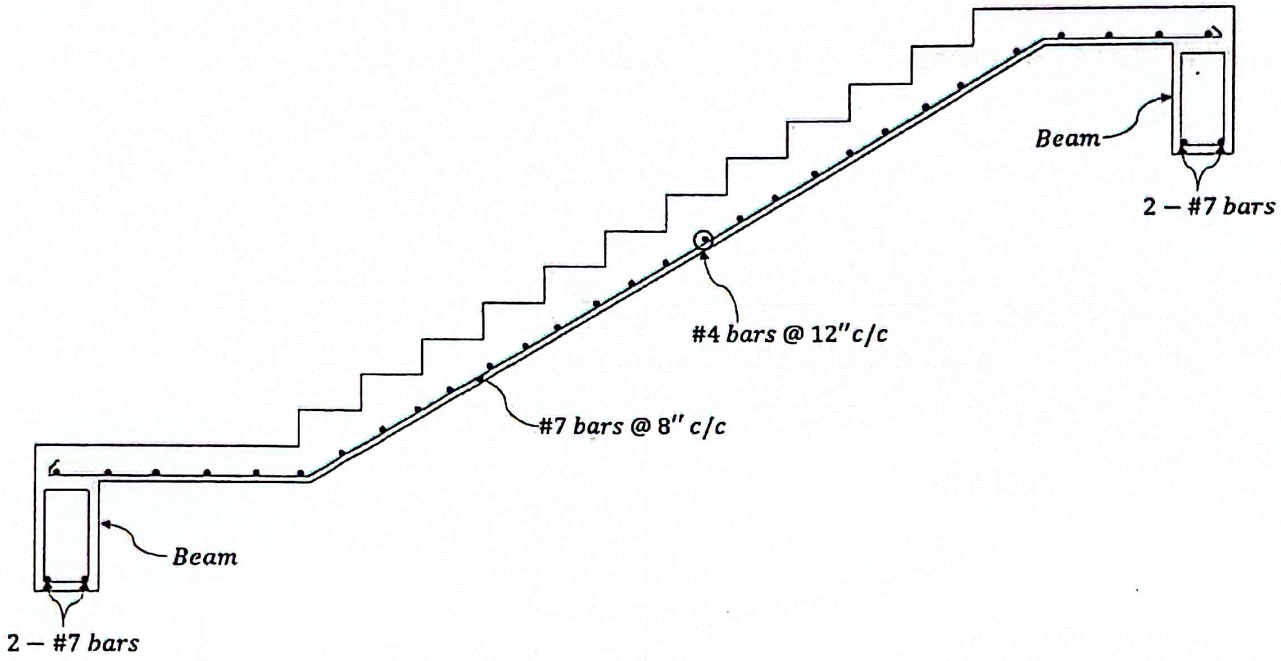
$d_{provided} = h - cc = 20 - 2 = 18\ in. \geq d\ \ (ok)$

Reinforcement:

$$\therefore A_s = \frac{M}{f_s jd} = \frac{37.19 \times 12}{24 \times 0.888 \times 18} = 1.163\ in^2$$

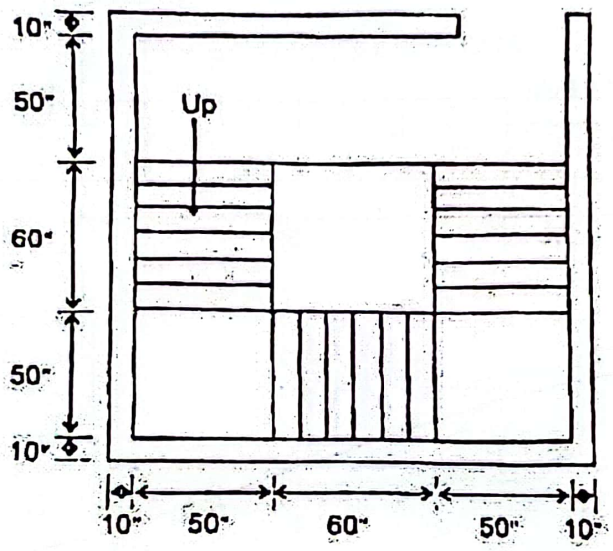
Provide 2 - #7 bars

Reinforcement Detailing:



4.4.1.2 Three Flight Stairs – WSD

Pb-4. Design and draw the reinforcement details of the staircase shown in the figure below. The stair is supported on 10 in. thick brick wall in each side. This staircase is for a commercial building. Given $f_c' = 3000$ psi, $f_y = 60000$ psi, and the floor to floor height is 10 ft. Follow WSD method. [*CE-2016*]



Sol^n:

Stair span length, $L = 6'' + 50'' + 60'' + 50'' + 6'' = 172'' = 14.33'$

Stair length, $S = 60'' = 5'$

Since the support condition of the stair is simply supported,

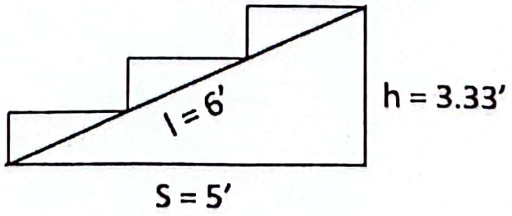
$\therefore$ Thickness of stair slab, $t = \frac{L}{25} = \frac{172''}{25} = 6.88'' \approx 7''$

Height of each flight, $h = \frac{\text{Floor height}}{\text{Flight no.}} = \frac{10'}{3} = 40'' = 3.33'$

Here, No. of trade (run) $= 6$ & No. rise $= 6 + 1 = 7$

$\therefore$ Riser $= \frac{40''}{7} = 5.7''$

$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2} = \sqrt{5^2 + 3.33^2} = 6'$



Load Calculation:

Dead load:

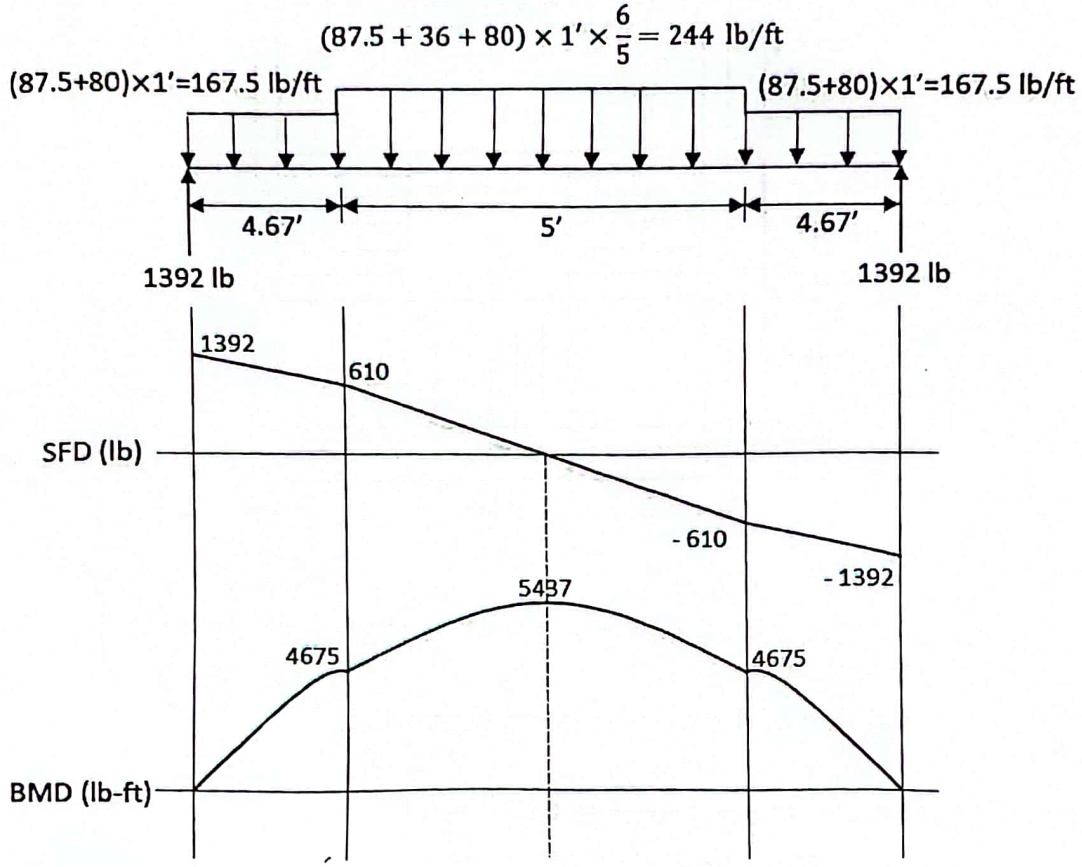
Self weight of stair slab $= \frac{7}{12} \times 150 = 87.5\ psf$

Weight due to stair step $= \frac{\left(\frac{5.7+0}{2}\right)}{12} \times 150 = 36\ psf$

Live load:

Live load $= 80\ psf$

Considering 1 ft. strip along stair span length,



Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \approx 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 3} = \frac{24}{1.35} = 17.78$$

$$k = \frac{n}{n+r} = 0.336$$

$$j = 1 - \frac{k}{3} = 0.888$$

Depth check:

Maximum moment = 5.437 k – ft.

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Rightarrow 5.437 \times 12 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 12 \times d^2$$

$$\therefore d = 5.2\ in. \approx 6\ in.$$

$$d_{provided} = t - cc = 7 - 1 = 6\ in. \geq d\ \ (ok)$$

Reinforcement:

Longitudinal bar:

$$\therefore A_s(+ve) = \frac{M}{f_s jd} = \frac{5.437 \times 12}{24 \times 0.888 \times 6} = 0.51\ in^2/ft$$

Here, A_s (min) $= 0.0018\ bt = 0.0018 \times 12 \times 7 = 0.1512\ \ in^2/ft$

Provide #5 bar @ 7 in. c/c

Distribution bar:

A_s (min) $= 0.1512\ \ in^2/ft$

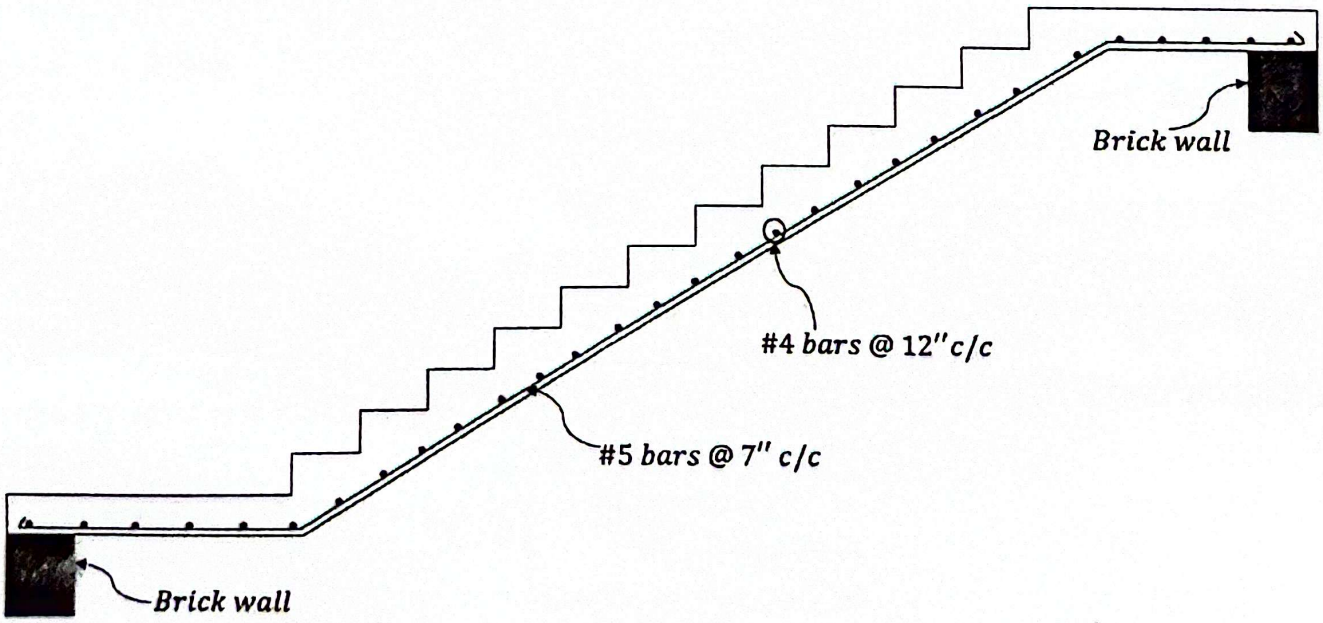
Provide #4 bar @ 12 in. c/c

Shear check:

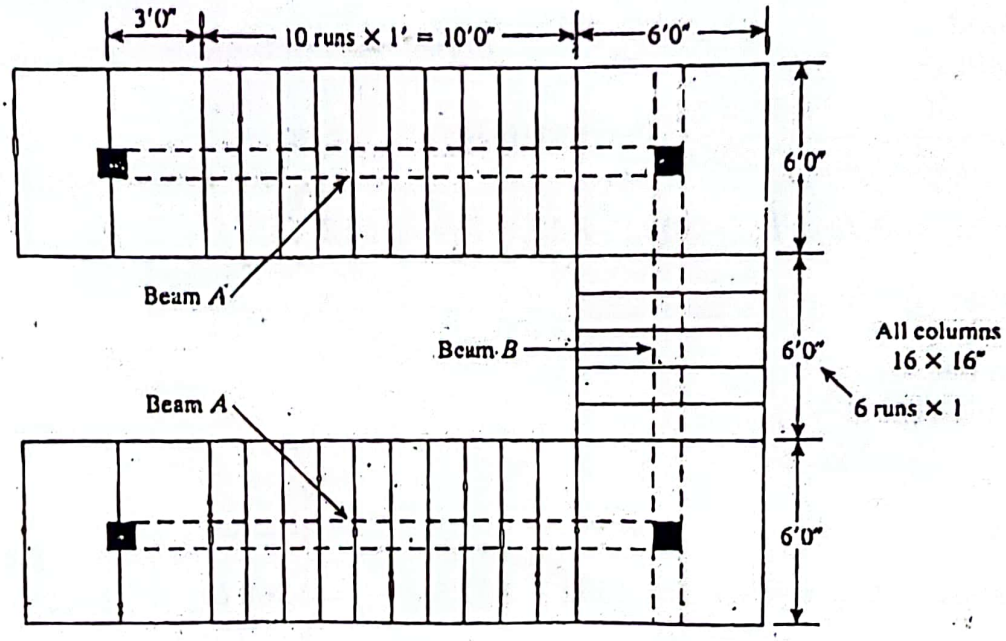
Developed shear, $V_{dev} = R - wd = 1392 - 167.5 \times \frac{6}{12} = 1308\ lb$

Allowable shear, $V_{all} = 1.1\sqrt{f_c'}\ bd = 1.1 \times \sqrt{3000} \times 12 \times 7 = 5061\ lb > V_{dev}\ \ (ok)$

Reinforcement Detailing:



Pb-5. Design a typical flight of stairs in a public building for the staircase arrangement shown in figure below. The stairs are supported by central beams, A and B. Design only one flight and the supporting beams A and B. The runs are 1 ft. width and rises are 6.5 in. high. Use $f_c' = 3$ ksi and $f_y = 60$ ksi, and a live load equal to 80 psf.



Sol^n:

The beam supports each stair flight and so stair acts as a cantilever member both sides of the beam. So, larger stair flight should be designed which rests on beam A.

Stair effective span length, $L = \frac{6'}{2} = 3' = 36''$

Stair length, $S = 10'$

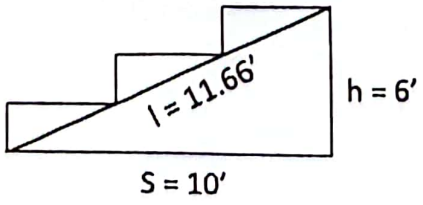
Since the support condition of the stair is cantilever,

$\therefore$ Thickness of stair slab, $t = \frac{L}{12} = \frac{36''}{12} = 3'' \approx 3.5''$

Here, No. of trade (run) $= 10$ & No. rise $= 10 + 1 = 11$

Stair height, $h =$ No. of rise $\times$ Riser $= 11 \times 6.5'' = 71.5'' \approx 72'' = 6'$

$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2} = \sqrt{10^2 + 6^2} = 11.66'$



Load Calculation:

Dead load:

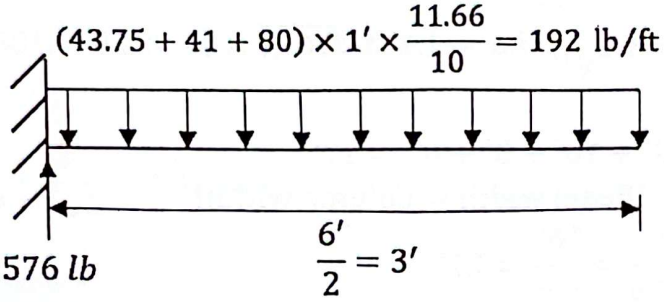
Self weight of stair slab $= \frac{3.5}{12} \times 150 = 43.75\ psf$

Weight due to stair step $= \frac{\left(\frac{6.5+0}{2}\right)}{12} \times 150 = 41\ psf$

Live load:

Live load $= 80\ psf$

Considering 1 ft. strip along cantilever length of stair,



$\therefore$ Design Moment, $M = \frac{wl^2}{2} = \frac{192 \times 3^2}{2} = 864\ lb - ft = 0.864\ k - ft$

Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \approx 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 3} = \frac{24}{1.35} = 17.78$$

$$k = \frac{n}{n+r} = 0.336$$

$$j = 1 - \frac{k}{3} = 0.888$$

Depth check:

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Rightarrow 0.864 \times 12 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 12 \times d^2$$

$\therefore d = 2.07\ in.$

$d_{provided} = t - cc = 3.5 - 1 = 2.5\ in. \geq d\ \ (ok)$

Reinforcement:

Longitudinal bar:

$$\therefore A_s = \frac{M}{f_s j d} = \frac{0.864 \times 12}{24 \times 0.888 \times 2.5} = 0.195\ in^2/ft$$

Here, A_s (min) $= 0.0018\ bt = 0.0018 \times 12 \times 3.5 = 0.0756\ \ in^2/ft$

Provide #3 bar @ 6 in. c/c

Distribution bar:

A_s (min) $= 0.0756\ in^2/ft$

Provide #3 bar @ ~~17~~ 12 in. c/c

Shear check:

Developed shear, $V_{dev} = R - wd = 576 - 192 \times \frac{2.5}{12} = 536\ lb$

Allowable shear, $V_{all} = 1.1\sqrt{f_c'}\, bd = 1.1 \times \sqrt{3000} \times 12 \times 2.5 = 1807\ lb > V_{dev}\ (ok)$

Beam design:

Beam span, $L = 6'' + 3' + 10' + 3' + 6'' = 17'$

Beam width, $b = 16''$ [Beam width = Column width]

$\therefore$ Beam height, $h = \frac{b}{60\%} = \frac{16''}{0.6} = 27''$

Load Calculation:

Dead load:

Self weight of beam $= \frac{16 \times 27}{144} \times 150 = 450\ lb/ft$

Load from landing slab or stair slab $= 75\ psf$

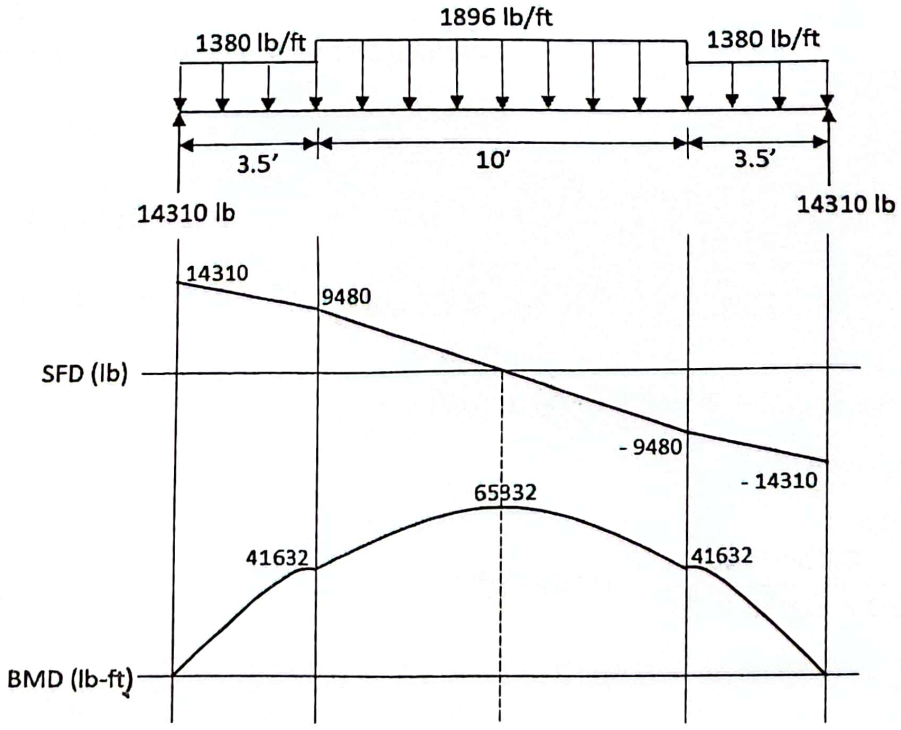
Load from stair $= 75 + 41 = 116\ psf$

Live load:

Live load $= 80\ psf$

$\therefore$ Load from landing on beam $= 450 + (75 \times 6) + (80 \times 6) = 1380\ lb/ft$

$\therefore$ Load from stair on beam $= \{450 + (116 \times 6) + (80 \times 6)\} \times \frac{11.66}{10} = 1896\ lb/ft$



Depth check:

Maximum moment = 65.332 k – ft.

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Longrightarrow 65.332 \times 12 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 16 \times d^2$$

$$\therefore d = 15.6\ in. \approx 16\ in.$$

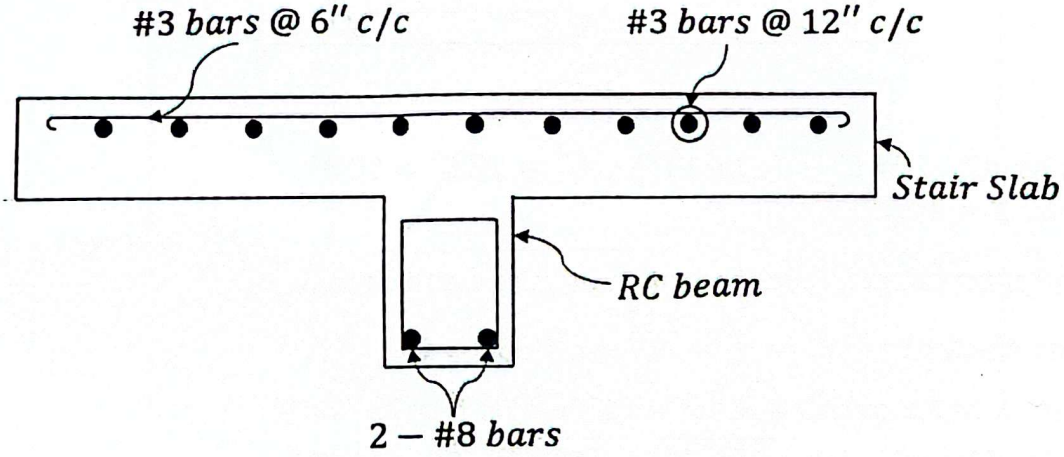
$$d_{provided} = h - cc = 27 - 2 = 25\ in. \geq d\ \ (Not\ ok)$$

Reinforcement:

$$\therefore A_s = \frac{M}{f_s j d} = \frac{65.332 \times 12}{24 \times 0.888 \times 25} = 1.471\ in^2$$

Provide 2 - #8 bars

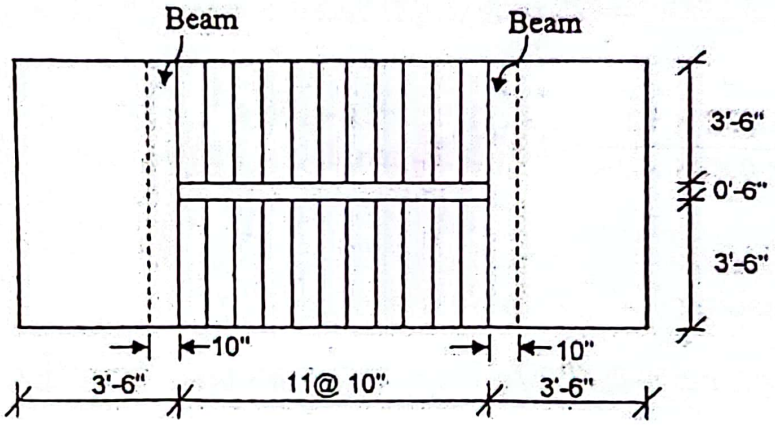
Reinforcement Detailing:



4.4.2 Stair Slab Spanning Longitudinally – USD

4.4.2.1 Double Flight Stairs / Dog-legged Stairs – USD

Pb-6. Design any flight of the following stair and show reinforcement detailing. The dimension of the beams are $10'' \times 18''$ and height of each riser is $6''$. Live load on stair is 80 psf. Use materials of $f_c' = 4000$ psi and $f_y = 60{,}000$ psi. Follow USD method.



Sol^n:

Stair span length, $L = 6'' + (11 * 10'') + 6'' = 122'' = 10.2'$

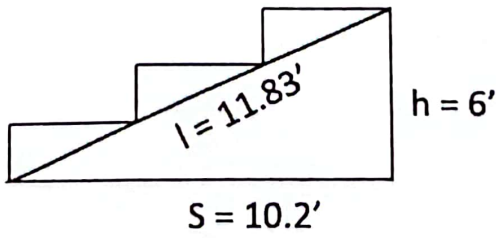
Stair length, $S = 10.2'$

Since the support condition of the stair is both end continuous,

$\therefore$ Thickness of stair slab, $t = \frac{L}{28} = \frac{122''}{28} = 4.36'' \approx 5''$

Stair height, $h = \text{No. of rise} \times \text{Riser} = (11 + 1) \times 6 = 72'' = 6'$

$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2} = \sqrt{10.2^2 + 6^2} = 11.83'$



Load Calculation:

Dead load:

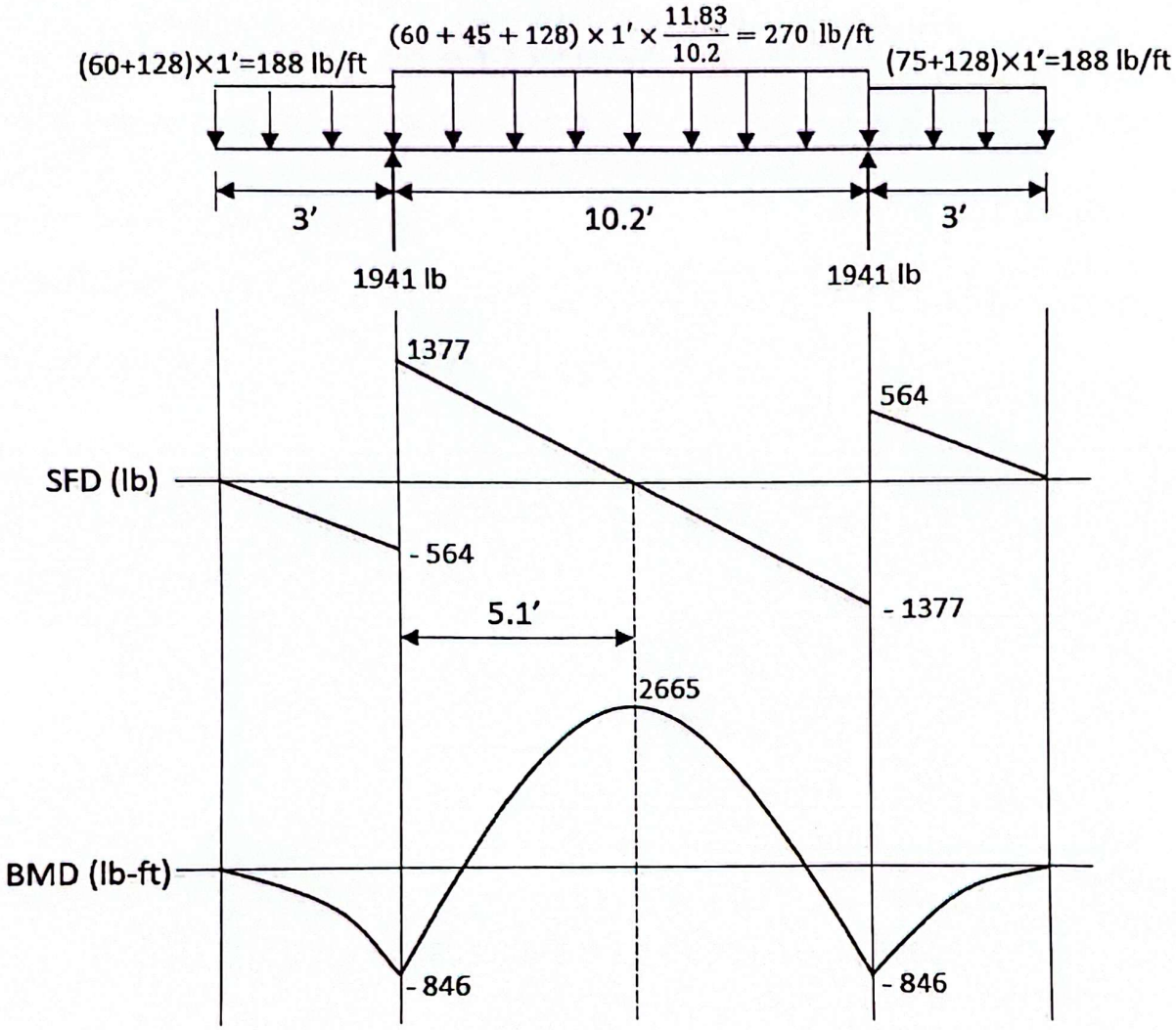
Self weight of stair slab $= 1.2 \times \frac{4}{12} \times 150 = 60\ psf$

Weight due to stair step $= 1.2 \times \frac{\left(\frac{6+0}{2}\right)}{12} \times 150 = 45\ psf$

Live load:

Live load $= 1.6 \times 80 = 128$ psf

Considering 1 ft. strip along stair span length,



Depth check:

Maximum moment = 2.665 k – ft.

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{4}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0181$$

Now, $M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$

$$\Rightarrow 2.665 \times 12 = 0.90 \times 0.0181 \times 60 \times 12 \times d^2 \times \left(1 - 0.59 \times 0.0181 \times \frac{60}{4}\right)$$

$$\Rightarrow 31.98 = 11.7288 \times d^2 \times 0.8398$$

$$\therefore d = 3.25\ in. \approx 3.5\ in$$

$$d_{provided} = t - cc = 5 - 1 = 4\ in. \geq d\ \ (ok)$$

Reinforcement:

Longitudinal bar:

$$A_s(+ve) = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{2.665 \times 12}{0.90 \times 60 \times \left(4 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 4 \times 12}$$

$$\therefore A_s = 0.148 \ in^2/ft$$

Here, A_s (min) $= \frac{200}{f_y} bd = \frac{200}{60000} \times 12 \times 4 = 0.16 \ in^2/ft$

Provide #3 bar @ 8 in. c/c

$$A_s(-ve) = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{0.846 \times 12}{0.90 \times 60 \times \left(4 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 4 \times 12}$$

$$\therefore A_s = 0.0484 \ in^2/ft$$

Here, A_s (min) $= 0.16 \ in^2/ft$

Provide #3 bar @ 8 in. c/c

Distribution bar:

A_s (min) $= 0.0018 \ bt = 0.0018 \times 12 \times 5 = 0.108 \ in^2/ft$

Provide #3 bar @ 12 in. c/c

Shear check:

Ultimate shear, $V_u = R - wd$

$$V_{u(L)} = R_1 - w_1 d = 1941 - 188 \times \frac{4}{12} = 1878 \ lb$$

$$V_{u(R)} = R_1 - w_2 d = 1941 - 270 \times \frac{4}{12} = 1851 \ lb$$

Allowable shear, $V_{all} = 2\varphi\sqrt{f_c'}\, bd = 2 \times 0.75 \times \sqrt{4000} \times 12 \times 4 = 4554 \ lb > V_{dev}$ (ok)

Beam design:

Here,

Beam width, $b = 10''$

Beam height, $h = 18''$

$\therefore$ Beam span, $L = 6'' + (3' + 6'' + 6'' + 3 + 6'') + 6'' = 102'' = 8.5'$

Load Calculation:

Dead load:

Self weight of beam $= 1.2 \times \frac{10 \times 18}{144} \times 150 = 225\ lb/ft$

Load from landing slab or stair slab $= 1.2 \times 50\ psf = 60\ psf$

Load from stair $= 1.2 \times (50 + 37.5) psf = 105\ psf$

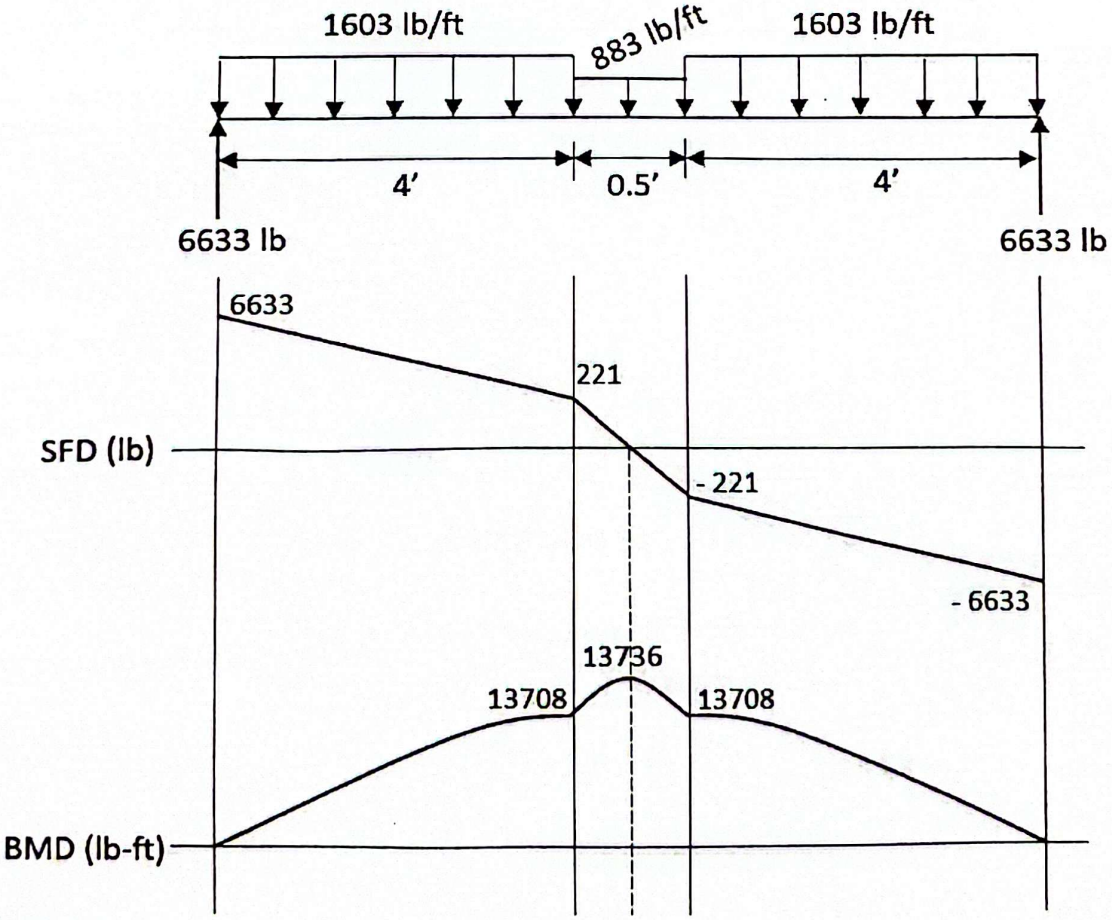
Live load:

Live load $= 1.6 \times 80\ psf = 128\ psf$

$\therefore$ Load from landing on beam $= 225 + (60 \times 3.5) + (128 \times 3.5) = 883\ lb/ft$

$\therefore$ Load from stair on beam $= 225 + (105 + 128) \times \frac{11.83}{2} = 1603\ lb/ft$

এখানে বলা রাখা ভালো, Beam এর load calculation করার সময় প্রতিটি psf load কে Beam এর transverse direction এর dimension দিয়ে গুণ করতে হবে। আর পুরো stair এর load half করে দুই পাশের beam তা carry করবে।



Depth check:

Maximum moment = 13.736 k – ft.

$\rho = 0.0181$

Now, $M = \varphi\rho f_y b d^2 \left(1 - 0.59\rho\frac{f_y}{f_c'}\right)$

$\Rightarrow 13.736 \times 12 = 0.90 \times 0.0181 \times 60 \times 10 \times d^2 \times \left(1 - 0.59 \times 0.0181 \times \frac{60}{4}\right)$

$\Rightarrow 164.832 = 9.774 \times d^2 \times 0.8398$

$\therefore d = 4.48\ in. \approx 5\ in$

$d_{provided} = h - cc = 18 - 2 = 16\ in. \geq d\ \ (ok)$

Reinforcement:

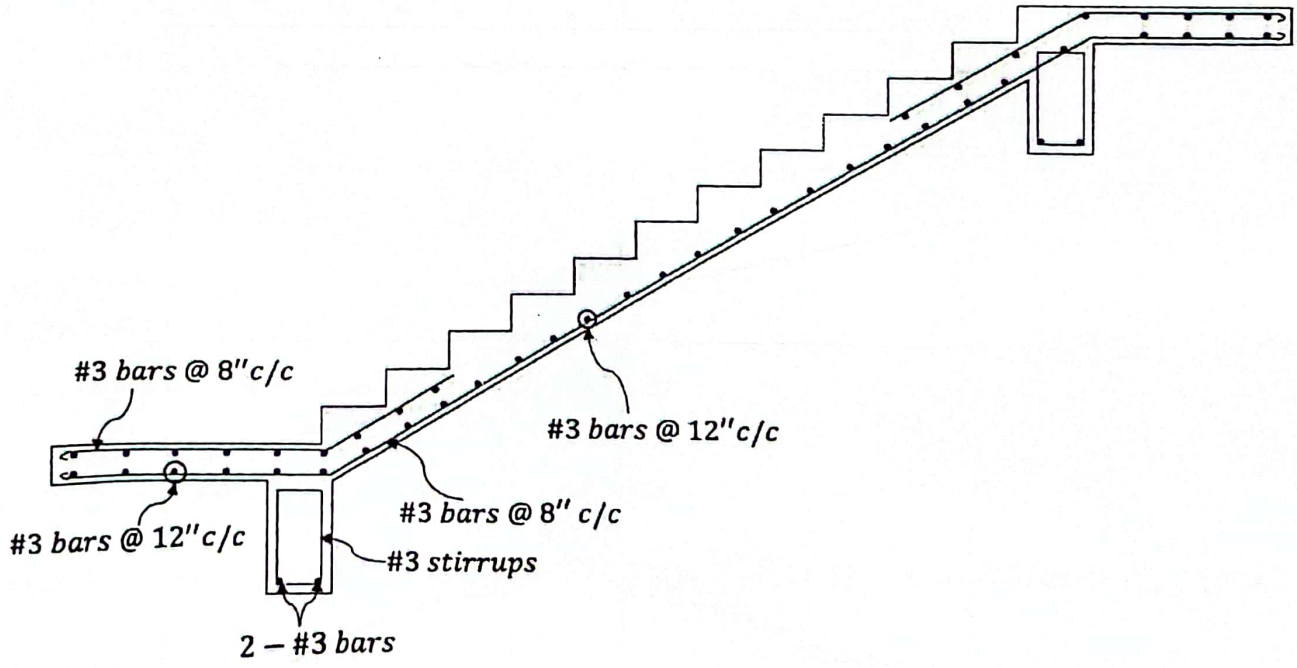
$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{13.736 \times 12}{0.90 \times 60 \times \left(16 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 4 \times 10}$$

$\therefore A_s = 0.191\ in^2$

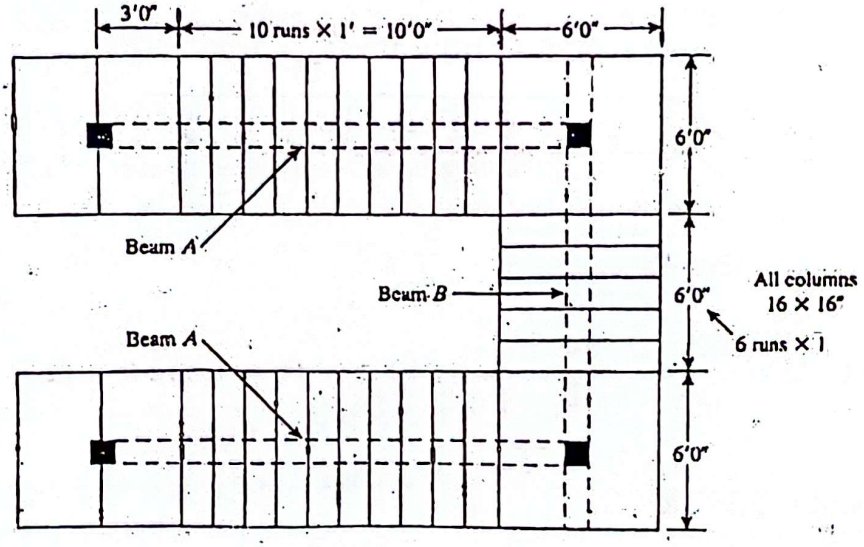
Provide 2 - #3 bars

Reinforcement Detailing:



4.4.2.2 Three Flight Stairs – USD

Pb-7. Design a typical flight of stairs in a public building for the staircase arrangement shown in figure below. The stairs are supported by central beams, A and B. Design only one flight and the supporting beams A and B. The runs are 1 ft. width and rises are 6.5 in. high. Use $f_c' = 3$ ksi and $f_y = 60$ ksi, and a live load equal to 80 psf.



Sol^n:

The beam supports each stair flight and so stair acts as a cantilever member both sides of the beam. So, larger stair flight should be designed which rests on beam A.

Stair effective span length, $L = \frac{6'}{2} = 3' = 36''$

Stair length, $S = 10'$

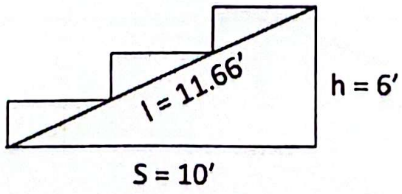
Since the support condition of the stair is cantilever,

$\therefore$ Thickness of stair slab, $t = \frac{L}{10} = \frac{36''}{10} = 3.6'' \approx 4''$

Here, No. of trade (run) $= 10$ & No. rise $= 10 + 1 = 11$

Stair height, $h =$ No. of rise $\times$ Riser $= 11 \times 6.5'' = 71.5'' \approx 72'' = 6'$

$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2} = \sqrt{10^2 + 6^2} = 11.66'$



Load Calculation:

Dead load:

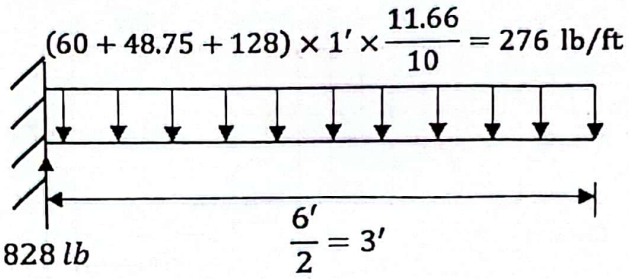
Self weight of stair slab $= 1.2 \times \frac{4}{12} \times 150 = 60\ psf$

Weight due to stair step $= 1.2 \times \frac{\left(\frac{6.5+0}{2}\right)}{12} \times 150 = 48.75\ psf$

Live load:

Live load $= 1.6 \times 80 = 128\ psf$

Considering 1 ft. strip along cantilever length of stair,



$\therefore$ Design Moment, $M = \frac{wl^2}{2} = \frac{276 \times 3^2}{2} = 1242\ lb - ft = 1.242\ k - ft$

Depth check:

Maximum moment = 1.242 k – ft.

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{3}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0135$$

Now, $M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$

$$\Rightarrow 1.242 \times 12 = 0.90 \times 0.0135 \times 60 \times 12 \times d^2 \times \left(1 - 0.59 \times 0.0135 \times \frac{60}{3}\right)$$

$$\Rightarrow 14.904 = 8.748 \times d^2 \times 0.8407$$

$\therefore d = 1.42\ in.$

$d_{provided} = t - cc = 4 - 1 = 3\ in. \geq d\ \ (ok)$

Reinforcement:

Longitudinal bar (top bar):

$$A_s(-ve) = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{1.242 \times 12}{0.90 \times 60 \times \left(3 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 12}$$

$\therefore A_s = 0.109\ in^2/ft$

Here, $A_s\ (\min) = \frac{200}{f_y} bd = \frac{200}{60000} \times 12 \times 3 = 0.12\ in^2/ft$

Provide #3 bar @ 10 in. c/c

Distribution bar:

A_s (min) $= 0.12\ \ in^2/ft$
Provide #3 bar @ 10 in. c/c

Shear check:

Developed shear, $V_{dev} = R - wd = 828 - 276 \times \frac{3}{12} = 759\ lb$

Allowable shear, $V_{all} = 1.1\sqrt{f_c'}\ bd = 1.1 \times \sqrt{3000} \times 12 \times 4 = 2892\ lb > V_{dev}\ \ (ok)$

Beam design:
Beam span, $L = 6'' + 3' + 10' + 3' + 6'' = 17'$
Beam width, $b = 16''$ [Beam width = Column width]
$\therefore$ Beam height, $h = \frac{b}{60\%} = \frac{16''}{0.6} = 27''$

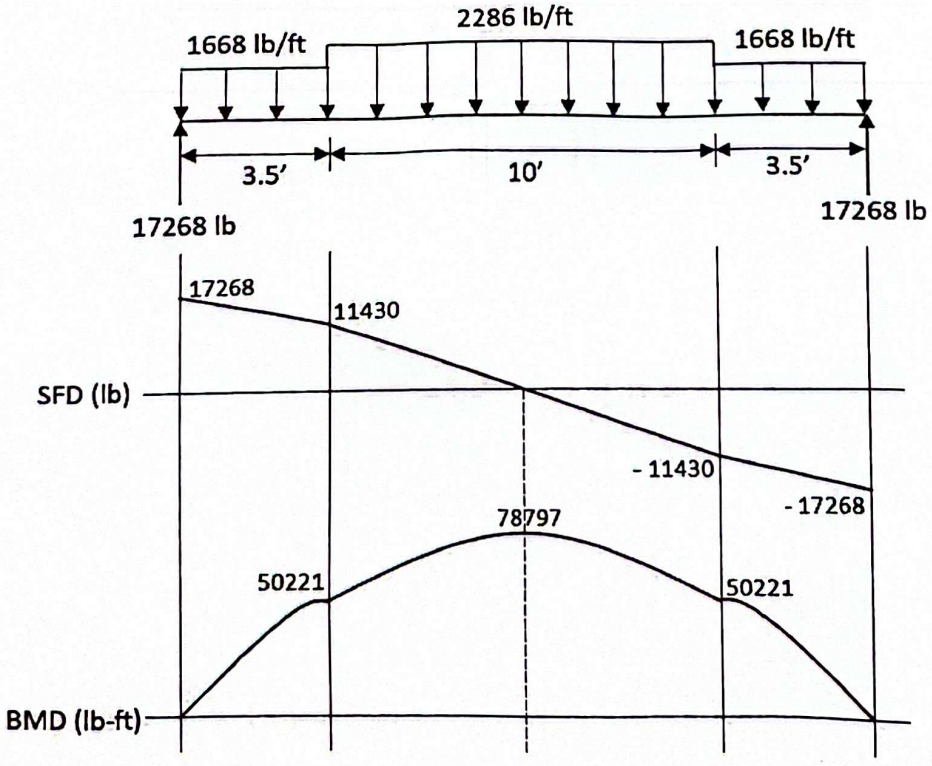
Load Calculation:
Dead load:

Self weight of beam $= 1.2 \times \frac{16 \times 27}{144} \times 150 = 540\ lb/ft$

Load from landing slab or stair slab $= 60\ psf$
Load from stair $= 60 + 48.75 = 108.75\ psf$
Live load:
Live load $= 1.6 \times 80 = 128\ psf$

$\therefore$ Load from landing on beam $= 540 + (60 \times 6) + (128 \times 6) = 1668\ lb/ft$

$\therefore$ Load from stair on beam $= \{540 + (108.75 \times 6) + (128 \times 6)\} \times \frac{11.66}{10} = 2286\ lb/ft$



Depth check:

Maximum moment = 78.797 k – ft.

$\rho = 0.0135$

Now, $M = \varphi\rho f_y b d^2 \left(1 - 0.59\rho\frac{f_y}{f_c'}\right)$

$\Rightarrow 78.797 \times 12 = 0.90 \times 0.0135 \times 60 \times 12 \times d^2 \times \left(1 - 0.59 \times 0.0135 \times \frac{60}{3}\right)$

$\Rightarrow 945.564 = 8.748 \times d^2 \times 0.8407$

$\therefore d = 11.34\ in.$

$d_{provided} = h - cc = 27 - 2 = 25\ in. \geq d\ \ (ok)$

Reinforcement:

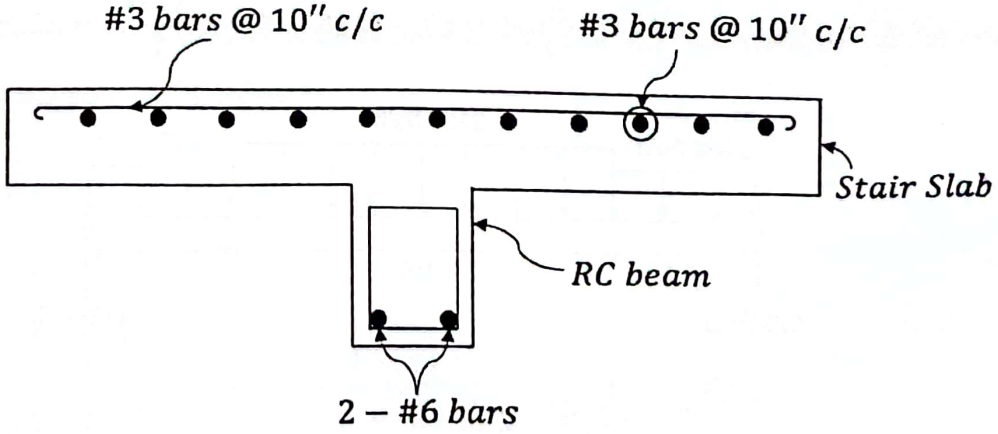
$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{78.797 \times 12}{0.90 \times 60 \times \left(25 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 16}$$

$\therefore A_s = 0.702\ in^2$

Provide 2 - #6 bars

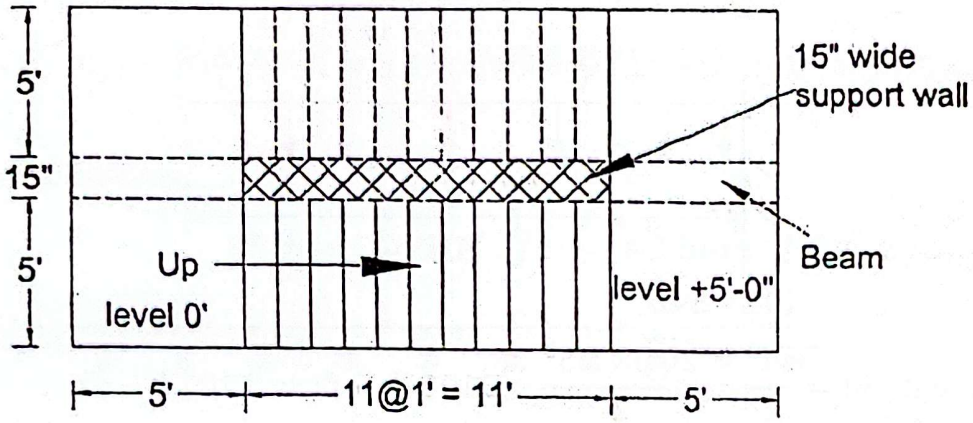
Reinforcement Detailing:



4.4.3 Stair Slab Spanning Transversely – WSD

4.4.3.1 Cantilever Slab Span from Edge Wall or Beam

Pb-8. Design and draw reinforcement detailing of the stair shown in figure below using $f_c' = 3$ ksi and $f_y = 60$ ksi. Live load on stair is 80 psf. [*CE-2018*]



Sol^n:

The brick wall supports each stair flight and so stair acts as a cantilever member both sides of the brick wall.

Stair effective span length, $L = 6'' + 5' = 66'' = 5.5'$

Stair length, $S = 11'$

Since the support condition of the stair is cantilever,

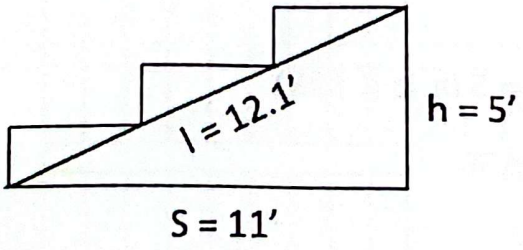
$\therefore$ Thickness of stair slab, $t = \frac{L}{12} = \frac{66''}{12} = 5.5'' \approx 6''$

Height of each flight, $h =$ Difference in level $= 5' - 0' = 5' = 60''$

Here, No. of trade (run) $= 11$ & No. rise $= 11 + 1 = 12$

$\therefore$ Riser $= \frac{60''}{12} = 5''$

$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2} = \sqrt{11^2 + 5^2} = 12.1'$



Load Calculation:

Dead load:

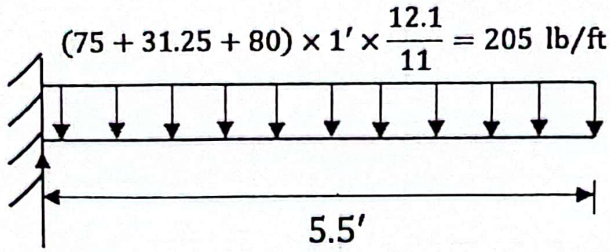
Self weight of stair slab $= \frac{6}{12} \times 150 = 75\ psf$

Weight due to stair step $= \frac{\left(\frac{5+0}{2}\right)}{12} \times 150 = 31.25\ psf$

Live load:

Live load $= 80\ psf$

Considering 1 ft. strip along cantilever span length of stair,



$\therefore$ Design Moment, $M = \frac{wl^2}{2} = \frac{205 \times 5.5^2}{2} = 3101\ lb - ft = 3.101\ k - ft$

Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \approx 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 3} = \frac{24}{1.35} = 17.78$$

$$k = \frac{n}{n+r} = 0.336$$

$$j = 1 - \frac{k}{3} = 0.888$$

Depth check:

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Rightarrow 3.101 \times 12 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 12 \times d^2$$

$\therefore d = 3.92\ in. \approx 4\ in.$

$d_{provided} = t - cc = 6 - 1 = 5\ in. \geq d\ \ (Ok)$

Reinforcement:

Longitudinal bar:

$$\therefore A_s = \frac{M}{f_s j d} = \frac{3.101 \times 12}{24 \times 0.888 \times 5} = 0.349\ \ in^2/ft$$

Here, A_s (min) $= 0.0018\ bt = 0.0018 \times 12 \times 6 = 0.1296\ \ in^2/ft$

Provide #5 bar @ 10 in. c/c

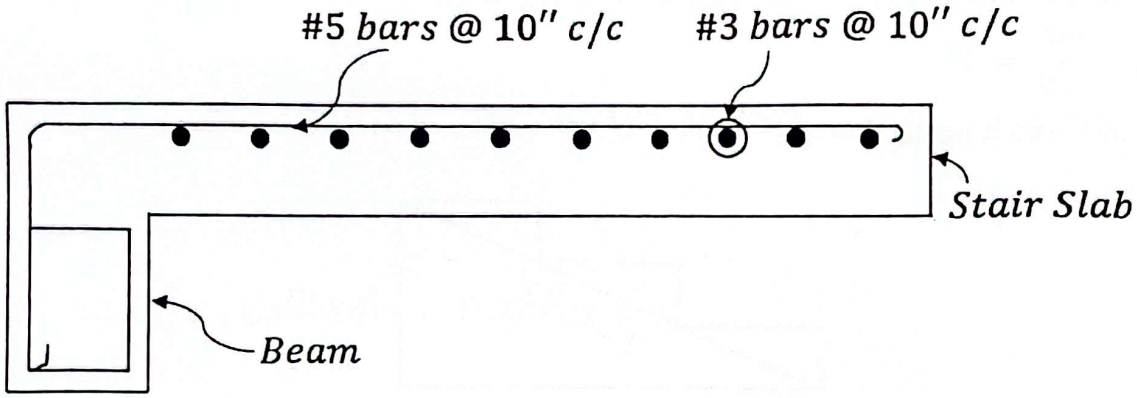
Distribution bar:
A_s (min) $= 0.1296\ in^2/ft$
Provide #3 bar @ 10 in. c/c

Shear check:

Developed shear, $V_{dev} = wl - wd = 1127.5 - 205 \times \frac{5}{12} = 1042\ lb$

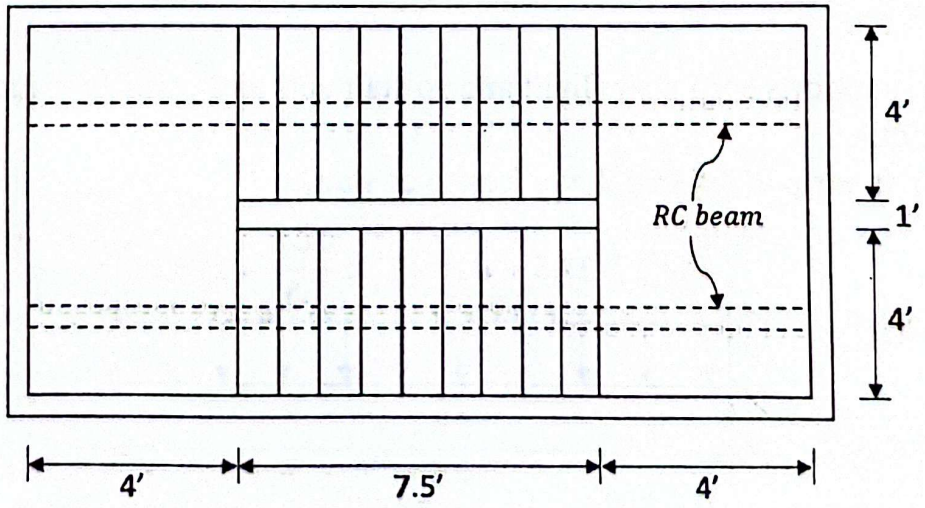
Allowable shear, $V_{all} = 1.1\sqrt{f_c'}\ bd = 1.1 \times \sqrt{3000} \times 12 \times 5 = 3615\ lb > V_{dev}\ (ok)$

Reinforcement Detailing:



4.4.3.2 Cantilever Slab Span from Central Beam

Pb-9. Design and draw reinforcement detailing of the following stair case. Live load on stair is 80 psf. Floor to floor height is 10 ft. Use $f_c' = 3$ ksi and $f_y = 60$ ksi.



Sol^{n}:

The RC beam supports each stair flight and so stair acts as a cantilever member both sides of the RC beam.

Stair effective span length, $L = 4' = 48''$

Stair length, $S = 7.5'$

Since the support condition of the stair is cantilever,

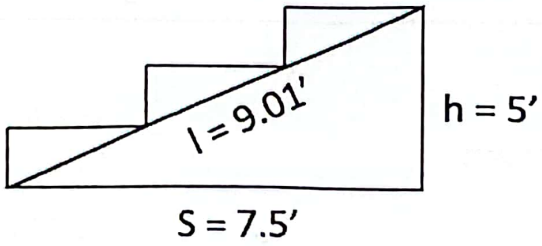
$\therefore$ Thickness of stair slab, $t = \frac{L}{12} = \frac{48''}{12} = 4'' \approx 5''$

Height of each flight, $h = \frac{10'}{2} = 5' = 60''$

Here, No. of trade (run) $= 9$ & No. rise $= 9 + 1 = 10$

$\therefore$ Riser $= \frac{60''}{10} = 6''$

$\therefore$ Stair inclined length, $l = \sqrt{S^2 + h^2} = \sqrt{7.5^2 + 5^2} = 9.01'$



Load Calculation:

Dead load:

Self weight of stair slab $= \frac{5}{12} \times 150 = 62.5\ psf$

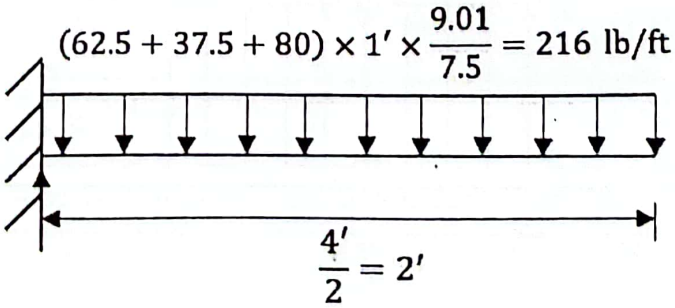
Weight due to stair step $= \frac{\left(\frac{6+0}{2}\right)}{12} \times 150 = 37.5\ psf$

Live load:

Live load $= 80\ psf$

The RC beam supports each stair flight and so stair acts as a cantilever member both sides of the RC beam.

Considering 1 ft. strip along cantilever length of stair,



$\therefore$ Design Moment, $M = \frac{wl^2}{2} = \frac{216 \times 2^2}{2} = 432\ lb - ft = 0.432\ k - ft$

Design constants:

$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \approx 9$

$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 3} = \frac{24}{1.35} = 17.78$

$k = \frac{n}{n+r} = 0.336$

$j = 1 - \frac{k}{3} = 0.888$

Depth check:

$M = \frac{1}{2} f_c k j b d^2$

$\Longrightarrow 0.432 \times 12 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 12 \times d^2$

$\therefore d = 1.46\ in. \approx 2\ in.$

$d_{provided} = t - cc = 5 - 1 = 4\ in. \geq d\ (ok)$

Reinforcement:

Longitudinal bar:

$\therefore A_s = \frac{M}{f_s j d} = \frac{0.432 \times 12}{24 \times 0.888 \times 4} = 0.0608\ in^2/ft$

Here, A_s (min) $= 0.0018\ bt = 0.0018 \times 12 \times 5 = 0.108\ \ in^2/ft$

Provide #3 bar @ 12 in. c/c

Distribution bar:

A_s (min) $= 0.108\ \ in^2/ft$

Provide #3 bar @ 12 in. c/c

Shear check:

Developed shear, $V_{dev} = wl - wd = 216 \times 2 - 216 \times \frac{4}{12} = 360\ lb$

Allowable shear, $V_{all} = 1.1\sqrt{f_c'}\ bd = 1.1 \times \sqrt{3000} \times 12 \times 4 = 2892\ lb > V_{dev}\ (ok)$

Beam design:

Beam span, $L = 6'' + 4' + 7.5' + 4' + 6'' = 16.5'$

Assume, Beam depth, $h = 16.5''$ [Beam depth (in) = Beam span (ft)]

Beam width, $b = 60\% \times 16.5'' = 10''$

Load Calculation:

Dead load:

Self weight of beam $= \frac{10 \times 16.5}{144} \times 150 = 172\ lb/ft$

Load from landing slab or stair slab $= 62.5\ psf$

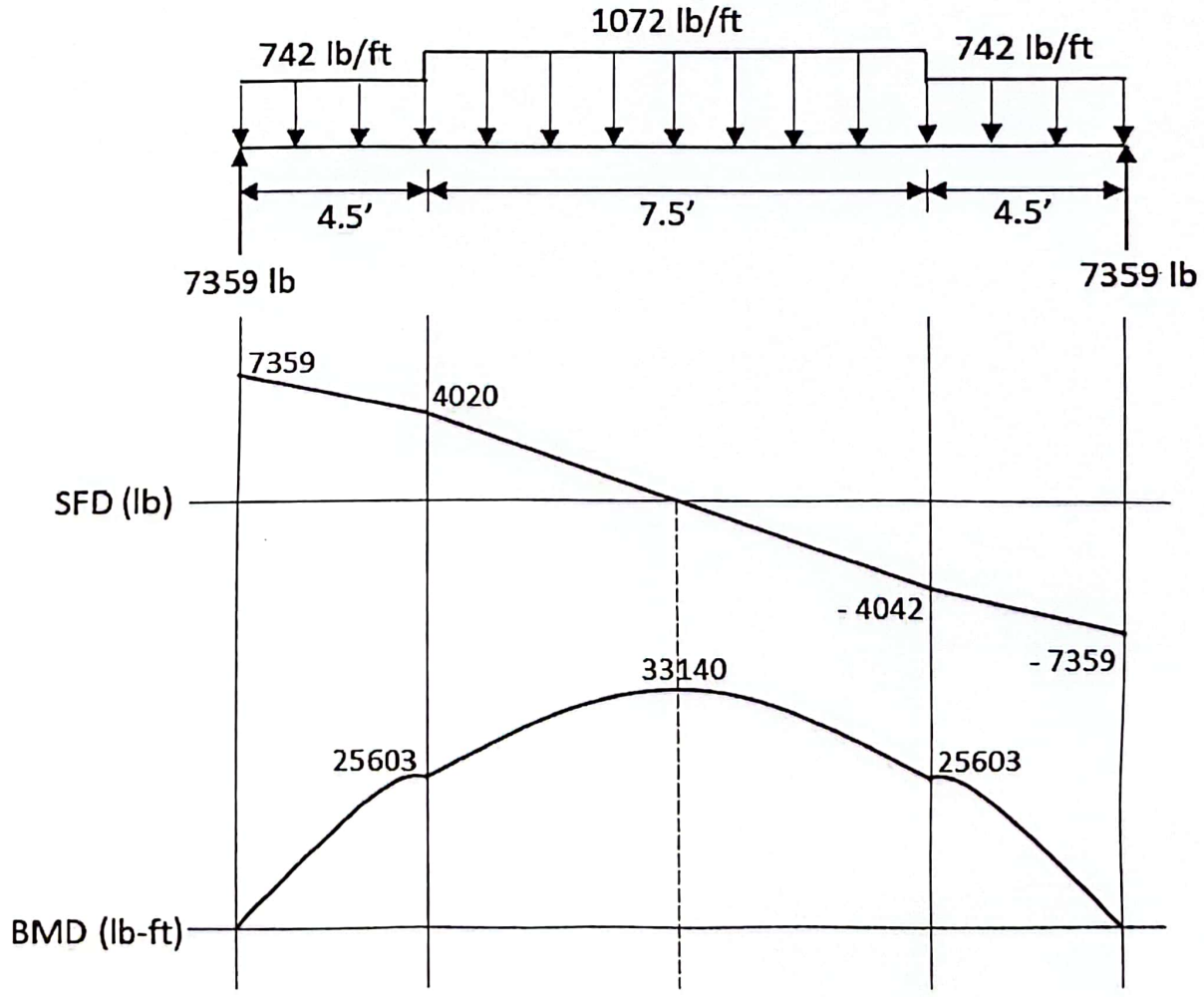
Load from stair $= 62.5 + 37.5 = 100\ psf$

Live load:

Live load $= 80\ psf$

$\therefore$ Load from landing on beam $= 172 + (62.5 \times 4) + (80 \times 4) = 742\ lb/ft$

$\therefore$ Load from stair on beam $= \{172 + (100 \times 4) + (80 \times 4)\} \times \frac{9.01}{7.5} = 1072\ lb/ft$



Depth check:

Maximum moment = 33.14 k – ft.

$M = \frac{1}{2} f_c k j b d^2$

$$\Longrightarrow 33.14 \times 12 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 10 \times d^2$$

$$\therefore d = 14.05\ in. \approx 14.5\ in.$$

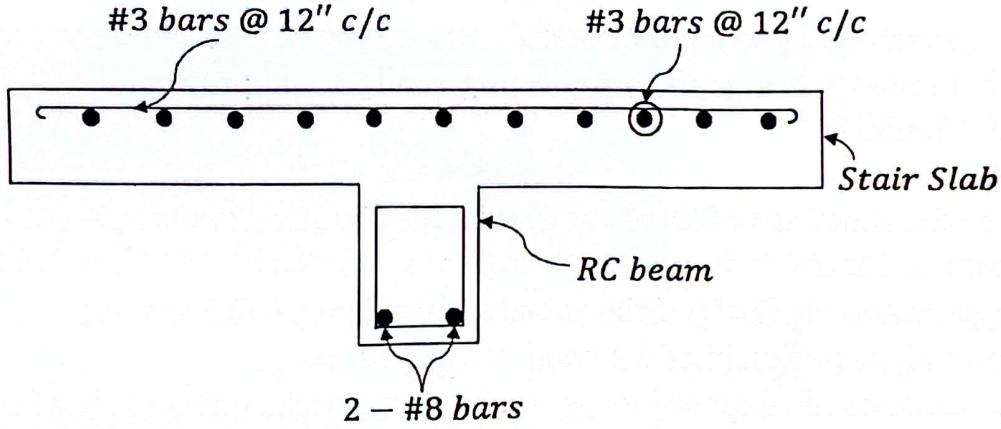
$$d_{provided} = h - cc = 16.5 - 2 = 14.5\ in. \geq d\ \ (Not\ ok)$$

Reinforcement:

$$\therefore A_s = \frac{M}{f_s j d} = \frac{33.14 \times 12}{24 \times 0.888 \times 14.5} = 1.287\ in^2$$

Provide 2 - #8 bars

Reinforcement Detailing:



Chapter - 5
Lintels and Sunshades

5.1 Theory Analysis

Q-1. Define lintel and sunshade?

Ans: Lintel: A lintel is a type of beam or structural horizontal support used to support the above wall when openings like doors, windows are necessary to provide in building construction. Lintels are generally used for load-bearing purposes, but they can also be decorative.

Sunshade: A sunshade is an architectural feature and horizontal or vertical component of a building that reduces heat gain by deflecting sunlight and protects the building insides from climatic elements.

Q-2. Describe the functions of lintel and sunshade in building structures. [*CE-2019*]

Ans: Functions of lintel:

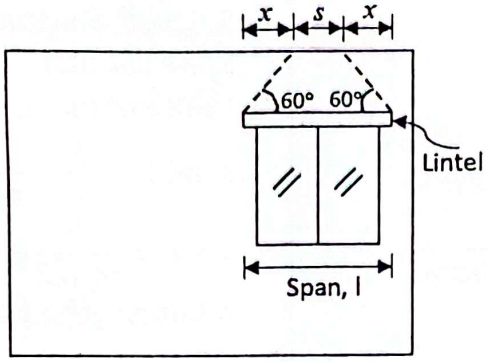
(i) Lintel supports the walls above the openings like doors, windows, etc.
(ii) Lintels provide a safeguard of the windows and doors.
(iii) Lintel withstands the imposed loads coming from bricks including roofing members.
(iv) The lintel is used to transfer all imposed loads to the side walls.
(v) Sometimes lintels are used as a decorative architectural element.

Functions of sunshade:

(i) Sunshade prevents glare from direct sunlight and protects from outside raindrops.
(ii) Sunshade provides privacy from outside observers.

5.2 Basic Concepts

5.2.1 Critical Shear Angle of Lintel



Lintel এর উপরে brick wall-এ shearing এর কারনে crack হয়। Lintel এর দুই end point থেকে উপরের দিকে 45° angle-এ shearing consider করা হয়। তবে maximum critical angle 60° consider করা হয়। এখানে ২টি বিষয় লক্ষ্য রাখতে হবেঃ

(১) যদি lintel এর দুই end point থেকে shearing angle arm উপরের slab কে intersect (contact) করে ($s \neq 0$), তাহলে মধ্যবর্তী slab portion থেকে UDL আকারে lintel এর উপর load আসবে।
(২) যদি shearing angle arm উপরের slab কে intersect (contact) না করে ($s = 0$) তাহলে slab portion থেকে UDL আকারে lintel এর উপর load আসবে না।

5.2.2 Design Consideration of Lintel and Sunshade

* আরেকটি বিষয় জেনে রাখা ভালো, Beam design এর মতো Lintel design করা হয় এবং Slab design এর মতো Sunshade design করা হয়।

* Lintel design করার সময়ে তাদের যে span দেওয়া থাকে, তার থেকে দুই পাশে support এর জন্য 6″ করে বাড়িয়ে ধরা হয়। অর্থাৎ effective span, given span থেকে always 1′ বেশি হবে।

5.3 Solution Techniques

5.3.1 Design of Lintel and Sunshade

WSD

Lintel Design

1. Lintel dimension

Span of lintel, $l =$

Width of lintel, $b = wall\ thickness =$

$$tan60° = \frac{y}{x} = \frac{wall\ height}{shearing\ arm\ in\ lintel}$$

$$\Rightarrow x = \frac{y}{tan60°}$$

$\therefore$ Cut length of slab, $s = l - 2x$

If $s \leq 0$; No UDL comes from slab on lintel
If $s > 0$; UDL comes from slab on lintel

2. Load calculation

Wall area over lintel, A_W
Load area over lintel, A_L
$\therefore$ Load on lintel from wall $= A_W \times A_L$
Slab area, $A_S =$
DL from slab $= A_S \times$ DL
LL from slab $= A_S \times$ LL
Total load from slab = LS
Here, wall length = WL

Load on wall from slab, $= \frac{LS}{WL} = w'$

$\therefore$ Load on lintel from slab $= w' \times s$
Total load on lintel, $W =$ from wall + slab

$\therefore$ UDL on lintel, $w = \frac{W}{l}$

$\therefore$ Design Moment, $M = \frac{wl^2}{8}$

3. Design constants: $n,\ r,\ k,\ j$

4. Depth check

$$M = \frac{1}{2} f_c k j b d^2$$

5. Reinforcement calculation

$$A_s = \frac{M}{f_s j d}$$

USD

Lintel Design

1. Lintel dimension

Span of lintel, $l =$

Width of lintel, $b = wall\ thickness =$

$$tan60° = \frac{y}{x} = \frac{wall\ height}{shearing\ arm\ in\ lintel}$$

$$\Rightarrow x = \frac{y}{tan60°}$$

$\therefore$ Cut length of slab, $s = l - 2x$

If $s \leq 0$; No UDL comes from slab on lintel
If $s > 0$; UDL comes from slab on lintel

2. Load calculation

Wall area over lintel, A_W
Load area over lintel, A_L
$\therefore$ Load on lintel from wall $= A_W \times A_L$
Slab area, $A_S =$
DL from slab $= A_S \times$ DL
LL from slab $= A_S \times$ LL
Total load from slab = LS
Here, wall length = WL

Dead Load on wall from slab, $= \frac{TDL}{WL} = w_1$

Dead Load on wall from slab, $= \frac{TLL}{WL} = w_2$

$\therefore$ Dead Load on lintel from slab $= w_1 \times s$
$\therefore$ Live Load on lintel from slab $= w_2 \times s$
Total ultimate load on lintel,
$W = 1.2DL + 1.6LL$

$\therefore$ UDL on lintel, $w = \frac{W}{l}$

$\therefore$ Design Moment, $M = \frac{wl^2}{8}$

3. Depth check

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t}$$

Now, $M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$

6. Shear Check

Developed shear, $V_{dev} = \frac{wl}{2} - wd$

Allowable shear, $V_{all} = 1.1\sqrt{f_c'}\, bd$

Sunshade Design

7. Sunshade design

i) Load calculation

ii) Moment calculation, $M = \frac{wl^2}{2}$

iii) Reinforcement calculation

Main bar:

$A_s = \frac{M}{f_s jd}$

Here, $A_s(min) = 0.0018\, bt$

Distribution bar: $A_s(min)$

Maximum spacing $= 5t$

8. Reinforcement deatiling

4. Reinforcement calculation

$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$

$a = \frac{A_s f_y}{0.85 f_c' b}$

5. Shear Check

Ultimate shear, $V_u = \frac{wl}{2} - wd$

Allowable shear, $V_{all} = 2\varphi\sqrt{f_c'}\, bd$

Sunshade Design

6. Sunshade design

i) Load calculation

ii) Moment calculation, $M = \frac{wl^2}{2}$

iii) Reinforcement calculation

Main bar:

$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$

$a = \frac{A_s f_y}{0.85 f_c' b}$

Here, $A_s(min) = 0.0018\, bt$

Distribution bar: $A_s(min)$

Maximum spacing $= 5t$

7. Reinforcement deatiling

5.4 Problem Solution

5.4.1 Design of Lintel and Sunshade – WSD

Pb-1. Design a lintel over a window opening of 6 ft. wide. The opening is left in the wall 2 ft. from one of its corners. The height of wall above opening is 6 ft. The thickness of wall the wall is 10 inch. Grade 50 reinforcing steel is to be used with 3000 psi concrete. [*CE-2015*]

Sol^n:

Span of lintel, $l = 6'' + 6' + 6'' = 7'$

Width of lintel, $b = wall\ thickness = 10''$

$$tan60° = \frac{y}{x} = \frac{wall\ height}{shearing\ arm\ in\ lintel}$$

$$\Rightarrow x = \frac{y}{tan60°} = \frac{6'}{1.732} = 3.46' \approx 3.5'$$

$\therefore$ Cut length of slab, $s = l - 2x = 7 - 2 \times 3.5' = 0$

So, no UDL comes from slab portion on the lintel.

Wall load on lintel = wall area × load area $= \left(\frac{1}{2} \times 7 \times 6\right) \times \left(\frac{10}{12} \times 120\right) = 2100\ lb$

$\therefore$ Total UDL on lintel $= \frac{2100}{7} = 300\ lb/ft$

$\therefore$ Design Moment, $M = \frac{wl^2}{8} = \frac{300 \times 7^2}{8} = 1837.5\ lb - ft = 1.8375\ k - ft$

Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \approx 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 50}{0.45 \times 3} = \frac{20}{1.35} = 14.81$$

$$k = \frac{n}{n+r} = 0.378$$

$$j = 1 - \frac{k}{3} = 0.874$$

Depth check:

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Rightarrow 1.8375 \times 12 = \frac{1}{2} \times 1.35 \times 0.378 \times 0.874 \times 10 \times d^2$$

$\therefore d = 3.14\ in. \approx 3.5\ in.$

Total thickness, $t = d + ec = 3.5 + 1.5 = 5\ in.$

Reinforcement:

$$\therefore A_s = \frac{M}{f_s jd} = \frac{1.8375 \times 12}{20 \times 0.874 \times 3.5} = 0.360\ in^2$$

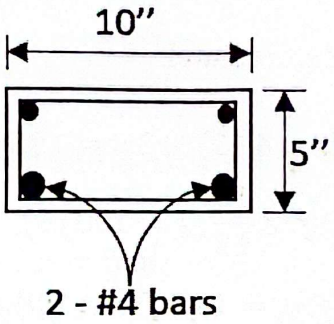
Provide 2 - #4 bars

Shear check:

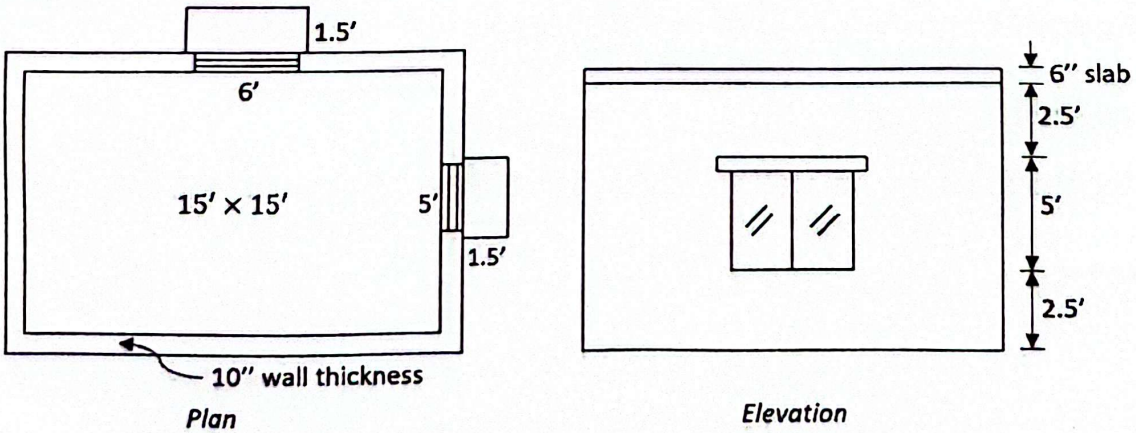
$$\text{Developed shear, } V_{dev} = \frac{wl}{2} - wd = \frac{300 \times 7}{2} - 300 \times \frac{3.5}{12} = 962.5\ lb$$

$$\text{Allowable shear, } V_{all} = 1.1\sqrt{f_c'}\ bd = 1.1 \times \sqrt{3000} \times 10 \times 3.5 = 2108.7\ lb > V_{dev}\ (ok)$$

Reinforcement Detailing:

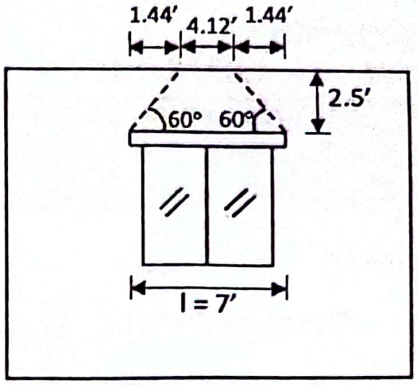


Pb-2. Design and show the reinforcement detailing of the lintel and sunshade shown in figure below. The thickness of load bearing wall is 10 in. and the live load on slab and sunshade is 40 psf. The height of wall above window opening is 2.5 ft. The projection length of sunshade is 1.5 ft. From the design data, material strengths are $f_c' = 3000$ psi and $f_y = 60{,}000$ psi.



Sol^{n}:

Lintel and sunshade should be designed for larger dimension.



Lintel Design:

Span of lintel, $l = 6'' + 6' + 6'' = 7'$

Width of lintel, $b = wall\ thickness = 10''$

$$tan60° = \frac{y}{x}$$

$$\Rightarrow x = \frac{y}{tan60°} = \frac{2.5'}{1.732} = 1.44'$$

∴ Cut length of slab, $s = l - 2x = 7' - 2 \times 1.44' = 4.12'$

So, UDL comes from slab portion on the lintel.

Wall area over lintel, $A_W = \frac{1}{2} \times (7 + 4.12) \times 2.5 = 13.9\ ft^2$

Load area over lintel, $A_L = 120\ pcf \times \frac{10}{12} ft = 100\ psf$

∴ Load on lintel from wall $= A_W \times A_L = 13.9\ ft^2 \times 100\ psf = 1390\ lb$

Slab area $= \frac{15 \times 15}{4} = 56.25\ ft^2$

Weight from slab $= \left(56.25 \times \frac{6}{12}\right) \times 150 = 4218.75\ lb$

Live load from slab $= 56.25 \times 40 = 2250\ lb$

∴ Total load from slab $= 4218.75 + 2250 = 6468.75\ lb$

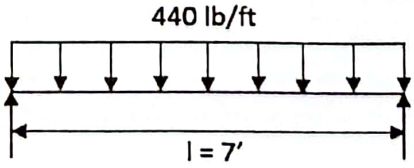
Here, wall length $= 15' + 5'' + 5'' = 15.83\ ft$

∴ Load on wall from slab $= \frac{6468.75}{15.83} = 408.64\ lb/ft$

∴ Load on lintel from slab $= 408.64 \times 4.12 = 1684\ lb$

∵ Total load on lintel = from wall + from slab = 1390 + 1684 = 3074 lb

∴ UDL on lintel $= \frac{3074}{7} = 440\ lb/ft$



$$\therefore \text{Design Moment, } M = \frac{wl^2}{8} = \frac{440 \times 7^2}{8} = 2695\ lb - ft = 2.695\ k - ft$$

Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \approx 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4 * 60}{0.45 \times 3} = \frac{24}{1.35} = 17.78$$

$$k = \frac{n}{n+r} = 0.336$$

$$j = 1 - \frac{k}{3} = 0.888$$

Depth check:

$$M = \frac{1}{2} f_c k j b d^2$$

$$\Longrightarrow 2.695 \times 12 = \frac{1}{2} \times 1.35 \times 0.336 \times 0.888 \times 10 \times d^2$$

$$\therefore d = 4.007\ in. \approx 4.5\ in.$$

Total thickness, $t = d + ec = 4.5 + 1.5 = 6\ in.$

Reinforcement:

$$\therefore A_s = \frac{M}{f_s j d} = \frac{2.695 \times 12}{24 \times 0.888 \times 4.5} = 0.337\ in^2$$

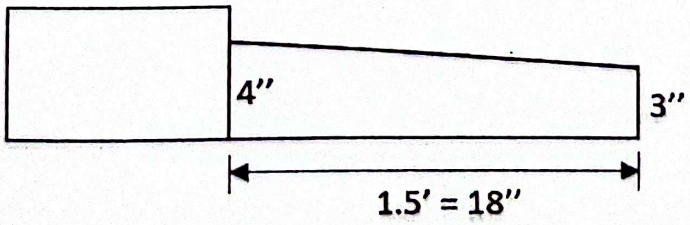
Provide 2 - #4 bars

Shear check:

$$\text{Developed shear, } V_{dev} = \frac{wl}{2} - wd = \frac{440 \times 7}{2} - 440 \times \frac{4.5}{12} = 1375\ lb$$

$$\text{Allowable shear, } V_{all} = 1.1\sqrt{f_c'}\, bd = 1.1 \times \sqrt{3000} \times 10 \times 4.5 = 2711\ lb > V_{dev}\ (ok)$$

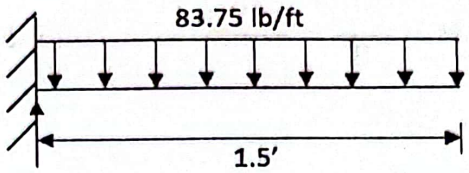
Sunshade Design:



Self weight of sunshade $= \frac{3.5}{12} \times 1' \times 150 = 43.75\ lb/ft$

Live load on sunshade $= 1' \times 40 = 40\ lb/ft$

Total load $= 83.75\ lb/ft$



$$\therefore M_{dev} = \frac{wl^2}{2} = \frac{83.75 \times 1.5^2}{2} = 94.22\ lb - ft = 0.09422\ k - ft$$

Reinforcement:

$$\therefore A_s = \frac{M}{f_s jd} = \frac{0.09422 \times 12}{24 \times 0.888 \times (4-1)} = 0.01768\ in^2/ft$$

Here, $A_s(min) = 0.0018\ bt = 0.0018 \times 12 \times 4 = 0.0864\ in^2/ft$

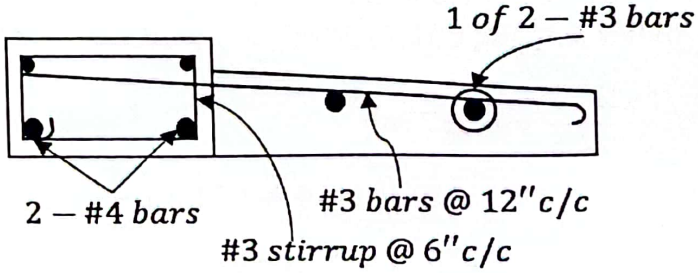
Provide #3 bar @ ~~15~~ 12 in. c/c

Distribution (shrinkage) bar:

Maximum spacing $= 5t = 5 \times 4 = 20\ in.$

Provide 1 of 2 nos #3 bar.

Reinforcement Detailing:



এখানে মনে রাখা আবশ্যকঃ

Spacing of stirrups of lintel: Least of –

$(i)\ 16\ d_{LB} = 16 \times \frac{4}{8} = 8''$

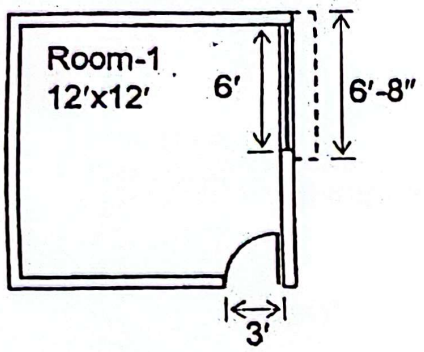
$(ii)\ 48\ d_S = 48 \times \frac{3}{8} = 18''$

$(iii)\ Least\ lintel\ dimension = 6''$

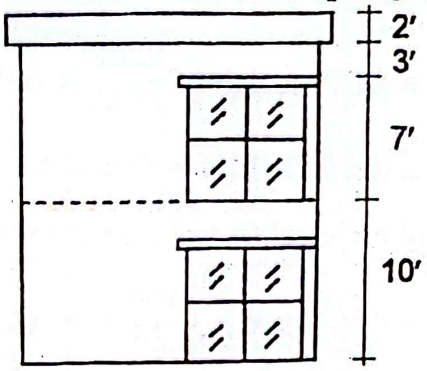
$So,\ maximum\ spacing = 6''$

Sunshade এর main bar এর spacing always lintel stirrup এর spacing এর গুনিতক আকারে রাখতে হবে, যেনো stirrup এর সাথে joining করা যায়।

Pb-3. Calculate the design load and draw loading diagram for the lintel and sunshade shown in figure below. Slab thickness is 6 in. and live load on slab is 40 psf. [*CE-2019*]



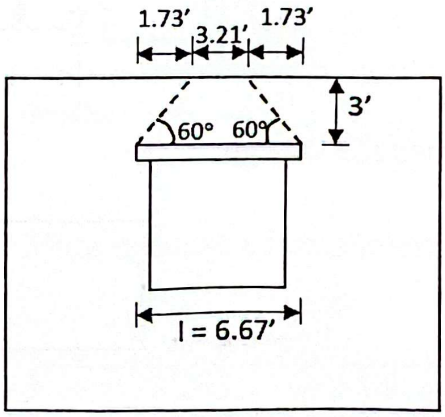
Plan



Right side Elevation

Sol^{n}:

Lintel and sunshade should be designed for 1st floor because of maximum load carrying capacity from all above structure.



Lintel Design:

Span of lintel, $l = 6' - 8'' = 6.67'$

Width of lintel, $b = wall\ thickness = 10''$

$$tan60° = \frac{y}{x}$$

$$\Rightarrow x = \frac{y}{tan60°} = \frac{3'}{1.732} = 1.73'$$

$\therefore$ Cut length of slab, $s = l - 2x = 6.67' - 2 \times 1.73' = 3.21'$

So, UDL comes from slab portion on the lintel.

Wall area over lintel of 1st floor, $A_W = \frac{1}{2} \times (6.67 + 3.21) \times 3 = 14.82\ ft^2$

Wall area over lintel of 2nd floor, $A_W = (10 + 2) \times 3.21 = 38.52\ ft^2$

Load area over lintel, $A_L = 120\ pcf \times \frac{10}{12} ft = 100\ psf$

$\therefore$ Load on lintel from wall $= A_W \times A_L = (14.82 + 38.52) \times 100 = 5334\ lb$

Slab area $= \frac{12 \times 12}{4} = 36\ ft^2$

Weight from slab $= 2 \times \left(36 \times \frac{6}{12}\right) \times 150 = 5400$ lb

Live load from slab $= 2 \times 36 \times 40 = 2880\ lb$

$\therefore$ Total load from slab $= 5400 + 2880 = 8280\ lb$

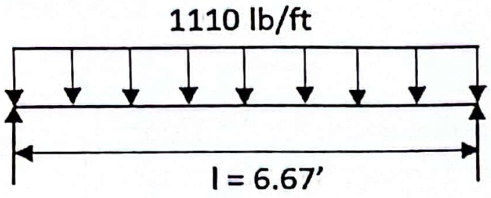
Here, wall length $= 12' + 5'' + 5'' = 12.83\ ft$

$\therefore$ Load on wall from slab $= \dfrac{8280}{12.83} = 645\ lb/ft$

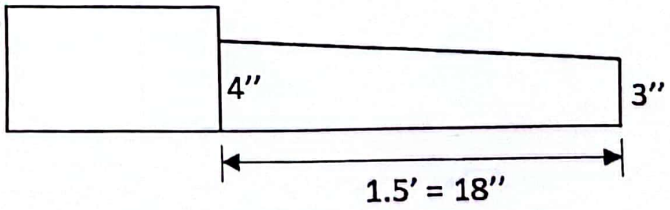
$\therefore$ Load on lintel from slab $= 645 \times 3.21 = 2070\ lb$

$\because$ Total load on lintel from 1st floor $= 5334 + 2070 = 7404$ lb

$\therefore$ Design UDL on lintel $= \dfrac{7404}{6.67} = 1110\ lb/ft$



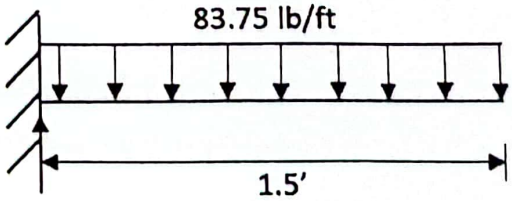
Sunshade Design:



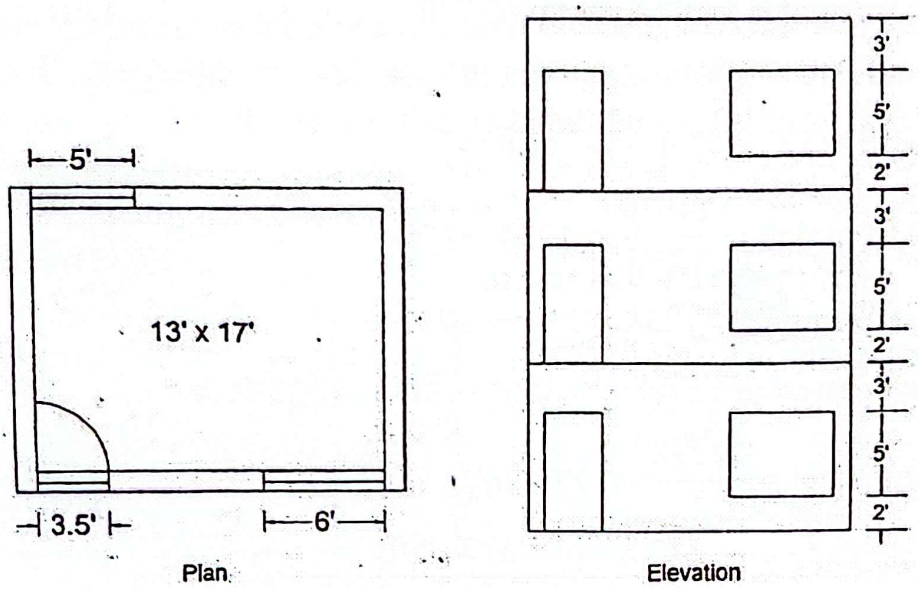
Self weight of sunshade $= \dfrac{3.5}{12} \times 1' \times 150 = 43.75\ lb/ft$

Live load on sunshade $= 1' \times 40 \quad = 40\ lb/ft$

Total load $= 83.75\ lb/ft$

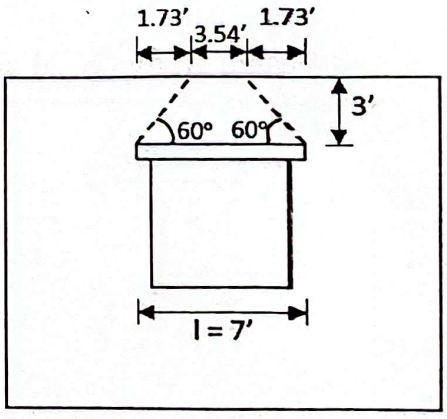


Pb-4. Compute the design moment for lintel and in the 3-storied building shown in figure below. Slab thickness is 6 in. and live load on slab is 40 psf. [*CE-2018*]



Sol^{n}:

Lintel should be designed for larger dimension and for 1st floor because of maximum load carrying capacity from all above structure.



Lintel Design:

Span of lintel, $l = 6'' + 6' + 6'' = 7'$

Width of lintel, $b = wall\ thickness = 10''$

$$tan60° = \frac{y}{x}$$

$$\Rightarrow x = \frac{y}{tan60°} = \frac{3'}{1.732} = 1.73'$$

$\therefore$ Cut length of slab, $s = l - 2x = 7' - 2 \times 1.73' = 3.54'$

So, UDL comes from slab portion on the lintel.

Wall area over lintel of 1st floor, $A_W = \frac{1}{2} \times (7 + 3.54) \times 3 = 15.81\ ft^2$

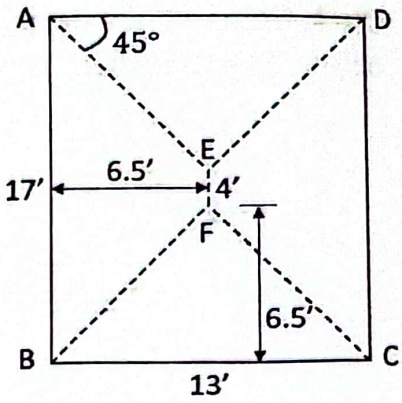
Wall area over lintel of 2nd, 3rd floor, $A_W = (10 + 10) \times 3.54 = 70.8\ ft^2$

Load area over lintel, $A_L = 120\ pcf \times \frac{10}{12} ft = 100\ psf$

$\therefore$ Load on lintel from wall $= A_W \times A_L = (15.81 + 70.8) \times 100 = 8661\ lb$

Slab area $= \frac{1}{2} \times 13 \times 6.5 = 42.25\ ft^2$

Weight from slab $= 3 \times \left(42.25 \times \frac{6}{12}\right) \times 150 = 9506$ lb



Live load from slab $= 3 \times 42.25 \times 40 = 5070\ lb$

$\therefore$ Total load from slab $= 9506 + 5070 = 14576\ lb$

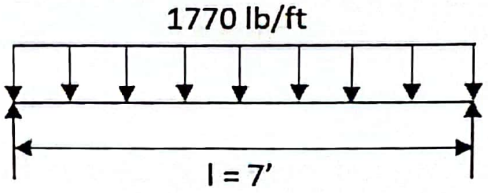
Here, wall length $= 13' + 5'' + 5'' = 13.83\ ft$

$\therefore$ Load on wall from slab $= \frac{14570}{13.83} = 1054\ lb/ft$

$\therefore$ Load on lintel from slab $= 1054 \times 3.54 = 3731\ lb$

$\because$ Total load on lintel from 1st floor $= 8661 + 3731 = 12392$ lb

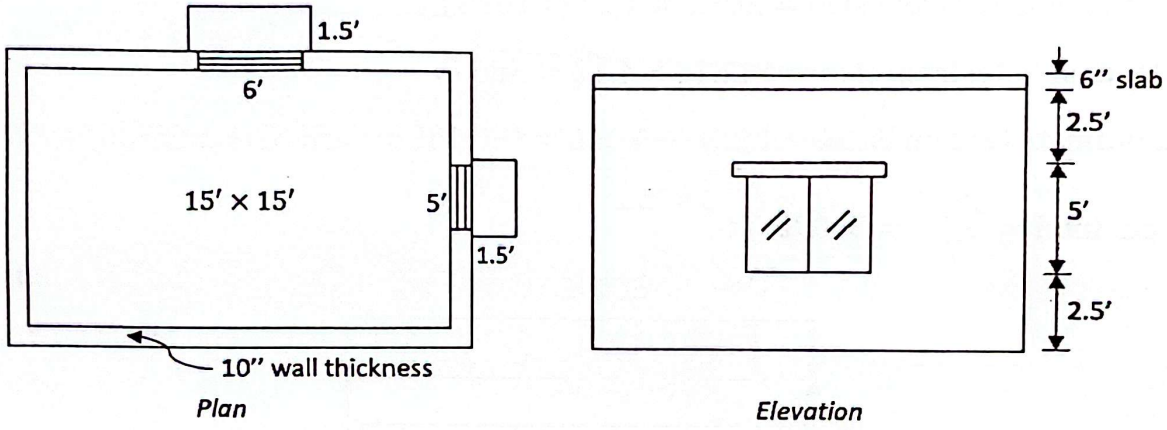
$\therefore$ Design UDL on lintel $= \frac{12392}{7} = 1770\ lb/ft$



$\therefore$ Design Moment, $M = \frac{wl^2}{8} = \frac{1770 \times 7^2}{8} = 10841\ lb - ft = 10.841\ k - ft$

5.4.2 Design of Lintel and Sunshade – USD

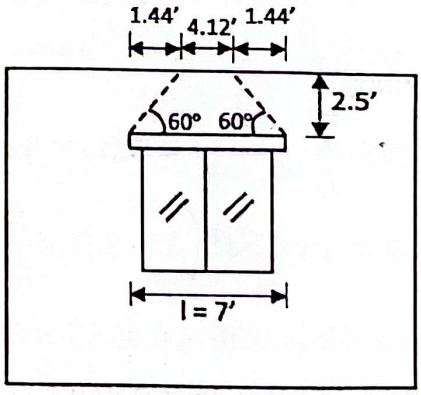
Pb-5. Design and show the reinforcement detailing of the lintel and sunshade shown in figure below. The thickness of load bearing wall is 10 in. and the live load on slab and sunshade is 40 psf. The height of wall above window opening is 2.5 ft. The projection length of sunshade is 1.5 ft. From the design data, material strengths are $f_c' = 3000$ psi and $f_y = 60{,}000$ psi.



Sol^n:

Lintel and sunshade should be designed for larger dimension.

Lintel Design:



Span of lintel, $l = 6'' + 6' + 6'' = 7'$

Width of lintel, $b = wall\ thickness = 10''$

$$tan60° = \frac{y}{x}$$

$$\Rightarrow x = \frac{y}{tan60°} = \frac{2.5'}{1.732} = 1.44'$$

$\therefore$ Cut length of slab, $s = l - 2x = 7' - 2 \times 1.44' = 4.12'$

So, UDL comes from slab portion on the lintel.

Wall area over lintel, $A_W = \frac{1}{2} \times (7 + 4.12) \times 2.5 = 13.9\ ft^2$

Load area over lintel, $A_L = 120\ pcf \times \frac{10}{12} ft = 100\ psf$

$\therefore$ Load on lintel from wall $= A_W \times A_L = 13.9\ ft^2 \times 100\ psf = 1390\ lb$

Slab area $= \frac{15 \times 15}{4} = 56.25\ ft^2$

Weight from slab $= \left(56.25 \times \frac{6}{12}\right) \times 150 = 4218.75$ lb

Live load from slab $= 56.25 \times 40 = 2250\ lb$

Here, wall length $= 15' + 5'' + 5'' = 15.83\ ft$

Dead load on wall from slab $= \dfrac{4218.75}{15.83} = 266.5\ lb/ft$

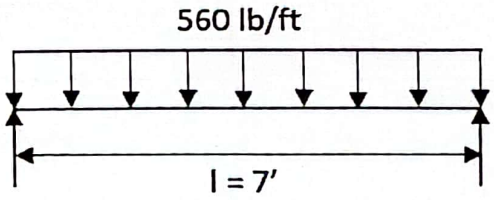
Live load on wall from slab $= \dfrac{2250}{15.83} = 142.14\ lb/ft$

$\therefore$ Dead load on lintel from slab $= 266.5 \times 4.12 = 1098\ lb$

$\therefore$ Live load on lintel from slab $= 142.14 \times 4.12 = 586\ lb$

$\because$ Total ultimate load on lintel $= 1.2DL + 1.6LL = 1.2(1390 + 1098) + 1.6(586) = 3923\ lb$

$\therefore$ UDL on lintel $= \dfrac{3923}{7} = 560\ lb/ft$



$\therefore$ Design Moment, $M = \dfrac{wl^2}{8} = \dfrac{560 \times 7^2}{8} = 3430\ lb - ft = 3.43\ k - ft$

Depth check:

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{3}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0135$$

Now, $M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \dfrac{f_y}{f_c'}\right)$

$$\Rightarrow 3.43 \times 12 = 0.90 \times 0.0135 \times 60 \times 10 \times d^2 \times \left(1 - 0.59 \times 0.0135 \times \frac{60}{3}\right)$$

$$\Rightarrow 41.16 = 7.29 \times d^2 \times 0.8407$$

$\therefore d = 2.59\ in. \approx 3\ in$

Total thickness, $t = d + ec = 3 + 1.5 = 4.5\ in.$

Reinforcement:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{3.43 \times 12}{0.90 \times 60 \times \left(3 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 10}$$

$$\therefore A_s = 0.286\ in^2$$

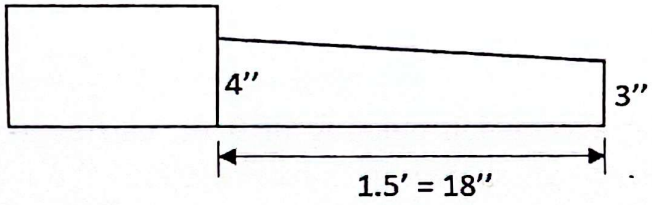
Provide 2 - #4 bars

Shear check:

$$\text{Ultimate shear, } V_u = \frac{wl}{2} - wd = \frac{560 \times 7}{2} - 560 \times \frac{3}{12} = 1820\ lb$$

$$\text{Allowable shear, } V_{all} = 2\varphi\sqrt{f_c'}\, bd = 2 \times 0.75 \times \sqrt{3000} \times 10 \times 3 = 2465\ lb > V_u\ (ok)$$

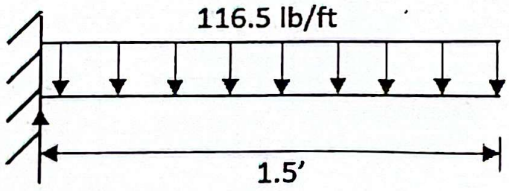
Sunshade Design:



$$\text{Self weight of sunshade} = 1.2 \times \frac{3.5}{12} \times 1' \times 150 = 52.5\ lb/ft$$

$$\text{Live load on sunshade} = 1.6 \times 1' \times 40 = 64\ lb/ft$$

$$\text{Total load} = 116.5\ lb/ft$$



$$\therefore M_u = \frac{wl^2}{2} = \frac{116.5 \times 1.5^2}{2} = 131\ lb - ft = 0.131\ k - ft$$

Reinforcement:

$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{0.131 \times 12}{0.90 \times 60 \times \left(3 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 12}$$

$$\therefore A_s = 0.0097\ in^2/ft$$

Here, $A_s(min) = 0.0018\, bt = 0.0018 \times 12 \times 4 = 0.0864\ in^2/ft$

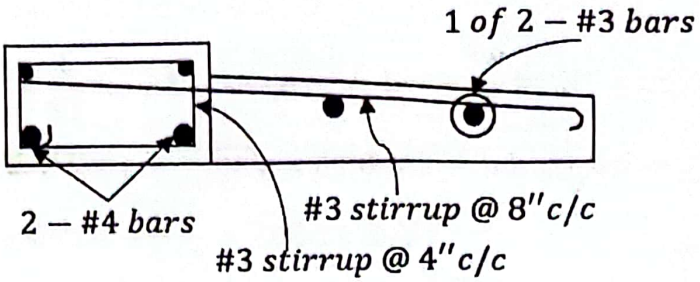
Provide #3 bar @ ~~15~~ 8 in. c/c

Distribution (shrinkage) bar:

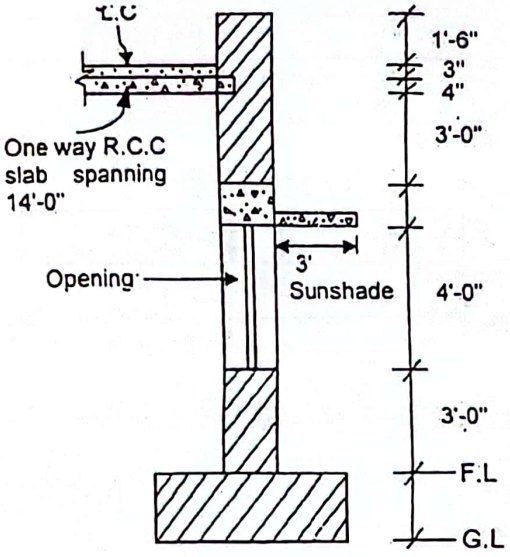
Maximum spacing $= 5t = 5 \times 4 = 20\ in.$

Provide 1 of 2 nos #3 bar.

Reinforcement Detailing:

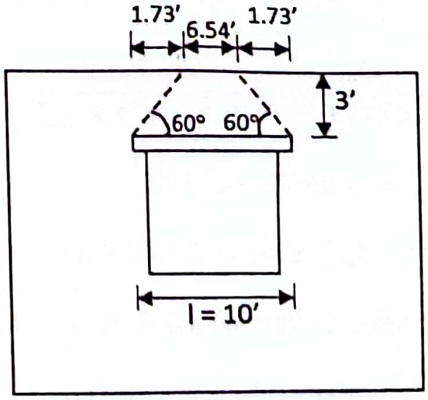


Pb-6. A lintel, covering a verandah opening of 9 ft.; carries a sunshade of 3 ft. wide, monolithic with the lintel as shown in figure below. The verandah roof carries a superimposed load of 30 psf. Design the lintel and sunshade by using material strengths are $f_c' = 3000$ psi, $f_y = 60{,}000$ psi and USD method. [*CE-2014*]



Sol^n:

Lintel Design:



Span of lintel, $l = 6'' + 9' + 6'' = 10'$

Width of lintel, $b = wall\ thickness = 10''$

$$tan60° = \frac{y}{x}$$

$$\Rightarrow x = \frac{y}{tan60°} = \frac{3'}{1.732} = 1.73'$$

$\therefore$ Cut length of slab, $s = l - 2x = 10' - 2 \times 73 = 6.54'$

So, UDL comes from slab portion on the lintel.

Wall area over lintel, $A_W = \frac{1}{2} \times (10 + 6.54) \times 3 = 24.81 ft^2$

Load area over lintel, $A_L = 120\ pcf \times \frac{10}{12} ft = 100\ psf$

$\therefore$ Load on lintel from wall $= A_W \times A_L = 24.81\ ft^2 \times 100\ psf = 2481\ lb$

Slab area $= \frac{14 \times 14}{4} = 49\ ft^2$

Weight from slab $= \left(49 \times \frac{4+3}{12}\right) \times 150 = 4287.5$ lb

Live load from slab $= 49 \times 30 = 1470\ lb$

Here, wall length $= 14' + 5'' + 5'' = 14.83\ ft$

Dead load on wall from slab $= \frac{4287.5}{14.83} = 289\ lb/ft$

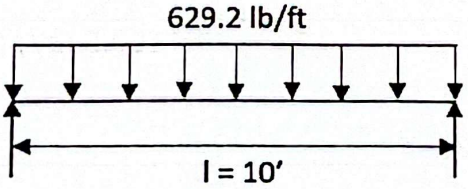
Live load on wall from slab $= \frac{1470}{14.83} = 100\ lb/ft$

$\therefore$ Dead load on lintel from slab $= 289 \times 6.54 = 1890\ lb$

$\therefore$ Live load on lintel from slab $= 100 \times 6.54 = 654\ lb$

$\because$ Total ultimate load on lintel $= 1.2DL + 1.6LL = 1.2(2481 + 1890) + 1.6(654) = 6292\ lb$

$\therefore$ UDL on lintel $= \frac{6292}{10} = 629.2\ lb/ft$



$\therefore$ Design Moment, $M = \frac{wl^2}{8} = \frac{629.2 \times 10^2}{8} = 7865\ lb - ft = 7.865\ k - ft$

Depth check:

$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{3}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0135$

Now, $M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$

$\Rightarrow 7.865 \times 12 = 0.90 \times 0.0135 \times 60 \times 10 \times d^2 \times \left(1 - 0.59 \times 0.0135 \times \frac{60}{3}\right)$

$\Rightarrow 94.38 = 7.29 \times d^2 \times 0.8407$

$\therefore d = 3.92\ in. \approx 4.5\ in$

Total thickness, t = d + ec = 4.5 + 1.5 = 6 *in.*

Reinforcement:

$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{7.865 \times 12}{0.90 \times 60 \times \left(4.5 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 10}$$

$$\therefore A_s = 0.439\ in^2$$

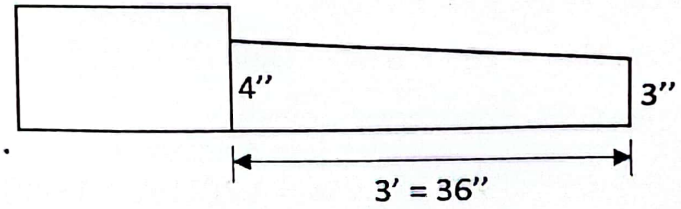
Provide 2 - #5 bars

Shear check:

Ultimate shear, $V_u = \frac{wl}{2} - wd = \frac{629.2 \times 10}{2} - 629.2 \times \frac{4.5}{12} = 2910\ lb$

Allowable shear, $V_{all} = 2\varphi\sqrt{f_c'}\, bd = 2 \times 0.75 \times \sqrt{3000} \times 10 \times 4.5 = 3697\ lb > V_u\ (ok)$

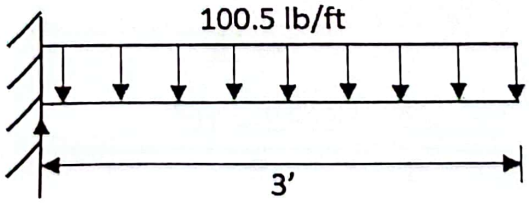
Sunshade Design:



Self weight of sunshade $= 1.2 \times \frac{3.5}{12} \times 1' \times 150 = 52.5\ lb/ft$

Live load on sunshade $= 1.6 \times 1' \times 30 = 48\ lb/ft$

Total load $= 100.5\ lb/ft$



$$\therefore M_u = \frac{wl^2}{2} = \frac{100.5 \times 3^2}{2} = 452\ lb - ft = 0.452\ k - ft$$

Reinforcement:

$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{0.452 \times 12}{0.90 \times 60 \times \left(3 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 12}$$

$$\therefore A_s = 0.0339\ in^2/ft$$

Here, $A_s(min) = 0.0018\,bt = 0.0018 \times 12 \times 4 = 0.0864\ in^2/ft$

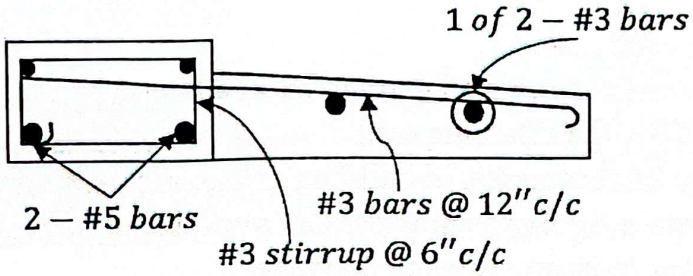
Provide #3 bar @ ~~15~~ 12 in. c/c

Distribution (shrinkage) bar:

Maximum spacing $= 5t = 5 \times 4 = 20\ in.$

Provide 1 of 2 nos #3 bar.

Reinforcement Detailing:



Chapter - 6
Retaining Walls

6.1 Theory Analysis

Q-1. Define retaining wall. What are the functions of retaining wall?
Ans: Retaining wall: The wall which is generally used to retain earth or such materials to maintain unequal levels on its two faces is called retaining wall.

Functions of retaining wall:
(i) Retaining wall prevents the retained earth to exert a lateral pressure on the wall tending to bend, overturn and slide the retaining wall.
(ii) It is used in bridges as abutments, in building as basements, and in embankments.
(iii) It is also used to retain liquids as in water tanks and sewage-treatment tanks.
(iv) Retaining wall helps in improving site drainage.

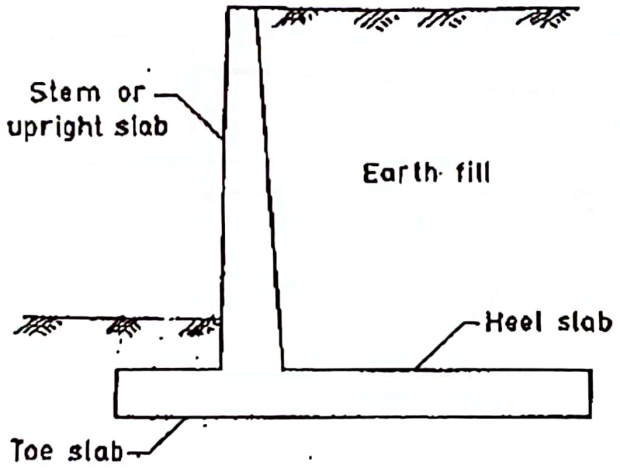


Figure 6.1: Different parts of a retaining wall

Q-2. What are the types of retaining walls? Describe them briefly.
Ans: There are two principle types of retaining walls as follows:
(1) Cantilever retaining wall
(2) Counterfort retaining wall

(1) Cantilever retaining wall: The retaining wall in which the vertical stem resisting earth pressure from one side and the slab bends like a cantilever is said to be a cantilever retaining wall. This type of retaining walls are adopted for small to medium heights from 8 to 20 ft.

(2) Counterfort retaining wall: The retaining wall in which a relatively large bending moment is developed for larger heights and providing the counterforts act as tension ties

supporting the vertical walls is said to be a counterfort retaining wall. This type of retaining walls are adopted for larger heights of higher than 20 ft.

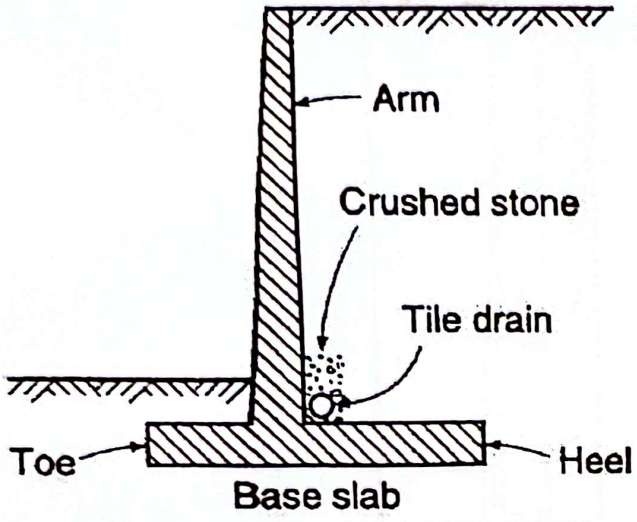


Figure 7.2: Cantilever retaining wall

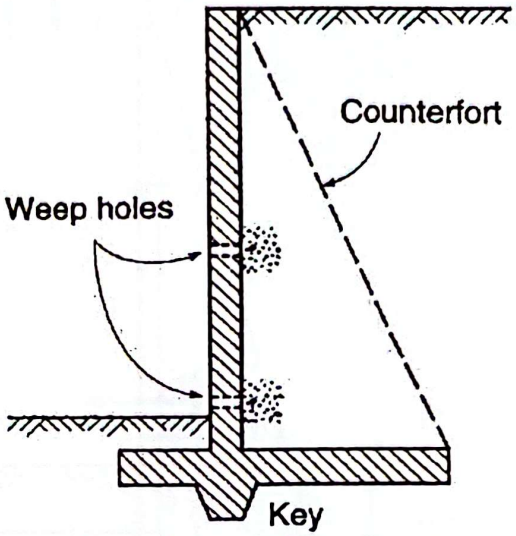


Figure 7.3: Counterfort retaining wall

6.2 Problem Solution

Pb-1. Design a cantilever retaining wall to support a bank of earth 16.5 ft. high. The top of the earth is to be level with a surcharge of 330 psf. The weight of the backfill is 110 pcf, the angle of internal friction is $\varphi = 35°$, the coefficient of friction between concrete and soil is $\mu = 0.5$, the coefficient of friction between soil layer is $\mu = 0.7$, allowable soil bearing capacity is 4 ksf, $f_c' = 3$ ksi and $f_y = 60$ ksi. [*CE-2008*]

Sol^n:

1. Dimensions of retaining wall

i) Height of wall, h = Height above soil + frost penetration = 16.5 + 3 = 19.5 ft.

ii) Base thickness, $t_b = 0.08\,h = 0.08 \times 19.5 = 1.56 \approx 1.5$ ft.

iii) Height of stem, $h_{st} = 19.5 - 1.5 = 18$ ft.

iv) Base length, $l_b = 0.53\,h = 0.53 \times 19.5 = 10.3 \approx 10.5$ ft.

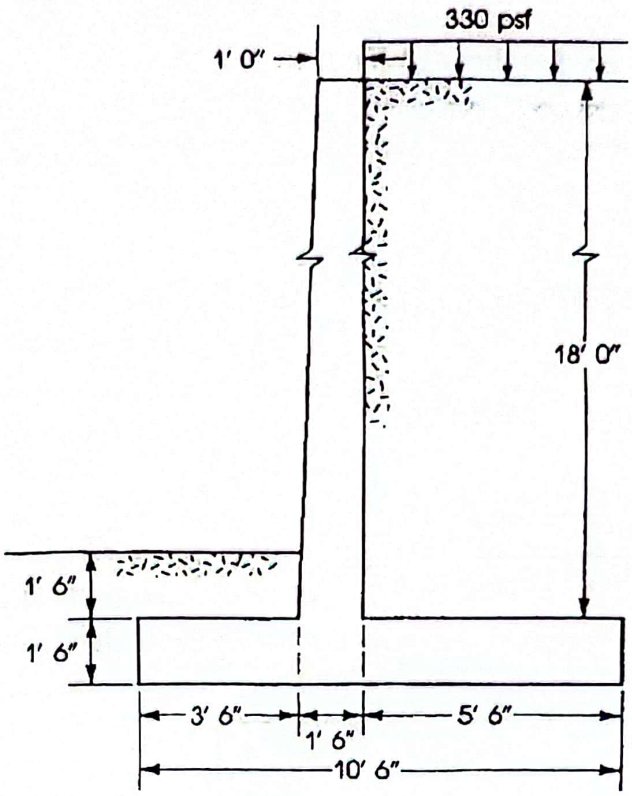
v) Projection of base in front of stem $= 0.17\,h = 0.17 \times 19.5 = 3.3 \approx 3.5$ ft.

vi) Stem thickness

Maximum stem thickness at bottom, $t_{sb} = 0.08\,h = 0.08 \times 19.5 = 1.56 \approx 1.5$ ft.

Minimum stem thickness at top, $t_{st} = 0.05\,h = 0.05 \times 19.5 = 0.975 \approx 1.0$ ft.

Minimum batter of the face of the wall $= 1.5 - 1.0 = 0.5$ ft. $= 6$ in.



2. Coefficient of active earth pressure, C_a

$$C_a = \frac{1 - sin\varphi}{1 + sin\varphi} = \frac{1 - 0.574}{1 + 0.574} = 0.271$$

3. Stability against overturning

i) Actual unfactored forces

Height of surcharge, $h_s = \frac{w_s}{w} = \frac{330}{110} = 3$ ft.

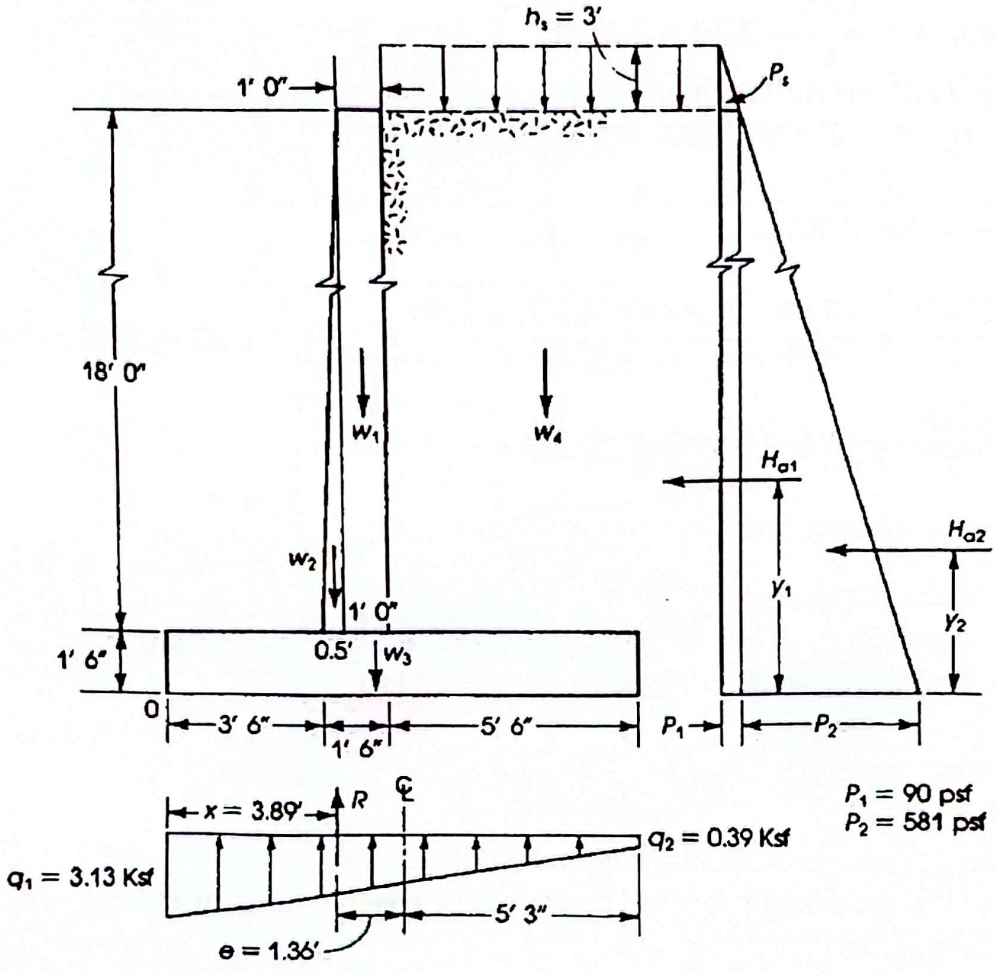
$\therefore P_1 = C_a w h_s = 0.271 \times 110 \times 3 = 90$ psf

$\therefore P_2 = C_a w h = 0.271 \times 110 \times 19.5 = 581$ psf

$H_{a1} = 90 \times 19.5 = 1755$ lb $= 1.755$ kip; arm $= \frac{19.5}{2} = 9.75$ ft.

$H_{a2} = \frac{1}{2} \times 581 \times 19.5 = 5665$ lb $= 5.665$ kip; arm $= \frac{19.5}{3} = 6.5$ ft.

ii) Overturning moment, $M_o = (1.755 \times 9.75) + (5.665 \times 6.5) = 53.93$ k – ft.



iii) Balancing moment (M_b) about the toe

Force (lb)	Arm (ft.)	Moment (k-ft.)
$w_1 = 1 \times 18 \times 150 = 2700$	4.50	12.15
$w_2 = \frac{1}{2} \times 18 \times 0.5 \times 150 = 675$	3.83	2.59
$w_3 = 10.5 \times 1.5 \times 150 = 2363$	5.25	12.41
$w_4 = 5.5 \times 21 \times 110 = 12705$	7.75	98.46
$\sum w = R = 18.44$ kip		$\sum M = M_b = 125.61$ k – ft.

$\therefore$ Factor of safety against overturning $= \frac{M_b}{M_o} = \frac{125.61}{53.93} = 2.33 > 2.0 \ (ok)$

4. Base soil pressure calculation

Location of the resultant R of the vertical forces, $x = \frac{M_b - M_o}{R} = \frac{125.61 - 53.93}{18.44} = 3.89$ ft.

Here, $x = 3.89$ ft. $> \frac{10.5}{3} = 3.5$ ft.; $\therefore x = 3.5$ ft.

$\therefore$ Eccentricity, $e = \frac{10.5}{2} - 3.89 = 1.36$ ft.

For 1 ft. strip length of the footing,

Area of footing, $A = B \times 1 = 10.5 \times 1 = 10.5\ \text{ft}^2$

$$I = \frac{1 \times 10.5^3}{12} = 96.47\ \text{ft}^4$$

$$\therefore q_1 = \frac{R}{A} + \frac{(Re)c}{I} = \frac{18.44}{10.5} + \frac{(18.44 \times 1.36) \times 5.25}{96.47} = 1.76 + 1.37 = 3.13\ \text{ksf} < 4\ \text{ksf}\ \ (\text{ok})$$

$$\therefore q_1 = \frac{R}{A} - \frac{(Re)c}{I} = 1.76 - 1.37 = 0.39\ \text{ksf}$$

So, soil pressure is adequate.

5. Stability against sliding

i) Force causing sliding, $H_S = H_{a1} + H_{a2} = 1.755 + 5.665 = 7.43$ kip

ii) Resisiting force, $H_R = \mu R = 0.5 \times 18.44 = 9.22$ kip

$$\therefore \text{Factor of safety against sliding} = \frac{H_R}{H_S} = \frac{9.22}{7.43} = 1.24 \not> 1.5\ \ (Not\ ok)$$

The resistance force is not adequate against sliding.

In this case, a key should be provided to develop a passive pressure to resist the excess force that cause sliding.

The key is placed about 6 in. from the back face of the stem.

Assume,

Height of top foot of the earth at toe side, $h' = 2$ ft.

Depth of key, $t_k = 1.5$ ft.

Width of key, $b_k = 1.5$ ft.

$$\therefore \text{Coefficient of passive earth pressure, } C_p = \frac{1 + sin\varphi}{1 - sin\varphi} = \frac{1}{C_a} = \frac{1}{0.271} = 3.69$$

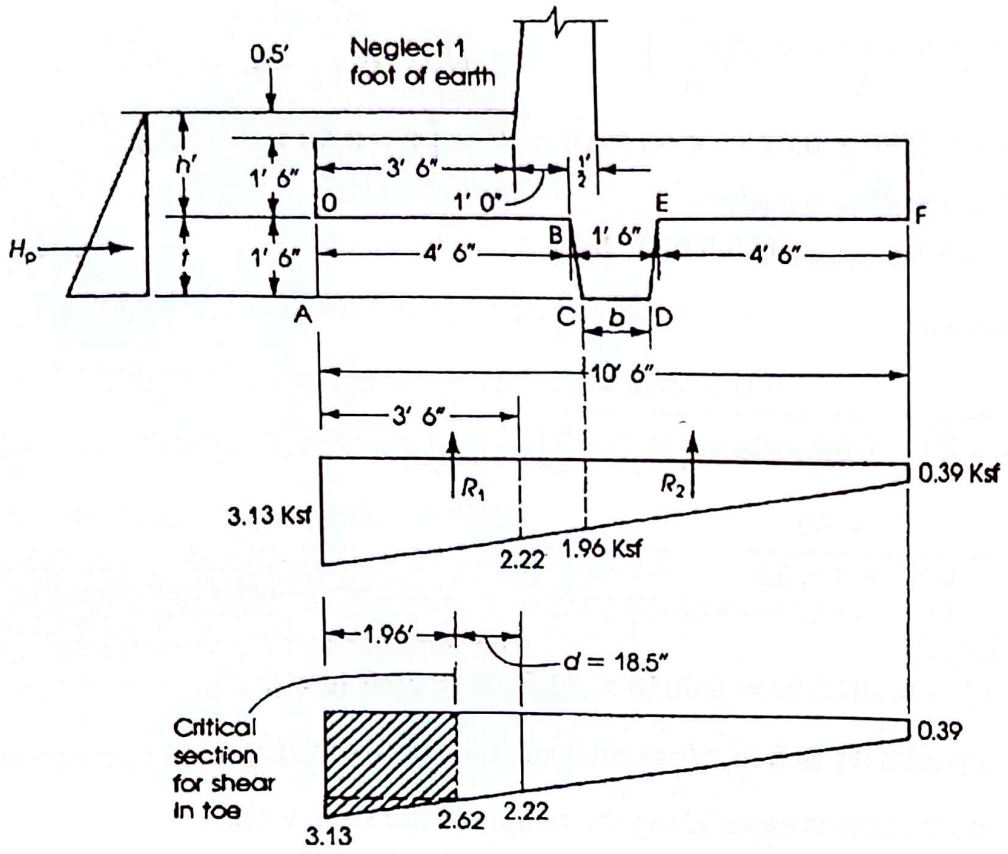
$$\therefore \text{Passive force, } H_p = \frac{1}{2} C_p w (h' + t_k)^2 = \frac{1}{2} \times 3.69 \times 110 \times (2 + 1.5)^2 = 2486\ \text{lb} = 2.49\ \text{kip}$$

$$R_1 = \text{Reaction of AC} = \left(\frac{3.13 + 1.96}{2}\right) \times 4.5 = 11.44\ \text{kip}$$

$$R_2 = \text{R} - R_1 = 18.44 - 11.44 = 7.0\ \text{kip}$$

$$R_2 = \text{Reaction of CDF} = \left(\frac{1.96 + 0.39}{2}\right) \times 6 = 7.05\ \text{kip}$$

$$\therefore \text{Frictional resistance, } F = \mu_1 R_1 + \mu_2 R_2 = (0.7 * 11.44) + (0.5 * 7.0) = 11.50\ \text{kip}$$



$\therefore$ Total resisting force, $H_R{}' = F + H_p = 11.50 + 2.49 = 13.99$ kip

$\therefore$ Factor of safety against sliding $= \dfrac{H_R{}'}{H_S} = \dfrac{13.99}{7.43} = 1.9 > 1.5 \ (ok)$

6. Design of stem (wall)

i) Main reinforcement

a) Ultimate forces, considering 1ft. or 12 in. strip along the length

$\therefore P_1 = 1.6(C_a w h_s) = 1.6 \times (0.271 \times 110 \times 3) = 143$ lb

$\therefore P_2 = 1.6(C_a w h) = 1.6 \times (0.271 \times 110 \times 18) = 858$ lb

$H_{a1} = 143 \times 18 = 2574$ lb $= 2.57$ kip; arm $= \dfrac{18}{2} = 9$ ft.

$H_{a2} = \dfrac{1}{2} \times 858 \times 18 = 7722$ lb $= 7.72$ kip; arm $= \dfrac{18}{3} = 6$ ft.

$\therefore$ Ultimate moment at bottom of stem, $M_u = 2.57 \times 9 + 7.72 \times 6 = 69.45$ k $-$ ft.

b) Depth check:

Here, effective depth, $d = t_{sb} - \text{cc} = 18 - 2.5 = 15.5$ in.

$$\rho = 0.85\beta_1 \frac{f_c{}'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{3}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0135$$

Now, $M = \varphi\rho f_y b d^2\left(1 - 0.59\rho\frac{f_y}{f_c'}\right)$

$\Rightarrow 69.45 \times 12 = 0.90 \times 0.0135 \times 60 \times 10 \times d^2 \times \left(1 - 0.59 \times 0.0135 \times \frac{60}{3}\right)$

$\Rightarrow 833.4 = 7.29 \times d^2 \times 0.8407$

$\therefore d = 11.66$ in. $> d_{provided}$ (15.5 in.) $[ok]$

c) Reinforcement:

$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{69.45 \times 12}{0.90 \times 60 \times \left(15.5 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 12}$$

$\therefore A_s = 1.04$ in^2/ft.

Here, $A_s(min) = 0.0018\, bt = 0.0018 \times 12 \times 18 = 0.39$ in^2/ft.

Provide #8 vertical bars @ 8 in. c/c at the back face of the wall

d) Again, the moment decreases along the height of the stem wall.
To calculate the moment at midheight of the wall, 9 ft. from the top,

$\therefore P_1 = 1.6(C_a w h_s) = 1.6 \times (0.271 \times 110 \times 3) = 143$ lb

$\therefore P_2 = 1.6(C_a w h) = 1.6 \times (0.271 \times 110 \times 9) = 429$ lb

$H_{a1} = 143 \times 9 = 1287$ lb $= 1.29$ kip; arm $= \frac{9}{2} = 4.5$ ft.

$H_{a2} = \frac{1}{2} \times 429 \times 9 = 1930$ lb $= 1.9$ kip; arm $= \frac{9}{3} = 3$ ft.

$\therefore$ Ultimate moment at bottom of stem, $M_u = 1.29 \times 4.5 + 1.9 \times 3 = 11.5$ k $-$ ft.

e) Depth check:

Here, effective depth, $d = \left(\frac{t_{sb} + t_{st}}{2}\right) - \text{cc} = \left(\frac{18 + 12}{2}\right) - 2.5 = 15 - 2.5 = 12.5$ in.

$\rho = 0.0135$

Now, $M = \varphi\rho f_y b d^2\left(1 - 0.59\rho\frac{f_y}{f_c'}\right)$

$\Rightarrow 11.5 \times 12 = 0.90 \times 0.0135 \times 60 \times 10 \times d^2 \times \left(1 - 0.59 \times 0.0135 \times \frac{60}{3}\right)$

$\Rightarrow 138 = 7.29 \times d^2 \times 0.8407$

$\therefore d = 4.75$ in. $> d_{provided}$ (12.5 in.) $[ok]$

f) Reinforcement:

$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{11.5 \times 12}{0.90 \times 60 \times \left(12.5 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 12}$$

$\therefore A_s = 0.22$ in^2/ft.
Here, $A_s(min) = 0.0018\, bt = 0.0018 \times 12 \times 15 = 0.32$ in^2/ft.

Provide #4 vertical bars @ 7 in. c/c at the back face of the wall

And, Provide #4 vertical bars @ 12 in. c/c at the front face of the wall

ii) Temperature and shrinkage reinforcement
$A_s(min) = 0.0018\, bt = 0.0018 \times 12 \times 18 = 0.389$ in^2/ft.
As the front face of the wall is mostly esposed to temperature changes, use one – half of the horizontal bars at the external and internal face.
$\therefore 0.5\, A_s = 0.0018\, bt = 0.5 \times 0.389 = 0.195$ in^2/ft.

Provide #4 horizontal bars @ 12 in. c/c

iii) Dowel bars
The anchorage length of #8 bars into the footing must be at least equal to the base thiclness (t_b)

iv) Shear check
The critical section for shear is at a distance $d = 15.5$ in. from the bottom of the stem
At this section, the distance from the top $= 18 - \frac{15.5}{12} = 16.7$ ft.

$\therefore P_1 = 1.6(C_a w h_s) = 1.6 \times (0.271 \times 110 \times 3) = 143$ lb

$\therefore P_2 = 1.6(C_a w h) = 1.6 \times (0.271 \times 110 \times 16.7) = 796$ lb

$H_{a1} = 143 \times 16.7 = 2388$ lb $= 2.39$ kip
$H_{a2} = \frac{1}{2} \times 796 \times 16.7 = 6646$ lb $= 6.6$ kip

$\therefore$ Total ultimate horizontal force, $H = 2.39 + 6.6 = 9.0$ kip

Concrete allowable shear force, $\varphi V_c = \varphi 2\lambda \sqrt{f_c'}\, bd$

$\therefore \varphi V_c = 0.75 \times 2 \times 1 \times \sqrt{3000} \times 12 \times 15.5 = 15281$ lb $= 15.28$ kip > 9.0 kip (ok)

7. Design of heel

i) Load and moment calculation
Total ultimate load on the heel, $V_u = 1.2\, DL + 1.6\, LL$
$\Rightarrow V_u = 1.2 \times [(18 \times 5.5 \times 110) + (1.5 \times 5.5 \times 150)] + 1.6 \times [(1.5 \times 5.5 \times 100)]$
$\therefore V_u = 17193$ lb $= 17.2$ kip

$\therefore$ Ultimate moment at face of wall, $M_u = V_u \times \frac{5.5}{2} = 17.2 \times \frac{5.5}{2} = 47.3 \text{ k} - \text{ft.}$

ii) Shear check

The critical section for shear is at a distance $d = 15.5$ in. from the face of wall

Here, $V_u = 17.5$ kip

$\therefore \varphi V_c = 15.28 \text{ kip} \not> V_u \ (Not\ ok)$

Now required, $d = \frac{17.2}{15.28} \times 15.5 = 17.4 \approx 18.5$ in.

$\therefore$ Total thickness of base slab $= d + cc = 18.5 + 3.5 = 22$ in.

iii) Depth check

$\rho = 0.0135$

Now, $M = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$

$\Longrightarrow 47.3 \times 12 = 0.90 \times 0.0135 \times 60 \times 10 \times d^2 \times \left(1 - 0.59 \times 0.0135 \times \frac{60}{3}\right)$

$\Longrightarrow 567.6 = 7.29 \times d^2 \times 0.8407$

$\therefore d = 9.62 \text{ in.} > d_{provided}$ (18.5 in.) $[ok]$

iv) Reinforcement

a) Main Reinforcement

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{47.3 \times 12}{0.90 \times 60 \times \left(18.5 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 12}$$

$\therefore A_s = 0.575 \text{ in}^2/\text{ft.}$

For shrinkage $A_s(min) = 0.0018\, bt = 0.0018 \times 12 \times 22 = 0.475 \text{ in}^2/\text{ft.}$

For flexural $A_s(min) = \frac{200}{f_y} bd = \frac{200}{60000} \times 12 \times 18.5 = 0.74 \text{ in}^2/\text{ft.}$

Provide #6 bars @ 7 in. c/c

b) Temperature and shrinkage Reinforcement

Not needed in the heel or toe, but it may be preferable to use minimum reinforcement.

Provide #4 bars @ 12 in. c/c

8. Design of toe

i) Load and moment calculation

Ultimate load on the heel, $V_u = 1.6 \times \left(\frac{3.13 + 2.62}{2}\right) \times 1.96 - 1.2 \times \left(\frac{22}{12} \times 0.150\right) \times 1.96$

$\therefore V_u = 8.37$ kip
$\therefore \varphi V_c = 15.28$ kip $> V_u$ (ok)

Ultimate moment at face of wall,

$$M_u = 1.6 \times \left[\frac{2.22}{2} \times (3.5)^2 + (3.13 - 2.22) \times 3.5 \times 0.5 \times \left(\frac{2}{3} \times 3.5\right)\right] - 1.2 \times \left[\left(\frac{22}{12} \times 0.150\right) \times \frac{(3.5)^2}{2}\right]$$

$\therefore M_u = 25.7$ k $-$ ft.

ii) Reinforcement

a) Main Reinforcement

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{25.7 \times 12}{0.90 \times 60 \times \left(18.5 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 12}$$

$\therefore A_s = 0.319$ in^2/ft.

For shrinkage $A_s(min) = 0.0018\, bt = 0.0018 \times 12 \times 22 = 0.475$ in^2/ft.

For flexural $A_s(min) = \frac{200}{f_y} bd = \frac{200}{60000} \times 12 \times 18.5 = 0.74$ in^2/ft.

Provide #6 bars @ 7 in. c/c

9. Design of shear key

Maximum ultimate moment between heel and toe, $M_u = 47.3$ k $-$ ft.
Depth of key, $t_k = 1.5$ ft. $= 18$ in.
Width of key, $b_k = 1.5$ ft. $= 18$ in.

Effective depth of key, $d = 18 - 2.5 = 15.5$ in.

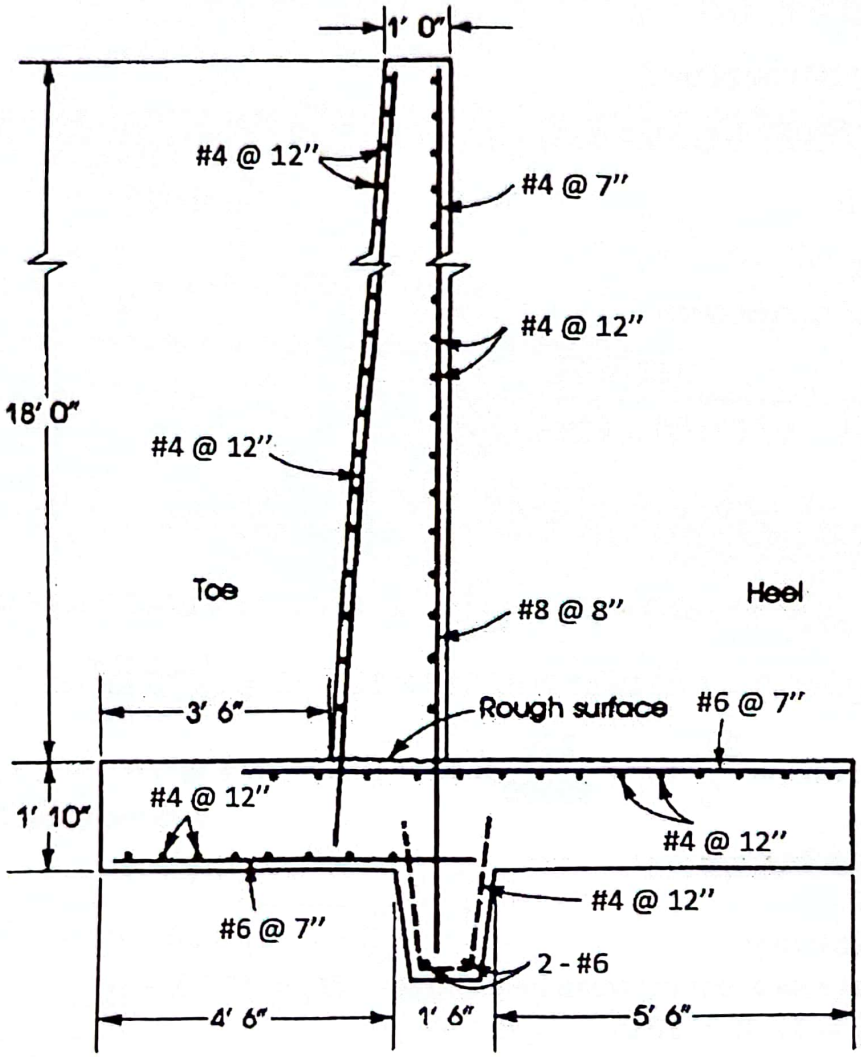
$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{47.3 \times 12}{0.90 \times 60 \times \left(15.5 - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s \times 60}{0.85 \times 3 \times 18}$$

$\therefore A_s = 0.69$ in^2

Provide 2 $-$ #6 bars
Also provide #4 bars @ 12 in. c/c as shear reinforcement as like external wall.

10. Reinforcement detailing



Chapter - 7
Introduction to Prestressed Concrete

7.1 Theory Analysis

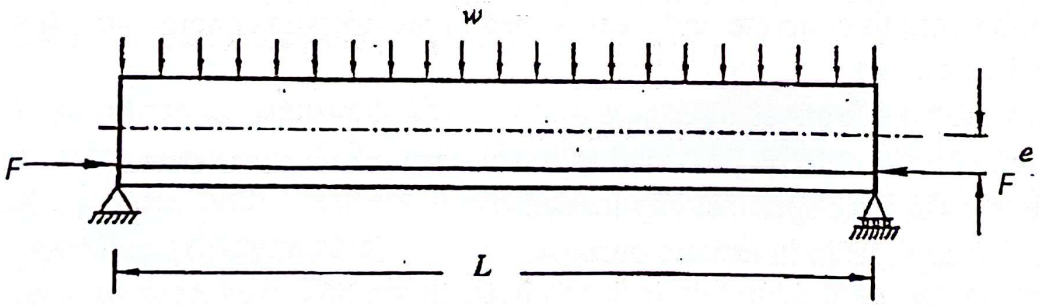
Q-1: What is prestressed concrete? [*CE-2018, 2017*]

Ans: **Prestressed concrete:** The concrete in which an initial compressive stress is introduced to the beam to offset or counteract the tensile stresses produced by the external loads is called prestressed concrete.

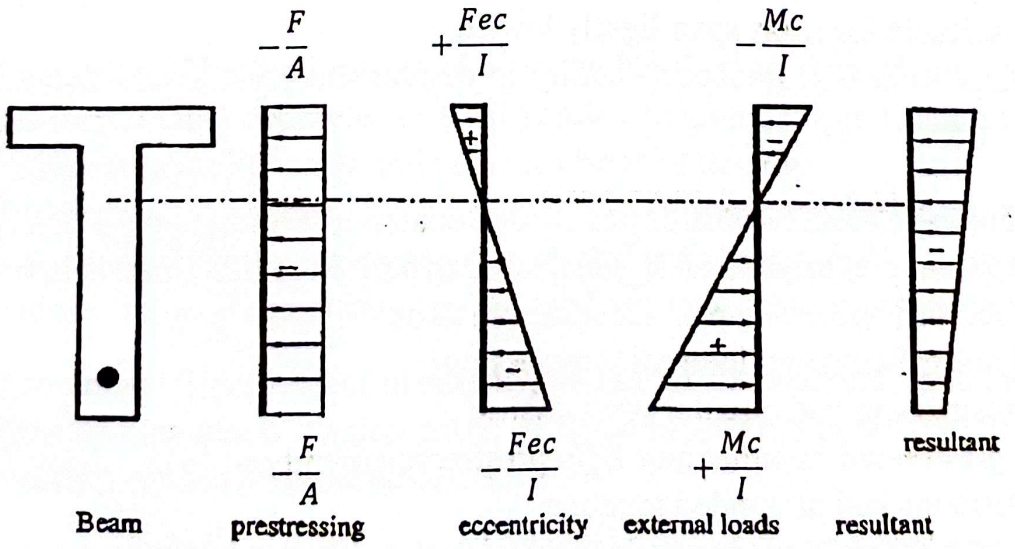
In reinforced concrete members, the prestress is commonly introduced by tensioning the steel reinforcement.

Q-2: Discuss the basic concepts to explain and analyze the behavior of prestressed concrete clearly. [*CE-2019*]

Ans:



a- Loading



b -Stresses

Figure 7.1: Distribution of stresses in a prestressed beam

Basic concepts of prestressed concrete:

(i) Prestressing to transform concrete into an elastic material.

(ii) Prestressing for combination of high strength steel with concrete.
(iii) Prestress to achieve load balancing.

Behavior of prestressed concrete:
(a) Flexural stresses in prestressed members are the result of internal prestressing force (F), the internal moment due to eccentric cable configuration (Fe), and the external applied moments (M).
(b) The prestressing force results in stresses that are opposite to those resulting from the external loads.
(c) The entire section is mainly subjected to compression stresses and is free from cracks.
(d) It is clear that the tensile stresses that result from the applied loads are eliminated by the compressive stresses due to prestressing (Figure 7.1).

Q-3: Write down advantages and disadvantages of prestressed concrete over reinforced concrete structures. [*CE-2019, 2017*]
Ans: Advantages of prestressed concrete:
(i) As the high strength concrete and steel is used in prestressed concrete, the lower amount of materials is required.
(ii) It reduces the tensile stress of concrete.
(iii) The stirrup need not to be provided as it can carry some shear stress.
(iv) It is suitable for long span heavily loaded.
(v) It can easily applicable in artistic purpose.

Disadvantages of prestressed concrete:
(i) It is not applicable and economical for simple structures.
(ii) It is not suitable for short span lightly loaded.
(iii) It requires sufficient transport facility to deliver the prestressing member from the factory.

Q-4: What are the classifications of prestressed concrete? Describe them briefly.
Ans: The different classifications of prestressed concrete are described below:
(1) Pre tensioned member and post tensioned member
(2) External prestressing and internal prestressing
(3) Linear prestressing and circular prestressing
(4) Partially prestressed member and fully prestressed member
(5) Bonded tendons and unbonded tendons
(6) Grouted member and non-grouted member

Brief description of the classification:
(1) Pre tensioned member: The member in which the tendons are tensioned before the concrete is placed and the prestress is transferred to the concrete after it has set is called pre tensioned member. [*CE-2019, 2015*]

Post tensioned member: The member in which the tendon is tensioned after the concrete has hardened and the prestressing is always performed against the hardened concrete and the tendons are anchored against it immediately after transfer is called post tensioned member. [*CE-2019, 2015*]

(2) External prestressing: The prestressing in which the prestressing force is applied by flat jacks placed between the concrete member ends and permanent rigid abutments is called external prestressing. [*CE-2015*]
Internal prestressing: The prestressing in which the tendons lie within the cross-section of the structure and prestressing force is transferred through anchorages and contact pressure against the surface of the duct is called internal prestressing.

(3) Linear prestressing: The prestressing in which the prestressed members are straight or flat in the direction of prestressing is called linear prestressing. The profile of the prestressing tendon may be curved. [*CE-2019*]
Circular prestressing: The prestressing in which the prestressed members are circular structures such as round tanks, pipes where the prestressing tendons are wound around circles is called circular prestressing. [*CE-2019*]

(4) Partially prestressed member: The prestressed member in which some tensile stress is produced under service load then the flexural cracking occurs and the main flexural tension steel includes both prestressed and non-prestressed steel is said to be partially prestressed member. [*CE-2019, 2018*]
Fully prestressed member: The prestressed member in which there are no tensile stress under service load and no flexural crack occurs in the concrete is said to be fully prestressed member. [*CE-2019*]

(5) Bonded tendons: The tendons (steel wire or bars) bond throughout the length of tendons to the surrounding concrete is called bonded tendons. In pre tensioning, concrete and tendons are cast monolithically and provides bonded tendons.
Unbonded tendons: The tendons which have no bond throughout the length of tendons to the concrete and tendons left without grout or coated with grease in the concrete is called unbonded tendons. In post tensioning, bonded tendons may achieve by grouting.

(6) Grouted member: The member in which grouting of the post tensioned tendons in the post or prestressed concrete is carried out to provide permanent protection to the tendons against corrosion is said to be grouted member.

Non-grouted member: The member in which grouting of the post tensioned tendons in the post or prestressed concrete is not required for the no void in between the tendons and concrete is said to be non-grouted member.

Q-5: What is loss of prestressing? [*CE-2013*]

Ans: Loss of prestressing: When the prestressing force is transferred from the jack to the concrete member, a continuous loss in the prestressing force occurs; the total loss is the reduction in the prestressing force during the lifespan of the structure. This loss is said to be the loss of prestressing.

The sources of prestress loss are given below:

(i) Elastic shortening of concrete
(ii) Shrinkage of concrete
(iii) Creep of concrete
(iv) Relaxation of steel tendons
(v) Friction
(vi) Anchorage set

Q-6: What is stress relaxation? [*CE-2018*]

Ans: Stress relaxation: Stress relaxation causes a time-dependent loss in the initial prestressing force, similar to creep in concrete. Stress relaxation is defined as the loss of stress under constant strain, while creep is defined as the change in strain under constant stress. This type of losses occurs under constant loading due to the elongation of the tendons with time.

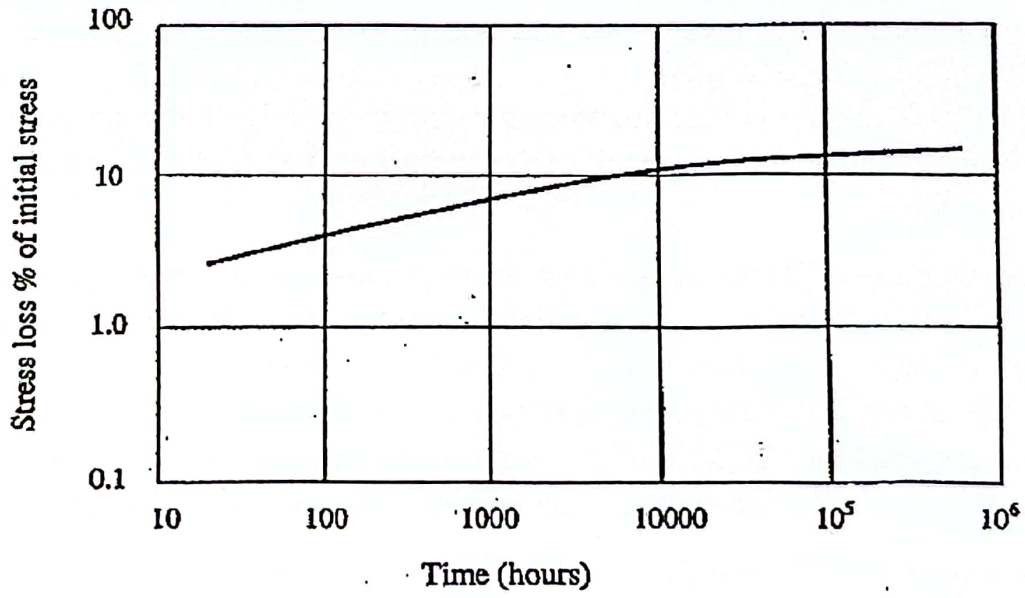
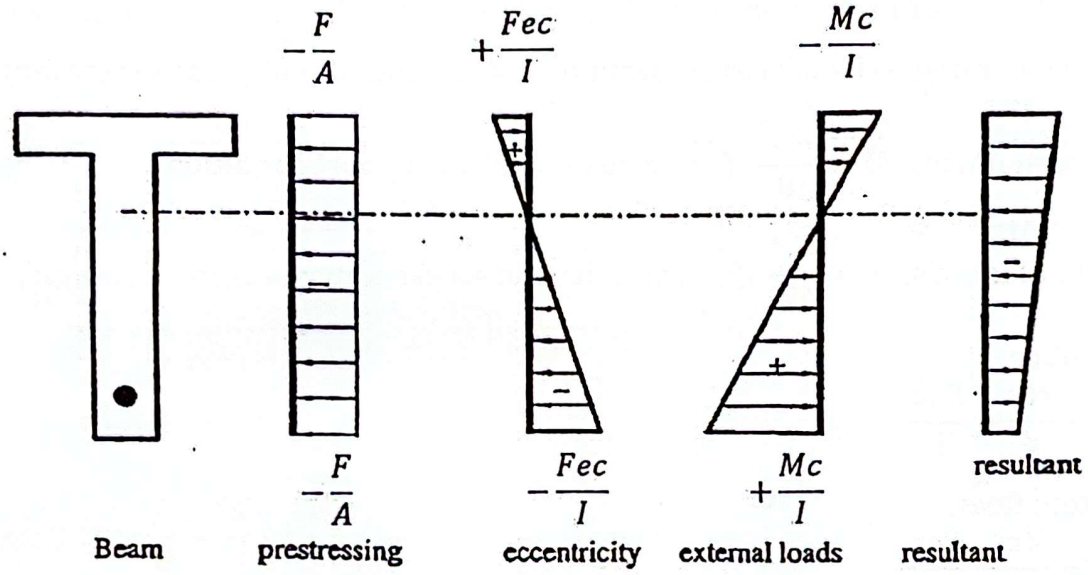


Figure 7.2: Typical stress relaxation losses

7.2 Basic Concepts

7.2.1 Fiber Stress and Sign Indication

Fiber stress, $f = -\frac{F}{A} \mp \frac{Mc}{I} \pm \frac{Fec}{I}$



i) Compression কে always (-ve) ধরা হয়েছে এবং tension কে always (+ve) ধরা হয়েছে।

ii) Prestressing force (F) এর কারনে top ও bottom-এ always compression কাজ করবে।

iii) Prestressing force এর eccentricity থাকলে internal moment (Fe) এর কারনে beam এর bottom-এ compression (-) এবং top-এ tension (+) কাজ করবে।

iv) External moment (M) এর কারনে beam এর bottom-এ tension (+) এবং top-এ compression (-) কাজ করবে।

∴ Fiber stress at top, $f_t = -\frac{F}{A} - \frac{Mc}{I} + \frac{Fec}{I}$

∴ Fiber stress at bottom, $f_t = -\frac{F}{A} + \frac{Mc}{I} - \frac{Fec}{I}$

7.3 Solution Techniques

7.3.1 Fiber stress at Top and Bottom

1. We have,

Fiber stress, $f = -\frac{F}{A} \mp \frac{Mc}{I} \pm \frac{Fec}{I}$

Here, c = center of gravity of cross − section, e = eccentricity of prestressing force

2. External moment, $M = \frac{wL^2}{8}$ (and also depend on support condition)

3. Moment of inertia, $I = \frac{bh^3}{12}$ (and also depend on geometry of cross − section)

4. At top fiber,

$f_t = -\frac{F}{A} - \frac{Mc}{I} + \frac{Fec}{I}$

5. At bottom fiber,

$f_b = -\frac{F}{A} + \frac{Mc}{I} - \frac{Fec}{I}$

6. Stress diagram

7.3.2 Prestressing Force Required Producing a Certain Stress

1. We have,

Fiber stress, $f = -\frac{F}{A} \mp \frac{Mc}{I} \pm \frac{Fec}{I}$

Here, c = center of gravity of cross − section, e = eccentricity of prestressing force

2. External moment, $M = \frac{wL^2}{8}$ (and also depend on support condition)

3. Moment of inertia, $I = \frac{bh^3}{12}$ (and also depend on geometry of cross − section)

4. Finding of Prestressing force (F)

At top fiber, $f_t = -\frac{F}{A} - \frac{Mc}{I} + \frac{Fec}{I}$

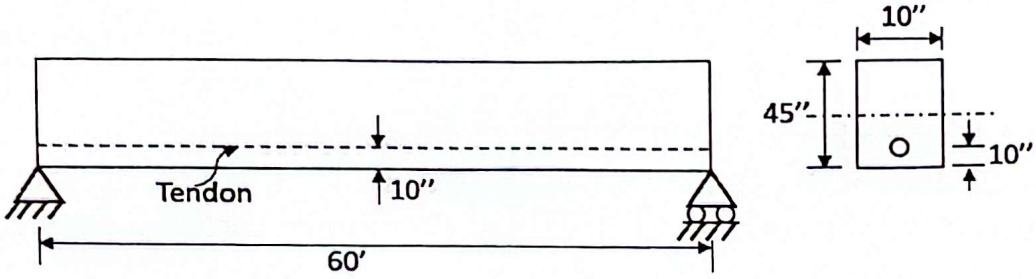
or,

At bottom fiber, $f_t = -\frac{F}{A} + \frac{Mc}{I} - \frac{Fec}{I}$

7.4 Problem Solution

7.4.1 Fiber Stress at Top and Bottom

Pb-1. A post tensioned beam of 60 ft. span length and the prestressing force amount is 1250 kip applied according to the figure. The beam carries a uniform load of 2.5 k/ft. including its self-weight. Calculate the stresses in the top and bottom fibers at the midspan section.



Sol^n:

We have,

Fiber stress, $f = -\frac{F}{A} \mp \frac{Mc}{I} \pm \frac{Fec}{I}$

Here, $c = \frac{45}{2} = 22.5''$, $e = 22.5 - 10 = 12.5''$

External moment, $M = \frac{wL^2}{8} = \frac{2.5 \times 60^2}{8} = 1125\ k-ft = 13.5 \times 10^6\ lb-in.$

Moment of inertia, $I = \frac{bh^3}{12} = \frac{10 \times 45^3}{12} = 75937.5\ in^4$

At top fiber,

$$f_t = -\frac{F}{A} - \frac{Mc}{I} + \frac{Fec}{I}$$

$$\Rightarrow f_t = -\frac{1250 \times 10^3}{45 \times 10} - \frac{13.5 \times 10^6 \times 22.5}{75937.5} + \frac{1250 \times 10^3 \times 12.5 \times 22.5}{75937.5}$$

$$\therefore f_t = -2148.15\ psi \text{ (compression)}$$

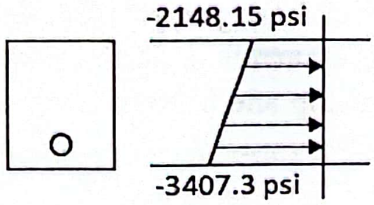
At bottom fiber,

$$f_b = -\frac{F}{A} + \frac{Mc}{I} - \frac{Fec}{I}$$

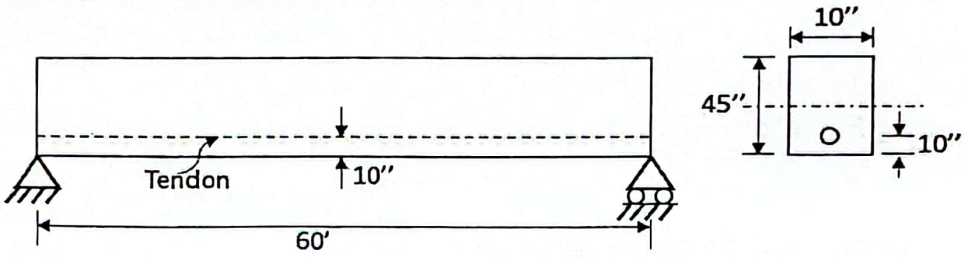
$$\Rightarrow f_b = -\frac{1250 \times 10^3}{45 \times 10} + \frac{13.5 \times 10^6 \times 22.5}{75937.5} - \frac{1250 \times 10^3 \times 12.5 \times 22.5}{75937.5}$$

$\therefore f_b = -3407.3\ psi$ (compression)

Stress diagram:



Pb-2. A post tensioned beam of 60 ft. span length and the prestressing force amount is 1250 kip applied according to the figure. The beam carries a uniform load of 2.5 k/ft. including its self-weight. If the stress in the top fibers to be zero at the midspan section, what will be changed in the beam?



Sol^n:

We have,

Fiber stress, $f = -\frac{F}{A} \mp \frac{Mc}{I} \pm \frac{Fec}{I}$

Here, $c = \frac{45}{2} = 22.5''$, $e = 22.5 - 10 = 12.5''$

External moment, $M = \frac{wL^2}{8} = \frac{w \times 60^2}{8} = 450 \times w\ k-ft = 5.4 \times 10^6 \times w\ lb-in.$

Moment of inertia, $I = \frac{bh^3}{12} = \frac{10 \times 45^3}{12} = 75937.5\ in^4$

At top fiber,

$$f_t = -\frac{F}{A} - \frac{Mc}{I} + \frac{Fec}{I}$$

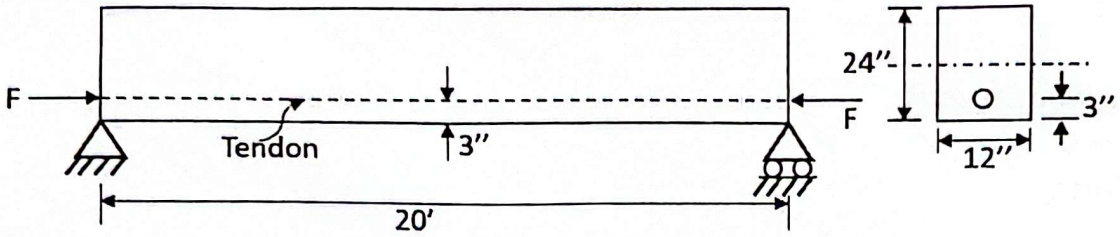
$$\Rightarrow 0 = -\frac{1250 \times 10^3}{45 \times 10} - \frac{5.4 \times 10^6 \times w \times 22.5}{75937.5} + \frac{1250 \times 10^3 \times 12.5 \times 22.5}{75937.5}$$

$$\Rightarrow 0 = -2777.78 - 1600\ w + 4629.63$$

$$\therefore w = 1.16\ k/ft$$

So, uniform load will be reduced $= 2.5 - 1.16 = 1.34\ k/ft$

Pb-3. A prestressed concrete beam of 20 ft. span length and the prestressing force amount is 250 kip applied according to the figure. The beam carries a uniform load of 3 k/ft. including its self-weight. Calculate the stresses in the top and bottom fibers at the midspan section and ends of the beam.



Sol^{n}:

We have,

Fiber stress, $f = -\frac{F}{A} \mp \frac{Mc}{I} \pm \frac{Fec}{I}$

Here, $c = \frac{24}{2} = 12''$, $e = 12 - 3 = 9''$

External moment, $M = \frac{wL^2}{8} = \frac{3 \times 20^2}{8} = 150\ k-ft = 1.8 \times 10^6\ lb-in.$

Moment of inertia, $I = \frac{bh^3}{12} = \frac{12 \times 24^3}{12} = 13824\ in^4$

Stresses at midspan:

At top fiber,

$$f_t = -\frac{F}{A} - \frac{Mc}{I} + \frac{Fec}{I}$$

$$\Rightarrow f_t = -\frac{250 \times 10^3}{12 \times 24} - \frac{1.8 \times 10^6 \times 12}{13824} + \frac{250 \times 10^3 \times 9 \times 12}{13824}$$

$\therefore f_t = -477.4\ psi$ (compression)

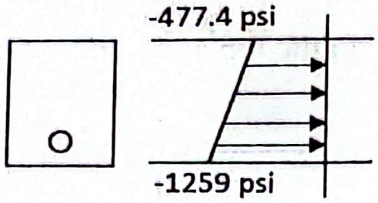
At bottom fiber,

$$f_b = -\frac{F}{A} + \frac{Mc}{I} - \frac{Fec}{I}$$

$$\Rightarrow f_b = -\frac{250 \times 10^3}{12 \times 24} + \frac{1.8 \times 10^6 \times 12}{13824} - \frac{250 \times 10^3 \times 9 \times 12}{13824}$$

$\therefore f_b = -1259\ psi$ (compression)

Stresses diagram at midspan:



Stresses at beam ends:

Here, $c = 0$

At top fiber,

$$f_t = -\frac{F}{A} + \frac{Fec}{I}$$

$$\Rightarrow f_t = -\frac{250 \times 10^3}{12 \times 24} + 0$$

$$\therefore f_t = -868\ psi \text{ (compression)}$$

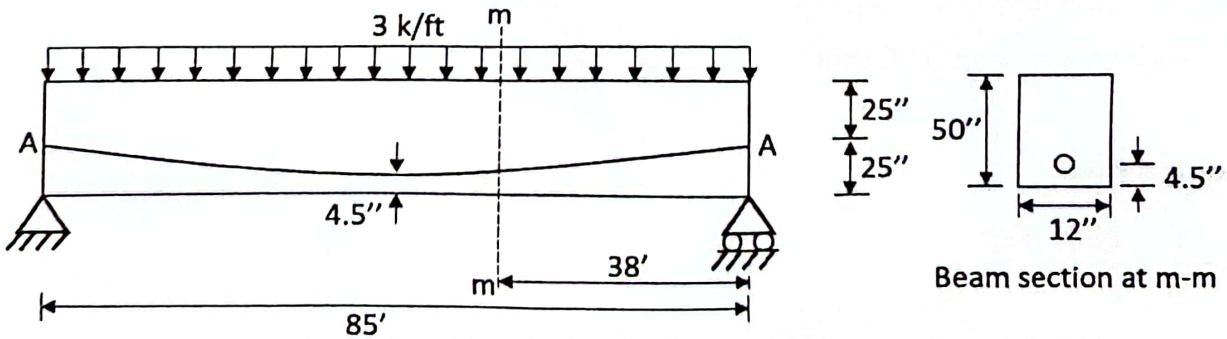
At bottom fiber,

$$f_b = -\frac{F}{A} - \frac{Fec}{I}$$

$$\Rightarrow f_b = -\frac{250 \times 10^3}{12 \times 24} - 0$$

$$\therefore f_b = -868\ psi \text{ (compression)}$$

Pb-4. A post tensioned beam shown in the figure given below stressed by 1000 kPa force at point A. Compute the extreme top and bottom fiber stress at m-m section. The beam is loaded by 3 k/ft. including its self-weight. [*CE-2017*]



Sol^n:

We have,

Fiber stress, $f = -\frac{F}{A} \mp \frac{Mc}{I} \pm \frac{Fec}{I}$

Here, $c = \frac{50}{2} = 25''$, $e = 25 - 4.5 = 20.5''$

Prestressing force, $F = P \times A = 1000\ kPa \times 12 * 50\ in^2$

$\therefore F = 1000 * 20.89\ psf \times \frac{12 * 50}{144} ft^2 = 87042\ lb$

External moment at m – m section, $M = \frac{wL}{2}x - \frac{wx^2}{2}$

$\therefore M = \frac{3 \times 85 \times 38}{2} - \frac{3 \times 38^2}{2} = 2679\ k - ft = 32.148 \times 10^6\ lb - in$

Moment of inertia, $I = \frac{bh^3}{12} = \frac{12 \times 50^3}{12} = 1.25 \times 10^5\ in^4$

At top fiber at m-m section,

$$f_t = -\frac{F}{A} - \frac{Mc}{I} + \frac{Fec}{I}$$

$$\Rightarrow f_t = -\frac{87042}{12 \times 50} - \frac{32.148 \times 10^6 \times 25}{1.25 \times 10^5} + \frac{87042 \times 20.5 \times 25}{1.25 \times 10^5}$$

$\therefore f_t = -6218\ psi$ (compression)

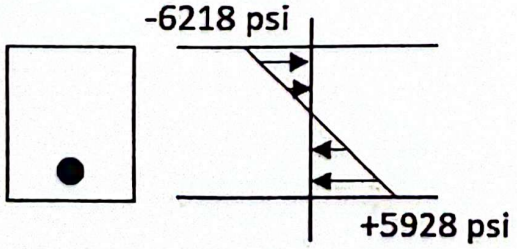
At bottom fiber,

$$f_b = -\frac{F}{A} + \frac{Mc}{I} - \frac{Fec}{I}$$

$$\Rightarrow f_b = -\frac{87042}{12 \times 50} + \frac{32.148 \times 10^6 \times 25}{1.25 \times 10^5} - \frac{87042 \times 20.5 \times 25}{1.25 \times 10^5}$$

$\therefore f_b = +5928\ psi$ (tension)

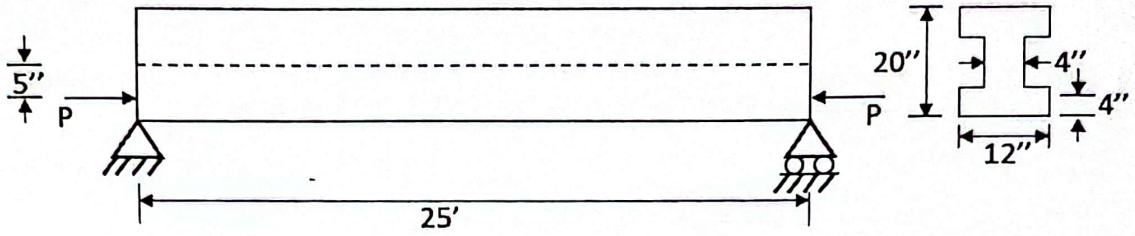
Stress diagram:



Pb-5. For the simply supported beam shown in the figure below, determine the maximum stresses at midspan section due to its own weight and the following cases of loading and prestressing:

(i) A uniform live load of 800 lb/ft and an axial centroidal longitudinal compressive force, P = 280 kip.

(ii) A uniform live load of 2200 lb/ft and an eccentric longitudinal compressive force, P = 280 kip. [*CE-2016*]



Sol^{n}:

(i) A uniform live load of 800 lb/ft and an axial centroidal longitudinal compressive force, P = 280 kip.

We have,

Fiber stress, $f = -\frac{F}{A} \mp \frac{Mc}{I} \pm \frac{Fec}{I}$

Here, $c = \frac{20}{2} = 10''$, e = 0, Area, $A = (12 \times 20) - (8 \times 12) = 144\ in^2$

Self weight of beam $= \frac{144}{144} \times 150 = 150\ lb/ft$

$\therefore$ Total uniform load, $w = (800 + 150) = 950\ lb/ft$

External moment, $M = \frac{wL^2}{8} = \frac{950 \times 25^2}{8} = 74219\ lb - ft = 8.9 \times 10^5\ lb - in.$

Moment of inertia, $I = \frac{12 \times 20^3}{12} - \frac{8 \times 12^3}{12} = 6848\ in^4$

At top fiber,

$$f_t = -\frac{F}{A} - \frac{Mc}{I} + \frac{Fec}{I}$$

$$\Rightarrow f_t = -\frac{280 \times 10^3}{144} - \frac{8.9 \times 10^5 \times 10}{6848}$$

$\therefore f_t = -3244\ psi$ (compression)

At bottom fiber,

$$f_b = -\frac{F}{A} + \frac{Mc}{I} - \frac{Fec}{I}$$

$$\Rightarrow f_b = -\frac{280 \times 10^3}{144} + \frac{8.9 \times 10^5 \times 10}{6848}$$

$$\therefore f_b = -645\ psi \text{ (compression)}$$

So, maximum stress at midspan section is – 3244 psi.

(ii) A uniform live load of 2200 lb/ft and an eccentric longitudinal compressive force, P = 280 kip.

We have,

$$\text{Fiber stress, } f = -\frac{F}{A} \mp \frac{Mc}{I} \pm \frac{Fec}{I}$$

Here, $c = \frac{20}{2} = 10''$, e = 5″, Area, A = $(12 \times 20) - (8 \times 12) = 144\ in^2$

∴ Total uniform load, $w = (2200 + 150) = 2350\ lb/ft$

$$\text{External moment, } M = \frac{wL^2}{8} = \frac{2350 \times 25^2}{8} = 1.84 \times 10^5\ lb - ft = 2.2 \times 10^6\ lb - in.$$

Moment of inertia, $I = 6848\ in^4$

At top fiber,

$$f_t = -\frac{F}{A} - \frac{Mc}{I} + \frac{Fec}{I}$$

$$\Rightarrow f_t = -\frac{280 \times 10^3}{144} - \frac{2.2 \times 10^6 \times 10}{6848} + \frac{280 \times 10^3 \times 5 \times 10}{6848}$$

$$\therefore f_t = -3113\ psi \text{ (compression)}$$

At bottom fiber,

$$f_b = -\frac{F}{A} + \frac{Mc}{I} - \frac{Fec}{I}$$

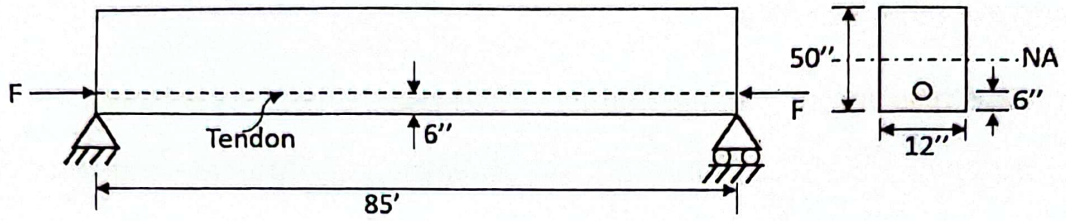
$$\Rightarrow f_b = -\frac{280 \times 10^3}{144} + \frac{2.2 \times 10^6 \times 10}{6848} - \frac{280 \times 10^3 \times 5 \times 10}{6848}$$

$$\therefore f_b = -776\ psi \text{ (compression)}$$

So, maximum stress at mid span section is – 3113 psi.

7.3.2 Prestressing Force Required Producing a Certain Stress

Pb-6. A pre-tensioned (prestressed) rectangular beam of 85 ft. span length having cross-section of 12 in. by 50 in. The beam carries a uniform load of 2 k/ft. including its self-weight. Calculate the prestressing force to produce -55 psi stress at bottom fiber at the midspan of the beam. [*Class Test-2019*]



Sol^n:

We have,

Fiber stress, $f = -\frac{F}{A} \mp \frac{Mc}{I} \pm \frac{Fec}{I}$

Here, $c = \frac{50}{2} = 25''$, $e = 25 - 6 = 21''$

External moment, $M = \frac{wL^2}{8} = \frac{2 \times 85^2}{8} = 1806.25\ k-ft = 21.675 \times 10^6\ lb-in.$

Moment of inertia, $I = \frac{bh^3}{12} = \frac{12 \times 50^3}{12} = 1.25 \times 10^5\ in^4$

At bottom fiber,

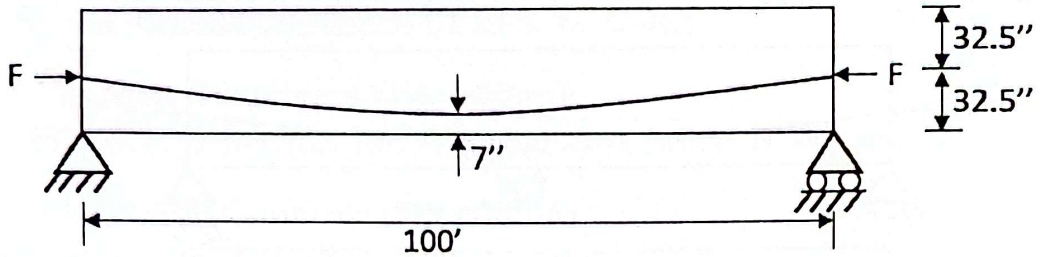
$$f_b = -\frac{F}{A} + \frac{Mc}{I} - \frac{Fec}{I}$$

$$\Rightarrow -55 = -\frac{F}{12 \times 50} + \frac{21.675 \times 10^6 \times 25}{1.25 \times 10^5} - \frac{F \times 21 \times 25}{1.25 \times 10^5}$$

$$\Rightarrow -55 = -1.67 \times 10^{-3} \times F + 4335 - 4.2 \times 10^{-3} \times F$$

$$\therefore F = 747870\ lb = 747.87\ kip \approx 750\ kip$$

Pb-7. A post tensioned concrete beam of span 100 ft. having $12'' \times 65''$ cross section. Find out the required amount of pre-stressing force to produce 60 psi tension in top fiber. Pre-stressed concrete beam details is shown in figure below. [*CE-2019*]



Sol^{n}:

We have,

Fiber stress, $f = -\frac{F}{A} \mp \frac{Mc}{I} \pm \frac{Fec}{I}$

Here, $c = \frac{65}{2} = 32.5''$, $e = 32.5 - 7 = 25.5''$

Self – weight of beam, $w = \frac{12 \times 65}{144} \times 150 = 812.5\ lb/ft = 0.8125\ k/ft$

External moment, $M = \frac{wL^2}{8} = \frac{0.8125 \times 100^2}{8} = 1015.625\ k-ft = 12.2 \times 10^6\ lb-in.$

Moment of inertia, $I = \frac{bh^3}{12} = \frac{12 \times 65^3}{12} = 2.75 \times 10^5\ in^4$

At top fiber,

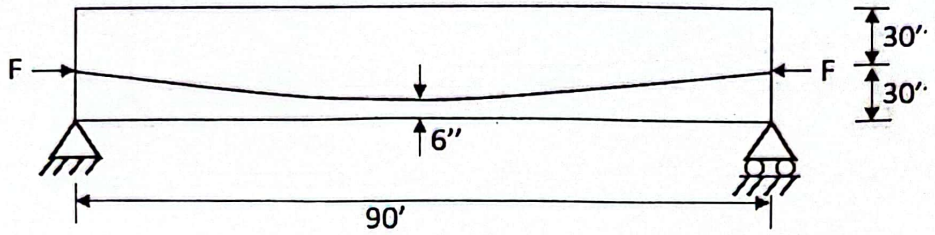
$$f_t = -\frac{F}{A} - \frac{Mc}{I} + \frac{Fec}{I}$$

$$\Rightarrow 60 = -\frac{F}{12 \times 65} - \frac{12.2 \times 10^6 \times 32.5}{2.75 \times 10^5} + \frac{F \times 25.5 \times 32.5}{2.75 \times 10^5}$$

$$\Rightarrow 60 = -1.282 \times 10^{-3} \times F + 1441.82 - 3.014 \times 10^{-3} \times F$$

$$\therefore F = 321680\ lb = 321.68\ kip \approx 322\ kip$$

Pb-8. A post tensioned beam is $10'' \times 60''$ in cross-section and $90'$ span length. Post tensioned cable follows a parabolic path shown in figure below. The beam carries a uniform load of 4 k/ft. including its self-weight. Find out the amount of pre-stressing force to produce "0" stress at bottom fiber of the beam in service condition. [*CE-2018*]



Sol^n:

We have,

Fiber stress, $f = -\frac{F}{A} \mp \frac{Mc}{I} \pm \frac{Fec}{I}$

Here, $c = \frac{60}{2} = 30''$, $e = 30 - 6 = 24''$

External moment, $M = \frac{wL^2}{8} = \frac{4 \times 90^2}{8} = 4050\ k-ft = 48.6 \times 10^6\ lb-in.$

Moment of inertia, $I = \frac{bh^3}{12} = \frac{10 \times 60^3}{12} = 1.8 \times 10^5\ in^4$

At bottom fiber,

$$f_b = -\frac{F}{A} + \frac{Mc}{I} - \frac{Fec}{I}$$

$$\Rightarrow 0 = -\frac{F}{10 \times 60} + \frac{48.6 \times 10^6 \times 30}{1.8 \times 10^5} - \frac{F \times 24 \times 30}{1.8 \times 10^5}$$

$$\Rightarrow 0 = -1.67 \times 10^{-3} \times F + 8100 - 4 \times 10^{-3} \times F$$

$$\therefore F = 1428572\ lb = 1428.572\ kip \approx 1430\ kip$$

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8. Building Code Requirements for Structural Concrete (ACI 318-08)

Appendix A: Necessary Tables and Graphs

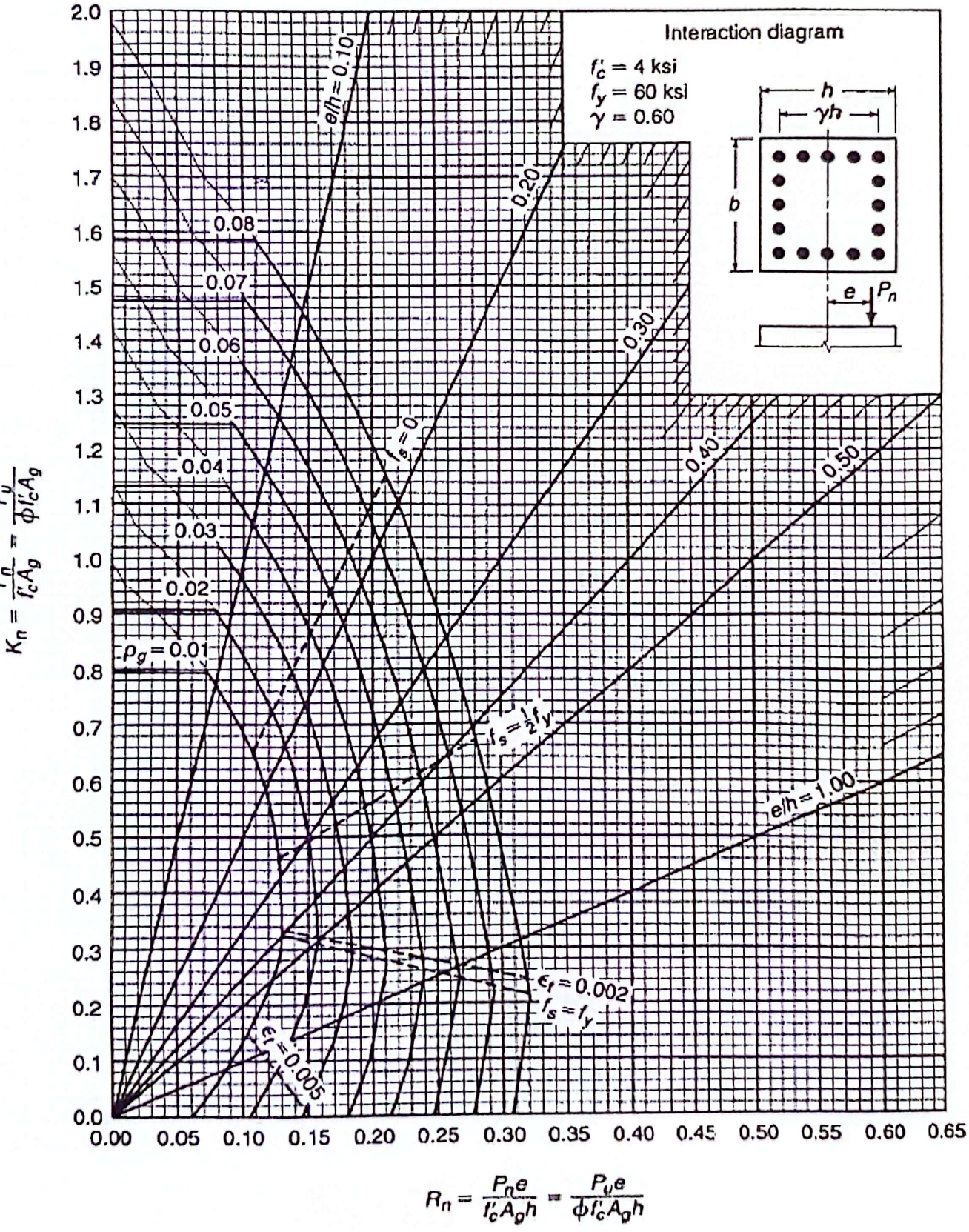
Table A: Designation, diameter and area of standard bars

Bar size		Diameter (in.)	Area (in^2)	Area (mm^2)
Bar No.	Bar (mm)			
3	10	0.375	0.11	78.5
4	13	0.5	0.20	132.7
5	16	0.625	0.31	201.0
6	19	0.75	0.44	283.5
7	22	0.875	0.60	380.0
8	25	1.0	0.79	491.0
9	29	1.128	1.00	660.5
10	32	1.27	1.27	804.0
11	36	1.41	1.56	1018.0
14	43	1.693	2.25	1452.0
18	57	2.25	4.00	2552.0

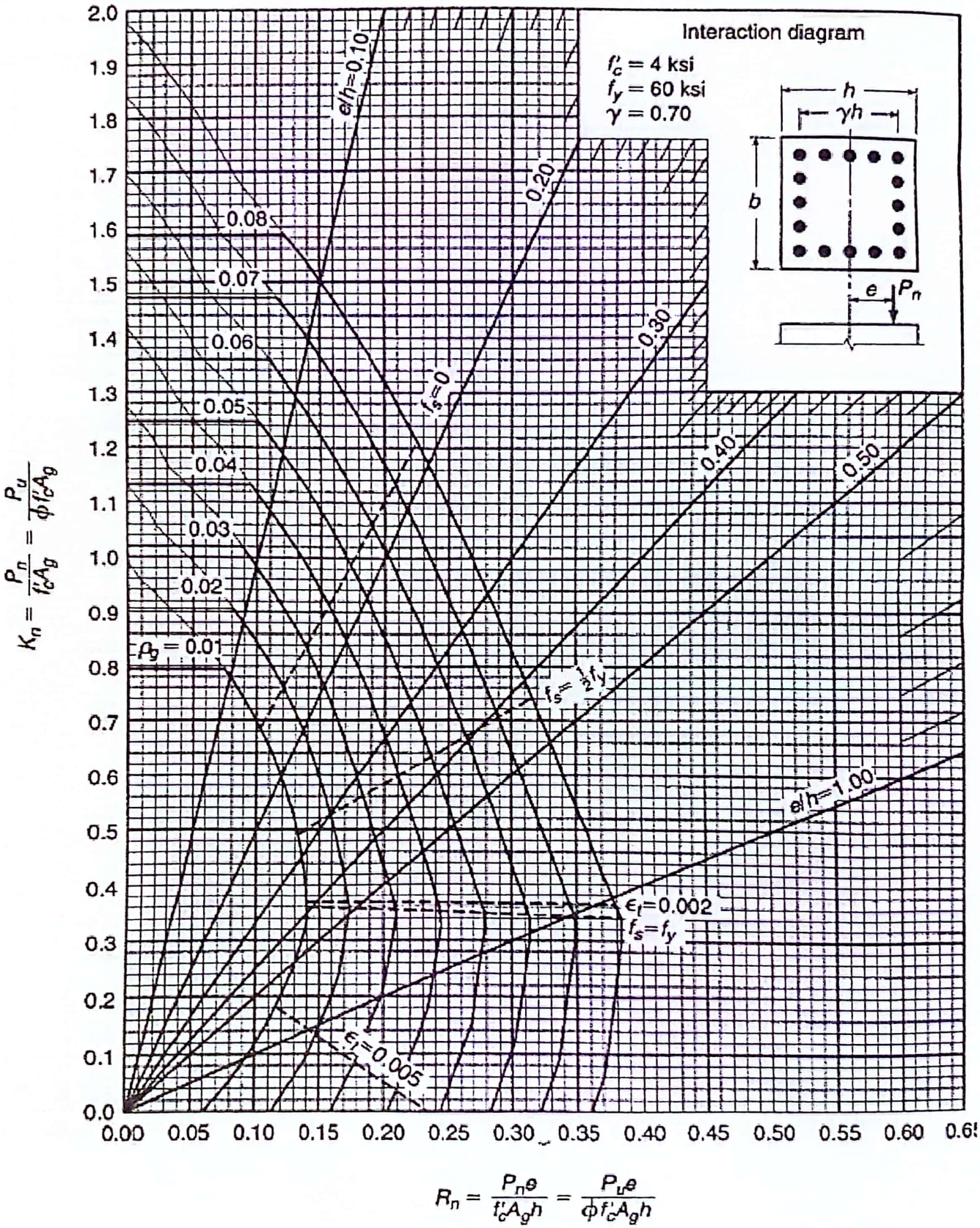
[*For finding bar diameter from* ***bar no. to mm (#3 to 10 mm)*** *& area by short techniques, follow* ***Appendix B*** *(Table B.1)*]

Graph A: Column strength interaction diagram for rectangular section with bars on four faces.

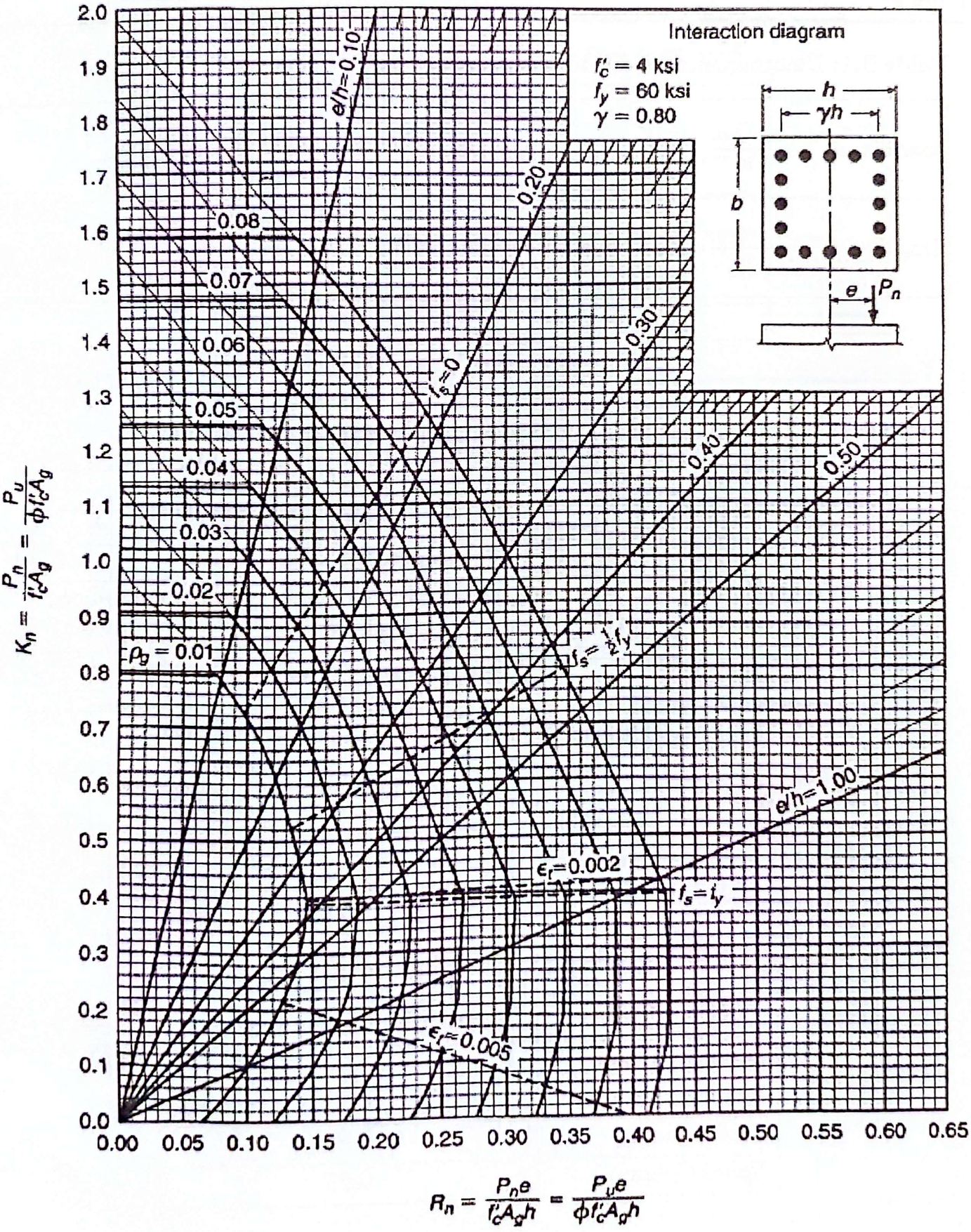
Graph A.5: for $\gamma = 0.60$



Graph A.6: for $\gamma = 0.70$



Graph A.7: for $\gamma = 0.80$



Appendix B: Special Short Techniques

Table B.1: Determination of diameter and area of bars

Bar dia, $d_b = \frac{Bar\ No.}{8}\ (in.)$	*e.g;* Dia. of #3 bar, $d_b = \frac{3}{8} = 0.375\ in.$
Bar dia, $d_b = \frac{Bar\ No.}{8} * 25.4\ (mm)$	*e.g;* Dia. of #3 bar, $d_b = \frac{3}{8} * 25.4 = 9.53 \approx 10\ mm$

Table B.2: When RCC member is to be designed in *WSD* and when in *USD*

When in WSD	***When in USD***
1. Directly *WSD* is said.	1. Directly *USD*/Strength Design Method is said.
2. To find allowable working moment.	2. To find factored moment/moment strength.
3. Given, f_s & f_c value.	3. Given, f_y & f_c' value.
4. Given, allowable working moment.	4. Given, factored moment.
	5. Given, service live load.

Table B.3: Strength Reduction Factor of RCC Structures

RCC Member	**Strength Reduction Factor, φ**
Beam for flexure	0.90
Beam for shear	0.75
Tied Column	0.65
Spiral Column	0.75

Table B.4: Clear Cover & Effective Cover of RCC Structures

RCC Structures	Clear Cover (cc)	Effective Cover $(ec) = cc + \frac{d_b}{2}$
Slab	0.75"	1"
Beam	2"	2.5"
Column	2"	2.5"
Foundation	3"	3.5"
Staircase	0.75"	1"
Lintel	1"	1.5"
Sunshade	0.75"	1"

Table B.5: Using of Reinforcement Bar in RCC Structures (normally used)

RCC Structures	Bar Size (No.)	Bar Size (mm)
Beam	#6, #7, #8, #9, #10	19, 22, 25, 29, 32 mm
Slab	#3, #4	10, 13 mm
Column	#5, #6, #7, #8	16, 19, 22, 25 mm
Wall Footing	#3, #4	10, 13 mm
Foundation	#7, #8, #9	22, 25, 29 mm
Staircase	#3, #4, #5	10, 13, 16 mm
Lintel	#3, #4, #5	10, 13, 16 mm
Sunshade	#3, #4	10, 13 mm

Appendix C: Necessary Unit Conversion

Table C.1: Necessary Unit Conversion

Length	**Pressure**
1 in. = 25.4 mm 1 m = 39.37 in. 1 m = 3.28 ft 1 m = 1.094 yd 1 mile = 1.61 km	1 kN/m^2 = 20.885 psf 1 MPa = 1 N/mm^2 = 145 psi 1 MPa = 10.197 kg/cm^2 1 tsf = 95.76 kN/m^2 (kPa) 1 bar = 14.5 psi
Area	**Force**
1 m^2 = 10.76 sft 1 acre = 4046.86 m^2 1 acre = 4840 yd^2 1 Dec = 435.6 sft 1 Katha = 720 sft	1 kN = 224.8 lb 1 ton = 2000 lb = 2 kips
Mass	**Moment**
1 kg = 2.2 lb 1 lb = 453 gm 1 ounce = 28.35 gm	1 kN. m = 0.737 kip − ft
Volume	**Unit Weight**
1 US gallon = 3.78 litre 1 m^3 = 100 litre 1 cft = 28.32 litre 1 m^3 = 35.31 cft	Steel: 488 lb/ft^3 = 78 kN/m^3 = 7850 kg/m^3 Concrete: 150 lb/ft^3 = 24 kN/m^3 = 2400 kg/m^3

About the Author

Md. Jaber Hossain, the son of Md. Humayun Kabir and Khorsheda Begum, was born at Matlab South in Chandpur district in 1996. He attends in Khulna University of Engineering & Technology (KUET) for receiving Bachelor's degree in Civil Engineering in 2017 and he is now in the final year. He has completed his secondary education from his village, Naogaon High School in 2013, and higher secondary education from Hajigonj Model Govt. College in 2016. He is rewarded the dean's award for his outstanding academic results in 2019 and received technical scholarship in 2018-2020 from KUET. He also received the primary and junior scholarship. He is still studying at Department of Civil Engineering in KUET. He wants to spread his acquired knowledge among all as well as he wants blessings from everyone so that he can become a skilled engineer and come forward for the welfare of the country and the nation.

Published Book (by the Author)

1. Standard Handbook of Surveying
2. A Handbook of Reinforced Concrete Structures

Publishing Book (by the Author)

1. Standard Handbook of Engineering Mechanics
2. Tactical Handbook of Fluid Mechanics
3. A Handbook of Engineering Hydraulics
4. Standard Handbook of Structural Analysis

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